

Optimal-Cost Scheduling of Electrical Vehicle Charging Under Uncertainty

Yaqin Zhou, David K. Y. Yau, *Senior Member, IEEE*, Pengcheng You, *Student Member, IEEE*, and Peng Cheng

Abstract—Electric vehicle (EV) charging stations are increasingly set up to meet the recharge demand of EVs, and the stations equipped with local renewable energy generation need to optimize their charging. A basic challenge for the optimization stems from inherent uncertainties such as intermittent renewable generation that is hard to predict accurately. In this paper, we consider a charging station for EVs that have deadline constraints for their requests and aim to minimize its supply cost. We use Lyapunov optimization to minimize the time-average cost under unknown renewable supply, EV mobility, and grid electricity prices. We model the unfulfilled energy requests as a novel system of queues, based on whose evolution we define the Lyapunov drift and minimize it asymptotically. We prove that our algorithm achieves at most $O(1/V)$ more than the optimal cost, where the parameter V trades off cost against unfulfilled requests by their deadlines, and its time complexity is linear in the number of EVs. Simulation results driven by real-world traces of wind power, EV mobility, and electricity prices show that, compared with a state-of-the-art scheduling algorithm, our algorithm reduces the respective charging costs by 12.48% and 51.98% for two scenarios.

Index Terms—Electrical vehicle charging, robust online scheduling, Lyapunov optimization.

I. INTRODUCTION

ELECTRICAL vehicles (EVs) have significantly smaller carbon footprints than traditional vehicles running on fossil fuel [1]. Because of government incentives for their adoption and lower fuel costs compared with gasoline, EVs have gained surging popularity in recent years. Commercial EV charging stations have sprung up to meet the recharge demand. These charging stations may similarly have been motivated to invest in renewable generation for selling electricity to their customers. In this case, they will prefer to use locally generated renewable power when it is available, and purchase electricity from the grid to supplement any shortfall only when needed. A basic challenge, however, is that the

supply of renewables can be extremely hard to predict, since it depends on micro-weather conditions that are volatile. Day-ahead prediction errors for wind, for instance, can easily exceed 20% [2]. Hence, it is important for charging stations to perform intelligent scheduling to optimize the charging for cost minimization while meeting the charging demand under uncertainty.

Related work on scheduling EV charging has sought to minimize peak loading, power loss, or load variance under various constraints for the grid (e.g., power capacity), vehicle (e.g., size of demand), and mobility (e.g., arrival and parking times of the EVs). A significant amount of work has treated the scheduling as a static offline problem, in which the EV owners are required to submit their charging demand in advance (e.g., for the next day) and the needed future electricity prices and renewable supply are assumed known in advance [3], [4]. Some other researchers [5]–[7] have formulated the optimization in the form of dynamic online scheduling, in which they utilize forecast approaches to obtain the future knowledge required to inform the scheduling decisions. The success of their algorithms, however, can be quite sensitive to prediction errors that are hard to avoid in practice. For example, our numerical results show that the performance of an existing optimal scheduler [5] deteriorates even under small prediction errors. The optimization objective is to minimize the total charging cost determined by an increasing function of the sum of the background load and the load of the charging station. Indeed, when we add a controlled error that varies from zero to 5% of the real basic load, the relative error of the optimized total cost grows quickly from 0.0363 to 0.3866 (see Table I). Fig. 1 compares the load profiles computed by the scheduling algorithm without prediction errors and with 5% prediction errors, respectively. It can be seen that the errors distort the optimal load profile significantly. Deviations from the optimal profile also worsen as uncertainty of the arrival process of charging demand grows [3]. Similarly, under 20% day-ahead prediction errors for wind, Zhao *et al.* [8] show that scheduling algorithms oblivious to these errors can have severely suboptimal performance.

Therefore, it is imperative to design a practical online scheduling algorithm that is robust against inherent uncertainties in the charging demand, grid electricity prices, and renewable energy supply. State-of-the-art algorithms that aim to address these uncertainties [8]–[10] cannot achieve optimal long-term performance (e.g., total charging cost over time), and have high computational complexity that increases with the scheduling time horizon [8], [9]. For example, research has sought to minimize the maximum peak consumption over time under a bounded level of uncertainty for future

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Y. Zhou is with the Singapore University of Technology and Design, Singapore (e-mail: yaqin_zhou@sutd.edu.sg).

D. K. Y. Yau is with SUTD and Advanced Digital Sciences Center, Singapore (e-mail: david_yau@sutd.edu.sg).

P. You and P. Cheng are with the State Key Laboratory of Industrial Control Technology, Zhejiang University, Hangzhou 310027, China (e-mail: pcyou@zju.edu.cn; pcheng@iipc.zju.edu.cn).

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TABLE I
IMPACT OF PREDICTION ERROR ON OPTIMAL SOLUTION

Relative error of predicted basic load	0.00	0.01	0.02	0.03	0.04	0.05
Relative error of total charging cost	0.0363	0.0363	0.0734	0.1658	0.2812	0.3866

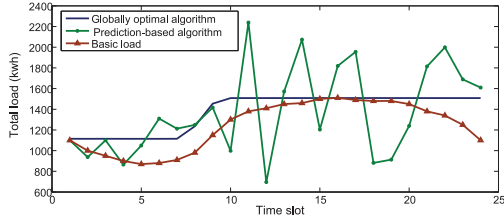


Fig. 1. Load profiles by the globally optimal algorithm [5] under accurate basic load and the prediction-based algorithm [5] under 0.05 prediction error for the basic load.

knowledge [8], [9]. Their algorithms are conservative in that they use the worst-case uncertainty in every scheduling decision; hence, the time-averaged performance can be severely suboptimal. In terms of computational overhead, they need to accumulate all prior history information as a basis for the prediction, and solve a convex optimization defined on that set of information in every time slot. The optimization becomes harder over time as the amount of history considered grows.

Recently, Lyapunov optimization has been applied to the scheduling of EV charging exploiting renewable energy [11], [12]. Lyapunov optimization has been initially well developed for dynamic control of queuing systems for wireless networks with unknown packet arrival processes, and generalized to stabilize queues in stochastic systems with provably near-optimal performance. Compared with other stochastic optimization techniques like Markov processes, Lyapunov optimization does not require a-priori knowledge, and is robust to non-i.i.d and non-ergodic processes. Thus, the model naturally fits in the charging scheduling problem under intermittent renewable energy. The focus of [11] is on the coupling between distributed charging stations, under the assumption that EVs can be guided by their optimization to go to different stations. In contrast, we assume the more likely scenario that EVs arrive at a charging station based on an extrinsic stochastic process that is neither known to nor controlled by the station, and the station needs to satisfy the demand subject to service deadlines of their customers. The important dimension of deadline constraints is not considered in [11]. Though Jin *et al.* [12] consider deadlines, they focus on minimizing the worst case time-averaged delay and implicitly assume that the EVs may wait indefinitely until they are fully charged. In practice, customers cannot tolerate more than a maximum waiting time, and we consider this realistic scenario.

In this paper, we analyze optimal scheduling of EV charging with deadlines that minimizes the long-term cost for the charging station under uncertain EV arrivals, electricity prices, and supply of renewable energy. Our **contributions** are as follows.

- We apply Lyapunov optimization to achieve our objective under inherent and significant uncertainties in the EV charging problem. Compared with state-of-the-art algorithms [5], [8], [9], ours does not require any methods to predict the future. We also do not need detailed (e.g., Markov [10]) assumptions about the stochastic

processes in question, but we can handle general unknown stochastic processes under mild assumptions.

- We develop an asymptotically optimal online scheduling algorithm, and prove that its total charging cost is at most $O(\frac{1}{V})$ more than the optimal, where V is a controllable parameter that balances between the charging cost and average fulfilment ratio of charging requests (i.e., ratio of the actual energy charged to the energy requested by the customer). The proposed algorithm is computationally efficient; its complexity is linear in the number of EVs.
- We present simulation results based on real-world traces of car mobility, electricity prices in Singapore, and available wind power. We use real-world arrival datasets of cars to carparks in Singapore [13] to drive our EV arrivals. We expect the travel patterns of EVs to be similar to those of traditional vehicles in the datasets [14], [15]. For electricity prices, we use real-time pricing data published by the Energy Market Company of Singapore [16]. The two real wind traces we use are from the Global Energy Forecasting Competition 2014 [17] and the U.K. National Grid status [18]. The results show that the proposed algorithm outperforms a state-of-the-art algorithm [5] with charging cost reductions of 12.48% and 51.98% for two usage scenarios, respectively.

II. SYSTEM MODEL

We consider an EV charging station that operates in discrete time with unit time slots $t \in \{0, 1, 2, \dots\}$. A renewable energy source, e.g., wind power generator, is operated by the station to provide $w(t)$ units of energy in each slot t . The renewable energy generation process $w(t)$ is intermittent and unpredictable. Installation and maintenance of the renewable generation have costs, which are justified by considerations like government subsidies, positive corporate image, and concerns for the environment. But once the infrastructure is in place and operational, using any harnessed renewable energy can be considered free, but the energy is not stored due to high costs of batteries and inefficiency of existing storage technologies. Therefore, the energy $w(t)$ must be used immediately or else it is wasted.

We focus on optimizing the charging schedule of a charging station. EVs arrive at the charging station according to a stochastic process $\lambda(t)$ at the beginning of each time slot t . Let $e \in \{1, 2, \dots\}$ be the index of EVs, and t_e^{arr} be the index of arrival time slot of e . When it arrives, e submits to the station a charging profile that specifies its maximum charging power P_e^{max} , initial energy level E_e^{ini} , final expected energy level E_e^{fin} , and charging delay tolerance d_e (in time slots). For each EV, the required charging energy, $E_e^{req} = E_e^{fin} - E_e^{ini}$, is bounded by a constant E_{max} . Whether the final expected energy level E_e^{fin} is reached or not, by the deadline (i.e., $t_e^{arr} + d_e$) e will leave the station. To assess the quality of the charging service, we define the *fulfilment ratio* for a customer to be the ratio of

the actual energy charged by the deadline to the total energy requested by the customer.

The available renewable energy $w(t)$ may not be enough to meet all the energy requests of the customers by their deadlines. Hence, in general in a time slot we need to purchase an amount of energy $x(t)$ from the grid. The amount $x(t)$ incurs a cost $x(t)\gamma(t)$, where $\gamma(t)$ is the real-time unit price of the grid's energy supply in slot t . Let $A(t)$ be the set of all unfulfilled EVs available in time slot t . For each $e \in A(t)$, the amount of energy charged in slot t , denoted by $y_e(t)$, consists of the renewable energy $w_e(t)$ and the purchased energy $x_e(t)$, i.e., $y_e(t) = x_e(t) + w_e(t)$. We use $y(t) = \sum_{e \in A(t)} y_e(t)$ to denote the total amount of energy supplied by the charging station in slot t ; we have $y(t) = x(t) + w(t)$. We assume that the EVs do not discharge power to the grid (i.e., $y_e(t) \geq 0$), since it is questionable whether EV owners will try to profit from selling electricity from their car batteries, and the infrastructure support required for charging stations to sell back energy is expensive [19].

A. Problem Formulation

The scheduling problem of EV charging consists in choosing $\{y_e(t), x(t)\}$ to minimize the time average of the cost $x(t)\gamma(t)$, while guaranteeing that all the EVs' charging requirements are met. Let \bar{C} denote the time-average charging cost achieved by our scheduling policy:

$$\bar{C} = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E\{\gamma(\tau)x(\tau)\}. \quad (1)$$

We need to decide $\{y_e(t)\}$ to solve

$$\begin{aligned} \mathbf{P1:} \quad & \min \bar{C} \\ \text{s.t.} \quad & x(t) = \sum_{e \in A(t)} y_e(t) - w(t) \quad \forall t \end{aligned} \quad (2)$$

$$0 \leq y_e(t) \leq P_e^{\max} \quad \forall e, t \quad (3)$$

$$\sum_{\tau=0}^{d_e-1} y_e(t_e^{\text{arr}} + \tau) = E_e^{\text{req}} \quad \forall e \quad (4)$$

In **P1**, the random processes of EV arrivals $\lambda(t)$, electricity prices $\gamma(t)$, and renewable energy supply $w(t)$ are all unknown in advance. Therefore, the problem cannot be solved directly. We henceforth consider a closely related but more flexible form of the problem, which admits a tradeoff between the cost minimization and the fulfilment ratio of the solution.

III. LYAPUNOV OPTIMIZATION

In this section, we use Lyapunov optimization to design an online control algorithm that achieves the optimal solution to **P1** asymptotically. We consider a variant of **P1** by relaxing the constraint (4):

$$\sum_{\tau=0}^{d_e-1} y_e(t_e^{\text{arr}} + \tau) \leq E_e^{\text{req}}, \quad \forall e, \quad (5)$$

and use a parameter V to control the amount of unfulfilled energy, where V is a penalty added to the objective of **P1**.

To map the problem to a Lyapunov optimization, we first assume that the renewable generation process $w(t)$, the EV arrival process $\lambda(t)$, and the market prices $\gamma(t)$ are of some unknown i.i.d. probability distribution over time. We will relax the i.i.d. assumption later in Section IV. The values $w(t)$, $\lambda(t)$, and $\gamma(t)$ are respectively upper bounded by finite constants w_{\max} , λ_{\max} , and γ_{\max} , so that

$$0 \leq w(t) \leq w_{\max}, 0 \leq \lambda(t) \leq \lambda_{\max}, 0 \leq \gamma(t) \leq \gamma_{\max}. \quad (6)$$

Then we build a novel queuing system to capture the dynamics of the requested energy and unfulfilled energy. Based on the evolution of the system of queues, we can define the Lyapunov optimization as a variant of **P1**.

A. Construction of Queuing System

We assume that the whole set of energy requests are stored in a queue. Let $Q(t)$ denote the total amount of unsatisfied energy of the EV set $A(t)$ at the beginning of slot t .

We group the EVs by their delay tolerance d_e . We assume that the delay tolerance falls within a finite set $D = \{1, 2, \dots, d_{\max}\}$. The assumption is natural; e.g., generally EVs stop at a charging station for less than one day or some not-too-long period of time. Consequently, we group the flow of arriving EVs $\lambda(t)$ into d_{\max} subflows, each of one delay tolerance. We use the subscript f as the subflow index of the EVs, where $f = 1, \dots, d_{\max}$.

Each subflow of EVs maintains conceptually a queue of energy requests $Q_f(t)$. We further divide each subflow into d_{\max} layers by the EVs' remaining time until they leave the charging station. Accordingly, we use $Q_{f,r}(t)$ to represent the total amount of requested energy by EVs with r time slots until the deadline, and $A_{f,r}(t)$ the corresponding set of unfulfilled EVs. In time slot t , the EVs that have r slots left until their deadlines will have $r-1$ slots left in the next slot $t+1$. Thus, in general unsatisfied energy requests will carry over to the next queue $Q_{f,r-1}(t+1)$. Let $Y_{f,r}(t)$ be the total energy supplied for layer (f, r) during the time slot t . We define $U_{f,r}(t)$ as the total remaining energy (yet to be supplied) to meet the total request at the end of the slot t , i.e., $U_{f,r}(t) = Q_{f,r}(t) - Y_{f,r}(t)$. For $r < f$, this remaining energy will go to the next layer, i.e., $Q_{f,r}(t+1) = U_{f,r+1}(t)$. In summary, the queues at the layers of subflows evolve as follows:

$$Q_{f,f}(t) = \sum_{e \in A_{f,f}(t)} E_e^{\text{req}}, \quad \text{for } r = f, \quad (7)$$

$$Q_{f,r-1}(t+1) = Q_{f,r}(t) - Y_{f,r}(t), \quad \forall 2 \leq r \leq f. \quad (8)$$

For each subflow f , the unfulfilled EVs that are in layer $r = 1$ leave the charging station at the end of the slot. The total unfulfilled energy for subflow f is stored in a debt queue Z_f . Z_f evolves as

$$Z_f(t+1) = Z_f(t) + Q_{f,1}(t) - Y_{f,1}(t). \quad (9)$$

In the later analysis, by minimizing the debt queues, we will gradually push the proposed algorithm's solution to satisfying the delay tolerance of every subflow.

Fig. 2 illustrates the queuing dynamics of a subflow with a 3-slot deadline, i.e., $f = d_e = 3$. At the beginning of time slot 1, the arriving energy request is $Q_{3,3}(t)$, and at the end of time slot 1, $Y_{3,3}(1)$ amount of energy has been charged,

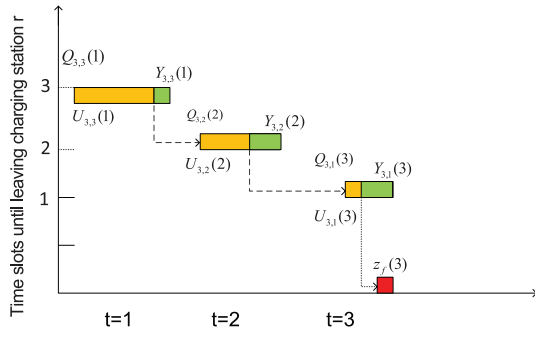


Fig. 2. An example showing the queuing system for a subflow f with a deadline of three time slots.

leaving $U_{3,3}(t)$ amount of energy unfulfilled. The unfulfilled energy moves into the queue of the next slot $t = 2$ with remaining time to leave $r = 2$. At the deadline (i.e., time slot 3), there is still $U_{3,1}(3)$ energy unfulfilled, which then moves into the debt queue $Z_f(3)$.

B. Solution Based on Lyapunov-Drift Minimization

Based on the specified evolution of queues, we define a quadratic Lyapunov function as

$$L(Q, Z, t) = \frac{1}{2} \sum_{f=1}^{d_{\max}} \left(\sum_{r=1}^f Q_{f,r}(t)^2 + Z_f(t)^2 \right) \quad (10)$$

where Q and Z are the vectors of all the requests and debt queues, respectively. Define the Lyapunov drift as follows:

$$\begin{aligned} \Delta L(Q, Z, \lambda, W) \\ = E \left[L(Q, Z, t+1) - L(Q, Z, t) \mid Q(t), Z(t), \lambda(t), W(t) \right] \end{aligned} \quad (11)$$

where λ and W are respectively the vectors of the EV arrivals and supplied renewable energy.

Following the drift-plus-penalty framework [20] to minimize the total cost over time subject to queue stability, we design our scheduling algorithm to observe the current queue states $Q(t)$ and $Z(t)$ and the current $w(t)$, $\lambda(t)$, and $\gamma(t)$, and make a decision $\{x_{f,r}(t)\}$ to minimize an upper bound on the solution to the following problem **P2** in every time slot t . The objective is to asymptotically achieve the optimal solution to the problem **P1**:

$$\begin{aligned} \mathbf{P2} : \min \quad & \Delta L((Q, Z, \lambda, W)) \\ & + VE \left[\gamma(t)x(t) \mid Q(t), Z(t), \lambda(t), W(t) \right] \\ \text{s.t.} \quad & (2), (3), (5). \end{aligned} \quad (12)$$

where V is a positive parameter that trades off between the total charging cost and the fulfilment ratio. The following lemma provides an upper bound for (12).

Lemma 1 (Drift Bound): For any scheduling policy that satisfies the constraints of **P2** for all t , the drift expression for all the time slots satisfies

$$\Delta L((Q, Z, \lambda, W)) + VE \left[\gamma(t)x(t) \mid Q(t), Z(t), \lambda(t), W(t) \right]$$

$$\begin{aligned} \leq B + \sum_f \sum_{r=1}^f (E[x_{f,r}(t)(V\gamma(t) + w_{f,r}(t) - Q_{f,r}(t))] \\ - E[Z_f(t)(x_{f,1}(t) + w_{f,1}(t) - Q_{f,1}(t))] \end{aligned} \quad (13)$$

where the constant $B = \frac{1}{2}(2\lambda_{\max}^2 E_{\max}^2 + w_{\max}^2)$.

Proof: See Appendix A. ■

C. Scheduling of EV Charging Requests

According to Lemma 1, **P2** is equivalent to the following dynamic control problem: (i) In every slot t , observe $\{Z_f(t)\}$, $\{Q_{f,r}(t)\}$, $\lambda(t)$, $\gamma(t)$, $w(t)$. (ii) Choose $\{x_{f,r}(t)\}$ to solve

$$\begin{aligned} \mathbf{P3} : \min \quad & \sum_f \sum_{r=1}^f x_{f,r}(t)(V\gamma(t) + w_{f,r}(t) - Q_{f,r}(t)) \\ & - Z_f(t)x_{f,1}(t) \end{aligned}$$

s.t.

$$x_{f,r}(t) + w_{f,r}(t) \leq Q_{f,r}(t), \quad (14)$$

$$0 \leq x_{f,r}(t) + w_{f,r}(t) \leq \sum_{e \in A_{f,r}(t)} P_e^{\max}, \quad (15)$$

$$x_{f,r}(t) \geq 0, \quad \forall f, r, t. \quad (16)$$

(iii) Allocate the power $x_{f,r}(t) + w_{f,r}(t)$ to each EV of this layer to satisfy the constraints (2), (3), and (5). (iv) Update the queues $Q_{f,r}(t)$ and $Z_f(t)$ according to (7)–(9). The above minimization of the $\{x_{f,r}(t)\}$ reduces to a simple threshold rule:

$$x_{f,r}(t) = \min \left(\sum_{e \in A_{f,r}(t)} P_e^{\max}, Q_{f,r}(t) - w_{f,r}(t) \right) \quad (17)$$

if $V\gamma(t) + w_{f,r}(t) - Q_{f,r}(t) < 0$ for $r > 1$ and $V\gamma(t) + w_{f,1}(t) - Q_{f,1}(t) - Z_f(t) < 0$ for $r = 1$; otherwise $x_{f,r}(t) = 0$. Here we assume that the renewable energy is allocated equally to each layer.

Therefore, a total energy of $y_{f,r}(t) = x_{f,r}(t) + w_{f,r}(t)$ is allocated to charge the unfulfilled EV set $A_{f,r}(t)$. We use a fair assignment of the energy to each EV, subject to the conditions that: (i) the charging rate does not exceed the maximum charging rate; (ii) $E_e(t)$, the energy level at the end of the slot t , does not exceed the final expected energy level. Specifically, each EV is charged at a rate of

$$y_e(t) = \min \left(P_e^{\max}, \frac{y_{f,r}(t)}{|A_{f,r}(t)|}, E_e^{\text{fin}} - E_e(t-1) \right). \quad (18)$$

We summarize the above online scheduling in Algorithm 1. As shown in the specification, the computational complexity is linear in the number of available EVs for charging.

D. Performance Analysis

Lemma 2 (Characterizing Optimality [20]): If the stochastic processes $w(t)$, $\lambda(t)$, $\gamma(t)$ are i.i.d. over the time slots, then there exists a randomized stationary scheduling policy that makes control decisions $\{x_{f,r}^*(t)\}$ at every time slot based on only the current state of $w(t)$, $\lambda(t)$, and $\gamma(t)$ (and independent of the queue backlogs and past system history) and satisfies:

$$E[\gamma(t)x^*(t)] = c^*, x^*(t) = \sum_f \sum_r x_{f,r}^*(t), \quad (19)$$

Algorithm 1 Online Scheduling Algorithm for the EVs' Charging

Initialize:

$$V, Q_{f,r}(1) = 0, A_{f,r}(1) = \emptyset, Z_f(1) = 0 \quad \forall 1 \leq r < f, f$$

for $t = 1, \dots$ **do**
for each subflow $f = 1, \dots, d_{\max}$ **do**

 Set $A_{f,f}(t)$ according to arrival EV set $\lambda(t)$

 Update $Q_{f,f}(t)$ as (7)

for each layer r of f **do**
if $V\gamma(t) + w_{f,r}(t) - Q_{f,r}(t) < 0 \quad \forall r > 1$ or
 $V\gamma(t) + w_{f,1}(t) - Q_{f,1}(t) - Z_f(t) < 0$ **then**

 Set $x_{f,r}(t)$ as (17)

else

 Set $x_{f,r}(t) = 0$
end if

 Set $y_{f,r}(t) = x_{f,r}(t) + w_{f,r}(t)$

 Charge each EV $e \in A_{f,r}(t)$ at

$$\min\left(P_e^{\max}, \frac{y_{f,r}(t)}{|A_{f,r}(t)|}, E_e^{\text{fin}} - E_e(t-1)\right)$$

end for

 Update $Q_{f,r}(t)$ as (8), and $A_{f,r}(t) \quad \forall 2 \leq r \leq f$

 Update $Z_f(t)$ as (9)

end for
end for

where c^* is the optimal solution, and the expectations are with respect to the stationary distributions of $w(t)$, $\lambda(t)$, and $\gamma(t)$, and the randomized scheduling policy.

Proof: The proof follows the framework of [20]; it is omitted for brevity. ■

Theorem 1: Let $w(t)$, $\lambda(t)$, and $\gamma(t)$ be i.i.d. over the time slots. The time-average cost under the proposed scheduling policy is at most $\frac{B}{V}$ more than the optimal:

$$\frac{1}{T} \sum_{\tau=0}^{T-1} E[\gamma(\tau)x(\tau)] \leq c^* + \frac{B}{V}. \quad (20)$$

Proof: The proposed algorithm is designed to minimize the drift bound (13) in Lemma 1, which holds for all scheduling policies, including the optimal and stationary policy given in Lemma 2. Therefore, we have the following:

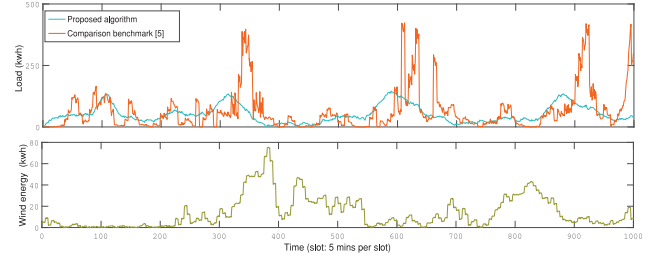
$$\begin{aligned} & \Delta L((Q, Z, \lambda, W)) + VE \left[\gamma(t)x(t) \middle| Q(t), Z(t), \lambda(t), W(t) \right] \\ & \leq B + \sum_f \sum_{r=1}^f \left(E \left[x_{f,r}^*(t) (V\gamma(t) + w_{f,r}(t) - Q_{f,r}(t)) \right] \right) \\ & \quad - E \left[(x_{f,1}^*(t) + w_{f,1}(t)) (Z_f(t) + Q_{f,1}(t) - Z_f(t)Q_{f,1}(t)) \right] \end{aligned}$$

From (19), we have:

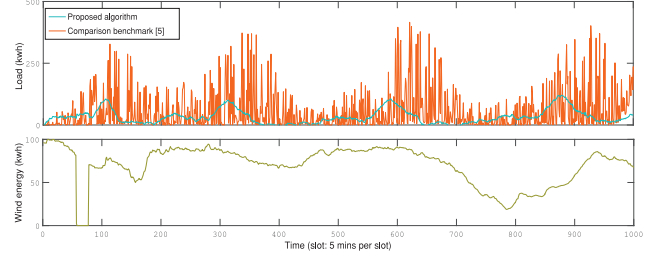
$$\begin{aligned} & \Delta L((Q, Z, \lambda, W)) + VE \left[\gamma(t)x(t) \middle| Q(t), Z(t), \lambda(t), W(t) \right] \\ & \leq B + Vc^*, \end{aligned} \quad (21)$$

where we use the fact that the proposed algorithm always minimizes **P3** so that $E[Z_f(t)(x_{f,1}^*(t) + w_{f,1}(t) - Q_{f,1}(t))] = 0$. Taking expectations of (21) and using the law of iterated expectations with the definition of ΔL in (11), we have

$$E[L(t+1) - E[L(t)] + VE[\gamma(t)x(t)]] \leq B + Vc^*. \quad (22)$$



(a) Trace of wind power from Global Energy Forecasting Competition 2014.



(b) Trace of wind power from U.K. National Grid Status.

Fig. 3. Comparison of load profiles under two real-world wind traces: $V = 100$.

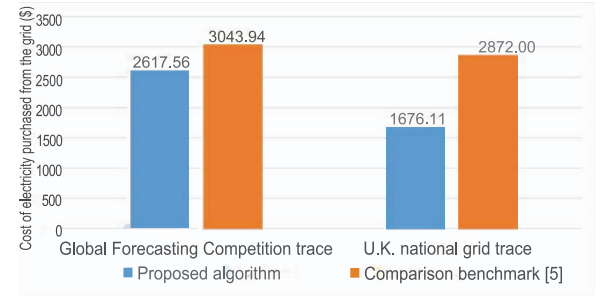


Fig. 4. Comparison of total charging costs.

The above inequality holds for all slots $t > 0$. Summing over $t \in \{0, 1, \dots, T-1\}$ for some positive integer T , we have:

$$E[L(T)] - E[L(0)] + \sum_{t=1}^{T-1} VE[\gamma(t)x(t)] \leq BT + VTc^*. \quad (23)$$

Because $L(0) = 0$ and $L(T) > 0$, dividing (23) by VT yields (19). ■

IV. NON-I.I.D. MODELS

In this section, we extend the analysis to arbitrary, possibly non-i.i.d., models of the processes $w(t)$, $\lambda(t)$, and $\gamma(t)$. We show that Algorithm 1 still achieves provable performance. To understand the optimal cost, we use the T -slot lookahead scheduling policy defined in *universal scheduling* [21] as a benchmark. The T -slot lookahead scheduling policy assumes that the scheduler has full knowledge of $w(t)$, $\lambda(t)$, and $\gamma(t)$ over the next future T consecutive slots. Let us define every T consecutive slots as a frame, and j be the index of frames. The T -slot lookahead policy is defined as an optimal solution

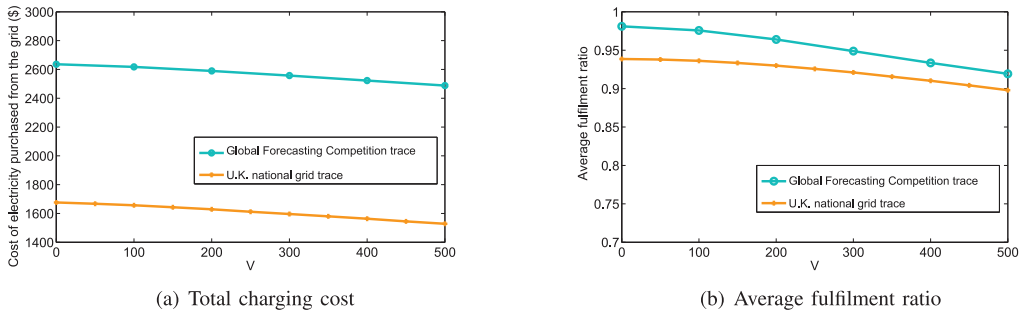


Fig. 5. Charging cost and average fulfilment ratio over all the EVs for different values of V under non-i.i.d. stochastic processes.

to the charging scheduling problem for the j -th frame:

$$\begin{aligned} \min \quad & \frac{1}{T} \sum_{\tau=jT}^{(j+1)T-1} \gamma(\tau)x(\tau) \\ \text{s.t.} \quad & x(\tau) = \sum_{e \in A(\tau)} y_e(\tau) - w(\tau) \quad \forall \tau \\ & 0 \leq y_e(\tau) \leq P_e^{\max} \quad \forall \tau \\ & \sum_{\tau=0}^{d_e-1} y_e(t_e^{\text{arr}} + \tau) = E_e^{\text{req}} \quad \forall e. \end{aligned} \quad (24)$$

Let c_j^* be the optimal cost that can be achieved over frame j , considering all possible allocations of $x(t)$ over this frame and all possible future values of $w(t)$, $\lambda(t)$, and $\gamma(t)$.

Theorem 2: For all positive integers T and J , under the assumptions given above, Algorithm 1 achieves a cost that satisfies:

$$\frac{1}{JT} \sum_{\tau=0}^{JT-1} \gamma(\tau) \sum_f \sum_{r=1}^f x_{f,r}(\tau) \leq \frac{B'}{V} + \frac{1}{J} \sum_{j=0}^{J-1} c_j^*, \quad (25)$$

where $B' = B + w_{\max}x_{\max}$ and B is defined in Theorem 1.

Proof: See Appendix B. ■

The above results indicate that the time-average cost over any interval of JT slots is within B'/V of the average of the c_j^* values.

V. SIMULATIONS

We evaluate the performance of the proposed algorithm on real-world datasets, and compare it with the forecast-based optimal scheduling algorithm by He *et al.* [5] as a benchmark. The benchmark algorithm [5] achieves near-optimal performance in experiments if the required near-future information is known accurately.

Experimental Settings: We set the length of a time slot to be five minutes. The total number of time slots is 1000. EVs are assumed to arrive at the beginning of each slot, and the delay tolerance of each request is chosen uniformly at random within [6, 12] hours. Though the primary concerns of the optimization are the total cost and average fulfilment ratio, the smoothness of the resulting load profiles is also important because sudden large changes in a profile could cause stability problems of the power system and damage the charging infrastructures. Thus we evaluate the algorithms in terms of the total cost, average fulfilment ratio, and load profile.

Datasets: We use real-world arrival datasets of cars to carparks in Singapore [13] to drive our EV arrivals. We

expect the travel patterns of EVs to be similar to those of traditional vehicles in the datasets [14], [15]. For electricity prices, we use real-time pricing data published by the Energy Market Company of Singapore [16]. We choose the Uniform Singapore Energy Price (USEP) in Nov 2015. The prices in our data range between [0.03311, 0.06274]S\$/kWh. We use two real-world traces of wind power. The first one is published by Global Energy Forecasting Competition 2014 [17]. This data is updated every period of 30 minutes. As the duration of a time slot in our simulations is five minutes, we assume that the wind power in the simulation time slots corresponding to the same update period in the data is the same. The second trace we use is from U.K. National Grid Status [18]. This data is updated every five minutes, which agrees with the length of our simulation time slot.

We first compare the profiles of grid loading (i.e., energy purchased from the grid by the charging station at different time slot t) for the two wind traces, under the same real-time prices of electricity in Fig. 3. The total requested energy by all the EVs is 58437kWh. The figure shows that the benchmark algorithm [5] exhibits a larger variance of the load profile than our proposed algorithm. In the U.K. wind trace especially, they have a much higher variance.

Fig. 4 compares the total cost for the charging station to purchase electricity from the grid. The costs of our proposed algorithm for the two wind traces are (2617.56, 1676.11)S\$, compared with (3043.94, 2872.00)S\$ for the benchmark algorithm. Our algorithm achieves average fulfilment ratios of 97.58% and 93.86% respectively for the two traces. Thus, the unfulfilled energy of the proposed algorithm in each case is respectively 1414kWh and 3588kWh. Assuming even the highest market price, the unfulfilled energy would cost at most (88.71, 225.11)S\$ respectively if it were purchased instead from the grid to make up for the shortfall. Therefore, under conservative estimates of its cost saving by assuming highest market prices for any shortfall, the proposed algorithm, without any predictions of the future, achieves costs 12.48% and 51.98% lower than the benchmark algorithm, and the latter algorithm does require good near-future knowledge of the wind power and electricity prices.

Figures 5(a) and 5(b) plot the total charging cost and average fulfilment ratio, respectively, under different values of V . Notice that both the total cost and the average fulfilment ratio decrease as V increases. It validates our analysis that V provides a tradeoff between the charging cost and fulfilment ratio, where a larger V lowers the cost, albeit at the expense of lower fulfilment.

VI. CONCLUSION

We presented an online algorithm for a charging station to schedule the requests of arriving EVs under deadline constraints. The algorithm is robust against inherent uncertainties in the future availability of renewable generation, arrivals and departures of EVs, and real-time grid electricity prices. We proved that the algorithm achieves a time-average charging cost that is at most $O(\frac{1}{V})$ more than the optimal, where V is a parameter that controls the tradeoff between cost and the average fulfilment ratio of requests. The algorithm is efficient and has time complexity linear in the number of EVs.

To validate and illustrate the performance of the proposed algorithm, we presented simulation results using real-world datasets. We showed that our algorithm outperforms a state-of-the-art scheduling algorithm [5] by 12.48% and 51.98% respectively for two wind power traces, while incurring a small unfulfilment only.

 APPENDIX A
 PROOF OF LEMMA 1

From the $Q_{f,r}(t)$ update rule (8) we have

$$\begin{aligned}
 & \frac{1}{2} \sum_f \left(\sum_{r=1}^f Q_{f,r}^2(t+1) - \sum_{r=1}^f Q_{f,r}^2(t) \right) \\
 &= \frac{1}{2} \sum_f \left(\sum_{r=1}^f (Q_{f,r+1}(t) - Y_{f,r+1})^2 - \sum_{r=1}^f Q_{f,r}^2(t) \right) \\
 &= \frac{1}{2} \sum_f \left(\sum_{r=1}^f (Q_{f,r+1}^2(t) - 2Q_{f,r+1}(t)Y_{f,r+1}(t) \right. \\
 & \quad \left. + Y_{f,r+1}^2(t)) - \sum_{r=1}^f Q_{f,r}^2(t) \right) \\
 &= \frac{1}{2} \sum_f \left(Q_{f,f+1}^2(t) + \sum_{r=1}^f Y_{f,r+1}^2(t) - Q_{f,1}^2(t) \right. \\
 & \quad \left. - 2 \sum_{r=1}^f Q_{f,r+1}(t)Y_{f,r+1}(t) \right) \quad (26)
 \end{aligned}$$

From the $Z_f(t)$ update rule (9), we have

$$\begin{aligned}
 & \frac{1}{2} \left(\sum_f Z_f^2(t+1) - \sum_f Z_f^2(t) \right) \\
 &= \frac{1}{2} \sum_f \left((Z_f(t) + Q_{f,1}(t) - Y_{f,1}(t))^2 - Z_f(t)^2 \right) \\
 &= \frac{1}{2} \sum_f \left(Q_{f,1}^2(t) + Y_{f,1}^2(t) + 2Z_f(t)Q_{f,1}(t) \right. \\
 & \quad \left. - 2(Z_f(t) + Q_{f,1}(t))Y_{f,1}(t) \right). \quad (27)
 \end{aligned}$$

Summing up (26) and (27), we have

$$\begin{aligned}
 \Delta L \leq & \sum_f \left(Q_{f,f+1}^2(t)/2 + \sum_{r=1}^f Y_{f,r}^2(t)/2 - \sum_{r=1}^f Q_{f,r}(t)Y_{f,r}(t) \right. \\
 & \left. - Z_f(t)(Y_{f,1}(t) - Q_{f,1}(t)) \right)
 \end{aligned}$$

$$\begin{aligned}
 \leq & \sum_f \left(Q_{f,f+1}^2(t)/2 + \sum_{r=1}^f (x_{f,r}^2(t)/2 + w_{f,r}^2(t))/2 \right. \\
 & \left. + \sum_{r=1}^f (x_{f,r}(t)(w_{f,r}(t) - Q_{f,r}(t))) \right. \\
 & \left. - Z_f(t)(x_{f,1}(t) + w_{f,1}(t) - Q_{f,1}(t)) \right)
 \end{aligned}$$

where we use $Y_{f,r}(t) = x_{f,r}(t) + w_{f,r}(t)$ in the last equation. Using upper bounds of the stochastic processes $\lambda(t)$, $w(t)$, i.e., λ_{\max} , w_{\max} , we have

$$\begin{aligned}
 L(t+1) - L(t) \leq & B + \sum_f \sum_{r=1}^f (E[x_{f,r}(t)(w_{f,r}(t) - Q_{f,r}(t))]) \\
 & - E[Z_f(t)(x_{f,1}(t) + w_{f,1}(t) - Q_{f,1}(t))]
 \end{aligned}$$

where $B = \frac{1}{2}(2\lambda_{\max}^2 E_{\max}^2 + w_{\max}^2)$. We now take conditional expectations of the above equation given $x(t)$, $w(t)$, $\lambda(t)$, and add $VE[\gamma(t)x(t)]$ to both sides. Consequently, we have (13).

 APPENDIX B
 PROOF OF THEOREM 2

We define the same Lyapunov function as (10), and define the T -slot Lyapunov drift as

$$\Delta_T L(Q, Z, \lambda, W) = L(Q, Z, t+T) - L(Q, Z, t). \quad (28)$$

This differs from the 1-slot conditional drift $\Delta L(Q, Z, \lambda, W)$.

Recall that the values of $w(t)$, $\lambda(t)$, $\gamma(t)$ and $x(t)$ are all upper bounded for all t . We have the following lemma.

Lemma 3: Fix any slot t , any queue state $Q(t)$, $Z(t)$, and any integer $T > 0$. Consider any $w(\tau)$, $\lambda(\tau)$, $\gamma(\tau)$ over the interval $\tau \in \{t, t+T-1\}$, which satisfies the upper bound constraints. The scheduling decisions by Algorithm 1 have that

$$\begin{aligned}
 \Delta_T L(t) + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x(\tau) \\
 \leq B'T + V \sum_{\tau=t}^{t+T-1} \sum_f \sum_{r=1}^f \gamma(\tau)x'_{f,r}(\tau) \quad (29)
 \end{aligned}$$

where $B' = B + w_{\max}x_{\max}$, $x'_{f,r}(\tau)$ are any alternative values that satisfy $0 \leq \sum_f \sum_{r=1}^f x'_{f,r}(\tau) \leq x_{\max}$ and $0 \leq x'_{f,r}(\tau)$, for all $\tau \in \{t, t+T-1\}$.

Proof: According to (11), we have that for all τ ,

$$\begin{aligned}
 L(\tau+1) - L(\tau) \leq & B + \sum_f \sum_{r=1}^f (x_{f,r}(\tau)(w_{f,r}(\tau) \\
 & - Q_{f,r}(\tau))) - C_f(\tau), \quad (30)
 \end{aligned}$$

where $C_f(\tau) = Z_f(\tau)(x_{f,1}(\tau) + w_{f,1}(\tau) - Q_{f,1}(\tau))$. Summing up (30) over all $\tau \in \{t, t+T-1\}$, we have

$$\begin{aligned}
 \Delta_T L(t) \leq & BT + \sum_{\tau=t}^{t+T-1} \sum_f \sum_{r=1}^f (x_{f,r}(\tau)(w_{f,r}(\tau) \\
 & - Q_{f,r}(\tau))) - C_f(\tau) \quad (31)
 \end{aligned}$$

Next we add the penalty term to both sides and get

$$\begin{aligned} \Delta_T L(t) + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x(\tau) \\ \leq BT + V \sum_{\tau=t}^{t+T-1} \sum_f \sum_{r=1}^f \gamma(\tau)x_{f,r}(\tau) \\ + \sum_{\tau=t}^{t+T-1} \sum_f \sum_{r=1}^f (x_{f,r}(\tau)(w_{f,r}(\tau) - Q_{f,r}(\tau))) - C_f(\tau) \end{aligned}$$

In Algorithm 1, for each slot τ , the value of $x_{f,r}(\tau)$ is chosen to minimize **P3** among all possible values of $x_{f,r}(\tau)$ such that $0 \leq \sum_f \sum_{r=1}^f x_{f,r}(\tau) \leq x_{\max}$, $x_{f,r}(\tau) \geq 0$. It indicates that, for any alternative value $x'_{f,r}(\tau)$ that satisfies these constraints, we have

$$\begin{aligned} \Delta_T L(t) + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x(\tau) \\ \leq BT + V \sum_{\tau=t}^{t+T-1} \sum_f \sum_{r=1}^f \gamma(\tau)x'_{f,r}(\tau) + \sum_{\tau=t}^{t+T-1} \sum_f \sum_{r=1}^f \\ (x'_{f,r}(\tau)(w_{f,r}(\tau) - Q_{f,r}(\tau))). \end{aligned}$$

Note that the available wind power is allocated equally. Therefore, we further have that

$$\begin{aligned} \Delta_T L(t) + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x(\tau) \\ \leq BT + V \sum_{\tau=t}^{t+T-1} \sum_f \sum_{r=1}^f \gamma(\tau)x'_{f,r}(\tau) \\ + \sum_{\tau=t}^{t+T-1} \frac{w(\tau)}{d} \sum_f \frac{1}{f} \sum_{r=1}^f x'_{f,r}(\tau) \\ \leq BT + V \sum_{\tau=t}^{t+T-1} \sum_f \sum_{r=1}^f \gamma(\tau)x'_{f,r}(\tau) + Tw_{\max}x_{\max} \\ \leq B'T + V \sum_{\tau=t}^{t+T-1} \sum_f \sum_{r=1}^f \gamma(\tau)x'_{f,r}(\tau) \end{aligned} \quad (32)$$

where $B' = B + w_{\max}x_{\max}$. ■

Now consider J successive frames of size T . Using the drift bound in Lemma 3, for each frame j , we have

$$\Delta_T L(jT) + V \sum_{\tau=jT}^{jT+T-1} \gamma(\tau) \sum_f \sum_{r=1}^f x_{f,r}(\tau) \leq B'T + VTc_j^*.$$

Summing the above equation over all $j \in \{0, 1, \dots, J-1\}$, and dividing the sum by JTV , we get

$$\frac{L(JT) - L(0)}{JTV} + \frac{1}{JT} \sum_{\tau=0}^{JT-1} \gamma(\tau) \sum_f \sum_{r=1}^f x_{f,r}(\tau) \leq \frac{B'}{V} + \frac{1}{J} \sum_{j=0}^{J-1} c_j^*.$$

Setting $L(0) = 0$ and $L(JT) > 0$ proves the lemma immediately.

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