Optimal Charging Scheduling by Pricing for EV Charging Station With Dual Charging Modes

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***Abstract*— With the increasing penetration of electric vehicles (EVs) and various user preferences, charging stations often provide several different charging modes to satisfy the various requirements of EVs. How to effectively utilize the charging capacity to minimize the service dropping rate is a pressing and open issue for charging stations. Given that EV owners are price-sensitive to the charging modes, we intend to design an optimal pricing scheme to minimize the service dropping rate of the charging station. First, we formulate the operation of a dual-mode charging station as a queuing network with multiple servers and heterogeneous service rates, and analyze the relationship between the service dropping rate of the charging station and the selections of EVs. Then, we formulate a customer attrition minimization problem to minimize the number of EVs that leave the charging station without being charged and propose an optimal pricing approach to guide and coordinate the charging processes of EVs in the charging station. The simulation has been conducted to evaluate the performance of the proposed charging scheduling scheme and show the efficiency of the proposed pricing scheme.**

***Index Terms*— Charging stations, charging modes, queuing theory, charging scheduling, pricing.**

I. Introduction

ELECTRIC vehicles (EVs) have been considered to be a key technology to cut down the massive greenhouse gas emissions from the transportation sector, and they are also expected to mitigate the fossil fuels scarcity problem [1]. Thanks to the policies and plans for promoting EVs from the regions and countries worldwide (e.g., the sales of EVs including PHEVs in US will reach 50% of total sales of mobile vehicles by 2030, and Europe has the similar targets [2]), the amount of EVs is expected to reach a sizable market share in the next decade.

However, due to limited cruising range, EVs may require frequent recharging when they travel to a faraway destination, and hence charging convenience is one of the most important concerns for EV owners. In addition, due to the high cost of battery replacement, battery lifetime-related cost is another important concern for EV owners. Given the slow evolution of battery technologies, the key issue to is how to address these challenges without waiting for new battery technologies.

Charging stations play an important role in providing charging services to EVs. Typically, there exist two different charging modes for conventional public chargers: i) AC Level II (charging power typically is 10-22 kW); and ii) Direct Current Quick Charging (DCQC) (charging power typically is 50-120 kW). Given the EV charging requirement, AC Level II has a longer charging duration while DCQC has a shorter charging duration, which may further reduce battery lifetime. Because of deployment cost concerns, both the number of charging stations and the number of chargers in charging stations are limited. How to use limited charging station resources to satisfy the various EV charging requirements has attracted attention, such as developing intelligent charging station architectures [3]–[6], and optimizing the location and sizing of charging stations [7]–[11]. Nevertheless, the non-cooperative charging behaviors of EVs will lead to congestion at charging stations and reduce their operational efficiency.

To guide and coordinate the charging behaviors of EVs, there have been extensive researches on developing charging scheduling schemes, for EV charging stations [12]–[17], for battery swapping stations [18]–[20], and for charging stations with renewable energy [21]–[24]. However, these works assumed that all the chargers in the charging station are using the similar charging mode, such as AC Level II or DCQC. Since different EVs may have different charging behaviors and charging service requirements, i.e., short charging duration or long battery lifetime or both of them, the charging station with single charging mode has a low flexibility and adaptability to satisfy the requirements of multi-class customers, and thus limits the service quality.

To deal with the various EV charging requirements, the charging station can install two types of chargers, such as part of them with the AC Level II mode and part of them with the DCQC mode. Consequently, the charging station can provide different charging services to different classes of

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customers based on their behaviors. Furthermore, guiding the EV owners to select adequate charging modes can reduce the congestion and improve the service quality of the charging station. This motives us to design an optimal scheduling scheme for the charging station with dual charging modes to minimize the service dropping rate. To the best of our knowledge, this is the first paper addressing the charging scheduling problem for the charging station with dual charging modes by designing an optimal pricing scheme.

Generally, the selections of EV owners depend on several factors, i.e., service fee, charging duration, waiting time, etc [12]. Given that EV owners are price sensitive, we analyze the relationship between the service dropping rate and the service rate of the charging station and then design an optimal pricing scheme to guide and coordinate the charging processes of EVs, such that the number of EVs that leave the charging station without being charged can be minimized and the operation efficiency of the charging station can be improved. The contributions of this papers can be summarized as follows:

- We model the operation processes of the charging station with dual charging modes as a queuing network with multiple servers and heterogeneous service rates.
- Based on the queuing theory, we analyze the relationship between the selections of EVs and the service dropping rate of the charging station, and prove that the service dropping rate is a convex function of the service rates.
- We formulate a customer attrition minimization problem for the charging station and propose an optimal pricing scheme to guide and coordinate the charging processes of EVs to minimize the service dropping rate.
- Simulation results show that the proposed pricing scheme can reduce the service dropping rate of the charging station with dual charging modes significantly comparing to the charging station with a single charging mode.

The rest of the paper is organized as follows: Section II presents the operation models for the charging station, and formulates the charging scheduling problem as a customer attrition minimization problem based on queuing theory. The relationship between the selections of EVs and the service dropping rate of the charging station is analyzed, and an optimal pricing scheme is designed by utilizing the price sensitiveness of EV owners in Section III. Section IV demonstrates the operational performance analysis based on simulation results. Finally, Section V concludes our work.

II. System Model and Problem Formulation

Considering a charging station, there are N_1 AC chargers and N_2 DC chargers to provide charging services for connected EVs.[1] Given the charging demand of one EV, both the service fee and the battery lifetime-related cost of the AC chargers are much lower than the DC chargers. But, the charging duration for the AC chargers is much longer. As shown in [12] and [25], the dominant factors that affect the selections

[1]For simplicity, we use the AC charger and the DC charger to denote the charger with the AC Level II charging mode and the charger with the DCQC charging mode, and the AC mode and the DC mode instead of the AC Level II charging mode and the DCQC charging mode in this paper, respectively.

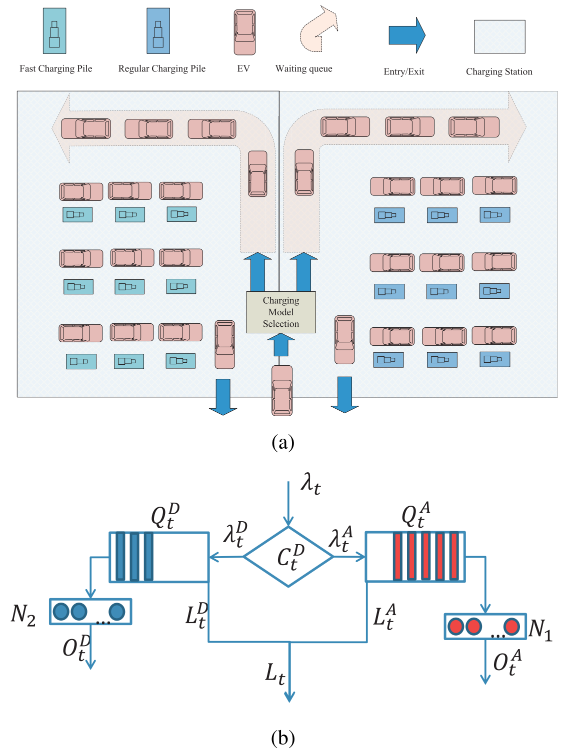


Fig. 1. Operation model and queuing model for the charging station. (a) Operation model. (b) Queuing model.

of EVs are service fee, waiting time, battery lifetime-related cost, etc. In this paper, we assume that the service fee and the battery lifetime-related cost will affect the EVs' selections while the waiting time will affect the service dropping rate of the charging station. When the corresponding queue length is too long, EV will leave the charging station without being charged. Otherwise, EV will be charged by one charger with the selected charging mode immediately or late.

For each EV, it needs to select one charging mode when it arrives at the charging station. Then, it will be charged when there is at least one available charger with the selected charging mode; otherwise, it needs to wait in the corresponding queue until one charger with the selected charging mode is available, or leaves the charging station directly without being charged if the corresponding queue length is too long. Here, the percentage of EVs that leave the charging station directly denotes the service dropping rate, which is one of the main factors related to the Quality of Service (QoS) of the charging station. Our objective is to design an optimal pricing scheme based on EV owners' preferences to guide them to select adequate charging modes, such that the service dropping rate of the charging station can be minimized. The operation model and the queuing model for the charging station are shown in Fig. 1. A summary of notations is given in Table I.

Note that, given the ubiquitous communication networks, in the future, EVs and charging stations can exchange pricing and waiting time information remotely. Such that, an EV can decide which charging mode it prefers and whether go to the charging station for charging service or not based on its QoS. In this scenario, the queuing model and the proposed

TABLE I
NOTATION DEFINITIONS

Symbol	Definition
t	The t-th time slot.
λ_t	Number of EVs that arrive at charging station during time slot t.
λ_t^A	Number of EVs that select the AC mode during time slot t.
λ_t^D	Number of EVs that select the DC mode during time slot t.
$\bar{p}_t^A$	The probability for EVs selecting the AC mode during time slot t.
$\bar{p}_t^D$	The probability for EVs selecting the DC mode during time slot t.
$\bar{p}_t^L$	The probability for EVs leaving the charging station without being charged during time slot t.
B^C	The expected battery capacity of each EV.
C_t^A	Service fee of the AC mode during time slot t.
C_t^D	Service fee of the DC mode during time slot t.
$\hat{C}_t^D$	Optimal service fee of the DC mode during time slot t.
$\underline{C}^D$	The lower bound of C_t^D during time slot t.
$\overline{C}^D$	The upper bound of C_t^D during time slot t.
C_B^A	Battery lifetime-related cost in the AC mode.
C_B^D	Battery lifetime-related cost in the DC mode.
E	The expected charging requirement of each EV.
L_t	Total number of EVs that leave the charging station without being charged during time slot t.
L_t^A	Number of EVs that select the AC mode and leave without being charged during time slot t.
L_t^D	Number of EVs that select the DC mode and leave without being charged during time slot t.
N_1	Total number of the AC chargers.
N_2	Total number of the DC chargers.
O_t^A	Number of EVs fully charged by the AC mode during time slot t.
O_t^D	Number of EVs fully charged by the DC mode during time slot t.
Q_t^A	The queue length for the AC mode during time slot t.
Q_t^D	The queue length for the DC mode during time slot t.
$\bar{Q}_t^A$	The maximal queue length for the AC mode during time slot t.
$\bar{Q}_t^D$	The maximal queue length for the DC mode during time slot t.
T	Total number of time slots in one period (i.e., one day).
T^A	Total time for recharging an empty battery to full in the AC mode.
T^D	Total time for recharging an empty battery to full in the DC mode.

pricing algorithm in this paper can still be applicable, using the concept of virtual queues.

A. Operation Model of Charging Stations

Considering the realistic scenarios that the average EV arrival rate during peak vs. off-peak hours may be different, we divide the whole day into T time slots (i.e., an hour is one time slot), and assume that the average EV arrival rate within the time slot remains the same, while that for different time slots may change. Let t, $t = 1, 2, \ldots, T$, denote t-th time slot. The arrival of EVs at each time slot follows a Poisson process with the average rate of λ_t during time slot t [26]. The probability for n EVs arriving at the charging station during time slot t is given by

$$P\{n\} = \frac{e^{\lambda_t}(\lambda_t)^n}{n!}, \quad n = 0, 1, 2, \cdots.$$

When EVs arrive at the charging station, they will select their charging modes based on their preferences and the corresponding service fee. Let $\bar{p}_t^A$ and $\bar{p}_t^D$ respectively denote the probability for one EV selecting the AC mode and the DC mode during time slot t, respectively. The values of $\bar{p}_t^A$ and $\bar{p}_t^D$ satisfy

$$\bar{p}_t^A + \bar{p}_t^D = 1, \tag{1}$$

$$0 \leq \bar{p}_t^A, \quad \bar{p}_t^D \leq 1. \tag{2}$$

Let λ_t^A and λ_t^D denote the expected number of EVs that select the AC mode and the DC mode during time slot t, respectively. We have

$$\lambda_t^A = \lambda_t \bar{p}_t^A \quad \text{and} \quad \lambda_t^D = \lambda_t \bar{p}_t^D. \tag{3}$$

Generally, based on the existing data analysis in [27], the EV charging requirements follow a lognormal distribution with a mean value E and a standard deviation of E_d. Thus, we assume that the expected charging requirement of each EV is E. Let T^A and T^D denote the expected service time for one AC charger and one DC charger to fully charge a battery with the capacity of B^C, respectively. $T^A > T^D$ always holds. The mean service times for one EV selecting the AC mode and the DC mode are ET^A/B^C and ET^D/B^C, respectively.

Based on the selections of EVs, they enter two separate queues: 1) one queue for EVs that select the AC mode, denoted by Q_t^A; 2) another queue for EVs that select the DC mode, denoted by Q_t^D. The queue lengths of Q_t^A and Q_t^D at time slot t can be given by the following equations:

$$Q_t^A = \max\{0, Q_{t-1}^A + \lambda_t^A - V_t^A - O_t^A - L_t^A\}, \tag{4}$$

$$Q_t^D = \max\{0, Q_{t-1}^D + \lambda_t^D - V_t^D - O_t^D - L_t^D\}, \tag{5}$$

where V_t^A and V_t^D respectively denote the total number of the available AC chargers and DC chargers at slot t, O_t^A and O_t^D respectively denote the number of EVs that have been fully recharged by the AC chargers and the DC chargers and leave the charging station during time slot t, and L_t^A and L_t^D respectively denote the number of EVs that select the AC mode and the DC mode and leave the charging station without being charged due to the long corresponding queue length.

In this paper, the maximum queue length is used to denote the maximal tolerable queue length of the EV owners, denoted by $\bar{Q}_t^A$ and $\bar{Q}_t^D$ respectively. If $Q_t^A \geq \bar{Q}_t^A$ ($Q_t^D \geq \bar{Q}_t^D$), the coming EVs that select the AC (DC) mode will leave the charging station without being charged; otherwise, they will be charged immediately or wait in the corresponding queue. Let L_t denote the service dropping rate of the charging station during time slot t. The expected value of L_t can be given by

$$L_t = L_t^A + L_t^D, \tag{6}$$

where

$$L_t^A = \lambda_t^A Pr\{Q_t^A | Q_t^A \geq \bar{Q}_t^A\}, \tag{7}$$

$$L_t^D = \lambda_t^D Pr\{Q_t^D | Q_t^D \geq \bar{Q}_t^D\}. \tag{8}$$

Obviously, the service dropping rate L_t depends on the selections of EVs λ_t^A and λ_t^D, and the queue lengths Q_t^A and Q_t^D, as well as the maximum queue lengths $\bar{Q}_t^A$ and $\bar{Q}_t^D$. Since the values of $\bar{Q}_t^A$ and $\bar{Q}_t^D$ are determined by EV owners, it is difficult for the charging station to change them. According to (4) and (5), Q_t^A and Q_t^D mainly depend on the selections of EVs λ_t^A and λ_t^D, respectively, which can be tuned to reduce the service dropping rate L_t.

B. Selection Model of EVs

Generally, due to the high construction cost of the DC mode, the service fee of the DC mode is much higher than that for the AC mode. Furthermore, the battery lifetime-related cost for the AC mode is much lower than that for the DC mode [28]. Different EVs may have the different preferences, but are price sensitive [16]. Since different charging stations always have the similar service fee of the AC mode, we set the service fee of the AC mode as a constant and adjust the service fee of the DC mode.

Let C_t^A and C_t^D denote the service fee per kWh during time slot t, and C_B^A and C_B^D denote the battery lifetime-related cost per kWh, with the AC mode and the DC mode, respectively. Here, $C_t^A < C_t^D$ and $C_B^A < C_B^D$ always hold. Generally, the service fee of the DC mode can only be adjusted in a given range. Let $\underline{C}^D$ and $\overline{C}^D$ denote the lower bound and upper bound on the service fee of the DC charger. Thus, the service fee C_t^D should always satisfy

$$\underline{C}^D \le C_t^D \le \overline{C}^D. \tag{9}$$

As mentioned above, the selections of EVs mainly depend on the service fees C_t^A and C_t^D and the battery lifetime-related costs C_B^A and C_B^D. Specifically, in this paper, we define $\bar{p}_t^A$ and $\bar{p}_t^D$ as follows:

$$\bar{p}_t^D = -aC_t^D + b, \tag{10}$$
$$\bar{p}_t^A = 1 - \bar{p}_t^D, \tag{11}$$

where $a = \frac{1}{C_t^A + \beta - (C_B^D - C_B^A)}$, $b = a\overline{C}^D$, and β denotes the service time reduction cost of the DC mode comparing with the AC mode. It can be seen that a is a price-dependent term and b is a price-independent term.

To ensure $0 \le \bar{p}_t^A, \bar{p}_t^D \le 1$, we set $\underline{C}^D = \overline{C}^D - (C_t^A + \beta - (C_B^D - C_B^A))$, which usually is larger than 0. Such that, when C_t^D equals its lower bound $\underline{C}^D$, all the coming EVs will select the DC mode, i.e., $\bar{p}_t^D = 1$ and $\bar{p}_t^A = 0$; when $C_t^D = \overline{C}^D$, all the coming EVs will select the AC mode, i.e., $\bar{p}_t^D = 0$ and $\bar{p}_t^A = 1$. Note that, our proposed algorithm also applies to other probability models, such as linear or concave ones.

C. Service Dropping Rate of Charging Station

Let $\bar{p}_t^L$ denote the probability for EVs leaving the charging station without being charged during time slot t. The expected values of $\bar{p}_t^L$ can be given by

$$\bar{p}_t^L = Pr\{Q_t^A | Q_t^A \ge \bar{Q}_t^A\}\bar{p}_t^A + Pr\{Q_t^D | Q_t^D \ge \bar{Q}_t^D\}\bar{p}_t^D. \tag{12}$$

Thus, the expected value of the service dropping rate L_t can be rewritten as

$$L_t = \lambda_t \bar{p}_t^L, \tag{13}$$

due to $\lambda_t^A = \lambda_t \bar{p}_t^A$ and $\lambda_t^D = \lambda_t \bar{p}_t^D$. It can be found that the service dropping rate L_t can be reduced by adjusting the service fee C_t^D, which affects the values of $\bar{p}_t^A$ and $\bar{p}_t^D$.

D. Customer Attrition Minimization Problem

The service dropping rate is not only an important parameter for the loss of benefit, but also one of the major factors for the satisfaction of EV owners. To minimize the service dropping rate of the charging station, we formulate a customer attrition minimization problem. Since EV arrives at different time slot follow a Poisson process, which has independent increments, we just need to minimize the service dropping rate of the charging station during each time slot, such that the total service dropping rate can be minimized. By now, the customer attrition minimization problem can be formulated as follows:

$$\textbf{P0:} \min_{C_t^D} \; L_t = \lambda_t \bar{p}_t^L,$$
$$s.t. \; \underline{C}^D \le C_t^D \le \overline{C}^D,$$
$$\bar{p}_t^D = -aC_t^D + b, \tag{14}$$
$$\bar{p}_t^A + \bar{p}_t^D = 1, \tag{15}$$
$$1 \le \bar{p}_t^A, \bar{p}_t^D \le 0, \tag{16}$$
$$(4), (5), \text{and } (12). \tag{17}$$

The objective is to minimize the service dropping rate L_t by adjusting the service fee C_t^D. The first constraint defines the available range of C_t^D. The constraints (14)-(16) describe the relationship between the service fee C_t^D and the selections of EVs, $\bar{p}_t^A$ and $\bar{p}_t^D$. The other constraints show relationships among $\bar{p}_t^L$, Q_t^A, Q_t^D, $\bar{p}_t^A$, and $\bar{p}_t^D$.

III. Optimal EV Charging Scheduling Scheme

To solve the customer attrition minimization problem, we first analyze the performance measure of the queue system to explore the possible range of optimal price and the relationship between the service dropping rate L_t and the service fee C_t^D. Then, we transform the primal problem into a convex optimization problem. At last, we propose an optimal pricing scheme to guide the EVs to select adequate charging modes, such that the total service dropping rate of the charging station can be minimized.

According to the definitions of $\bar{p}_t^A$ and $\bar{p}_t^D$, it can be found that the relationships among λ_t^A, λ_t^D and C_t^D are linear. For any given service fee C_t^D and arrival rate λ_t, λ_t^A and λ_t^D can be calculated according to (3), (10) and (11). Also, C_t^D and λ_t^A can be obtained when the value of λ_t^D is given. Hence, in the following sections, we can replace the variable C_t^D by λ_t^D for easy computation.

A. Performance Measures Based on Queue Theory

Let μ^A and μ^D denote the average service rate for one AC charger and that for one DC charger, respectively. Based on the system model, the values of μ^A and μ^D, and their relationship can be given by

$$\mu^A = \frac{B^C}{ET^A}, \tag{18}$$
$$\mu^D = \frac{B^C}{ET^D}, \tag{19}$$
$$\mu^A = \frac{T^D}{T^A}\mu^D. \tag{20}$$

Obviously, $\mu^A < \mu^D$ since $T^D < T^A$.

The charging processes of EVs at the charging station can be formulated as two independent queuing networks, i.e., $M/M/N_1$ with $\rho_t^A = \lambda_t^A/(N_1\mu^A)$ and $M/M/N_2$ with

$\rho_t^D = \lambda_t^D/(N_2\mu^D)$, respectively. Here, ρ_t^A and ρ_t^D are the corresponding utilization factors for different charging modes. For the optimal service fee $\hat{C}_t^D$, we have the following Theorem:

Theorem 1: For any given arrival rate λ_t, the available range for the optimal service fee $\hat{C}_t^D$ can be given by

$$\begin{cases} \frac{N_1\mu^A + b\lambda_t - \lambda_t}{a\lambda_t} \\ \quad \le \hat{C}_t^D \le \frac{b\lambda_t - N_2\mu^D}{a\lambda_t}, & \text{if } \lambda_t \ge N_1\mu^A + N_2\mu^D; \\ \frac{b\lambda_t - N_2\mu^D}{a\lambda_t} \\ \quad < \hat{C}_t^D < \frac{N_1\mu^A + b\lambda_t - \lambda_t}{a\lambda_t}, & \text{otherwise.} \end{cases}$$

Proof: According to the queuing theory, if with infinite buffer, the necessary conditions for these two queue systems to be stable are $\rho_t^A < 1$ and $\rho_t^D < 1$. If the mean arrival rate is greater than the mean service rate, the necessary conditions for the stable queue systems cannot be satisfied, which makes the server keeping busy and the queue growing without bound[2] [29]. Thus, based on the arrival rate λ_t and the service rate, we classify the available range of $\hat{C}_t^D$ into the following two cases:

CASE I: If $\lambda_t \ge N_1\mu^A + N_2\mu^D$, there is no available service fee satisfying $\frac{\lambda_t \bar{p}_t^A}{N_1\mu^A} < 1$ and $\frac{\lambda_t \bar{p}_t^D}{N_2\mu^D} < 1$ simultaneously. If $\rho_t^A \ge 1$, the chargers with the AC mode will keep busy since their queuing system becomes overloaded and the service dropping rate will increase with respect to λ_t^A, and $L_t^A \approx \lambda_t^A - N_1\mu^A$; otherwise, the service dropping rate $L_t^A \approx \lambda_t^A Pr\{Q_t^A|Q_t^A \ge \bar{Q}_t^A\} = \lambda_t^A(1 - Pr\{Q_t^A|Q_t^A < \bar{Q}_t^A\})$. The parameters for L_t^D are similar to those for L_t^A. Hence, the service dropping rate L_t can be given by the following cases:

- If $\lambda_t^A > N_1\mu^A$ and $\lambda_t^D < N_2\mu^D$, we have
$$L_t \approx \lambda_t - N_1\mu^A - \lambda_t^D Pr\{Q_t^D|Q_t^D < \bar{Q}_t^D\}; \quad (21)$$
- If $\lambda_t^A \ge N_1\mu^A$ and $\lambda_t^D \ge N_2\mu^D$, we have
$$L_t \approx \lambda_t - N_1\mu^A - N_2\mu^D; \quad (22)$$
- If $\lambda_t^A < N_1\mu^A$ and $\lambda_t^D > N_2\mu^D$, we have
$$L_t \approx \lambda_t - N_2\mu^D - \lambda_t^A Pr\{Q_t^A|Q_t^A < \bar{Q}_t^A\}. \quad (23)$$

Since $Pr\{Q_t^A|Q_t^A < \bar{Q}_t^A\} < 1$ and $Pr\{Q_t^D|Q_t^D < \bar{Q}_t^D\} < 1$, (22)<(21) and (22)<(23) always hold. To minimize the service dropping rate L_t, $\rho_t^A \ge 1$ and $\rho_t^D \ge 1$ should be satisfied simultaneously. Thus, the available range for the optimal service fee $\hat{C}_t^D$ is

$$\lambda_t \bar{p}_t^A \ge N_1\mu^A \Rightarrow \hat{C}_t^D \ge \frac{N_1\mu^A + b\lambda_t - \lambda_t}{a\lambda_t},$$
$$\lambda_t \bar{p}_t^D \ge N_2\mu^D \Rightarrow \hat{C}_t^D \le \frac{b\lambda_t - N_2\mu^D}{a\lambda_t}.$$

CASE II: If $\lambda_t < N_1\mu^A + N_2\mu^D$, it means that there exists an optimal service fee $\hat{C}_t^D$ that can satisfy $\rho_t^A < 1$ and $\rho_t^D < 1$ simultaneously. If $\rho_t^A < 1$ and $\rho_t^D < 1$, it means that $\lambda_t^A < N_1\mu^A$ and $\lambda_t^D < N_2\mu^d$ always hold, and the service dropping rate L_t can be calculated by

$$\begin{aligned} L_t &= \lambda_t^A Pr\{Q_t^A|Q_t^A \ge \bar{Q}_t^A\} + \lambda_t^D Pr\{Q_t^D|Q_t^D \ge \bar{Q}_t^D\} \\ &= \lambda_t - \lambda_t^A Pr\{Q_t^A|Q_t^A < \bar{Q}_t^A\} - \lambda_t^D Pr\{Q_t^D|Q_t^D < \bar{Q}_t^D\}. \end{aligned} \quad (24)$$

It can be found that (24)<(21) and (24)<(23) always hold. Hence, the optimal service fee $\hat{C}_t^D$ should satisfy

$$\lambda_t \bar{p}_t^A < N_1\mu^A \Rightarrow \hat{C}_t^D < \frac{N_1\mu^A + b\lambda_t - \lambda_t}{a\lambda_t},$$
$$\lambda_t \bar{p}_t^D < N_2\mu^D \Rightarrow \hat{C}_t^D > \frac{b\lambda_t - N_2\mu^D}{a\lambda_t}.$$

The available range for the optimal service fee $\hat{C}_t^D$ is obtained. ■

According to the available range for the optimal service fee $\hat{C}_t^D$ given by Theorem 1, if the arrival rate λ_t satisfies $\lambda_t \ge N_1\mu^A + N_2\mu^D$, we have the following Lemma for the optimal service fee $\hat{C}_t^D$:

Lemma 1: The optimal service fee $\hat{C}_t^D$ can be any value in the available range $[\frac{N_1\mu^A + b\lambda_t - \lambda_t}{a\lambda_t}, \frac{b\lambda_t - N_2\mu^D}{a\lambda_t}]$ when $\lambda_t \ge N_1\mu^A + N_2\mu^D$.

Proof: For the service dropping rate L_t given by (22), it can be found that the minimal value of the service dropping rate L_t is a constant when $\lambda_t \ge N_1\mu^A + N_2\mu^D$, and the only condition for the optimal service fee $\hat{C}_t^D$ is to make sure of that $\rho_t^A \ge 1$ and $\rho_t^D \ge 1$ are satisfied simultaneously. Thus, the optimal service fee $\hat{C}_t^D$ can be any value in the available range $[\frac{N_1\mu^A + b\lambda_t - \lambda_t}{a\lambda_t}, \frac{b\lambda_t - N_2\mu^D}{a\lambda_t}]$. ■

Therefore, we only need to design the optimal pricing scheme for the problem when $\lambda_t < N_1\mu^A + N_2\mu^D$.

Let n_1 and n_2 denote the steady state of the charging processes of the AC mode and the DC mode, in which there are n_1 and n_2 customers in their corresponding systems, include the customers in service, respectively. Let $p_t^A(n_1)$ denote the probability that in steady state the number of customers present in the AC mode is n_1 during time slot t and $p_t^D(n_2)$ denote the probability that in steady state the number of customers present in the DC mode is n_2 during time slot t. Then, we have

$$p_t^A(n) = \begin{cases} p_t^A(0)\frac{(N_1\rho_t^A)^n}{n!}, & \text{if } n \le N_1; \\ p_t^A(0)\frac{(\rho_t^A)^n N_1^{N_1}}{N_1!}, & \text{if } n \ge N_1; \end{cases} \quad (25)$$

$$p_t^D(n) = \begin{cases} p_t^D(0)\frac{(N_2\rho_t^D)^n}{n!}, & \text{if } n \le N_2; \\ p_t^D(0)\frac{(\rho_t^D)^n N_2^{N_2}}{N_2!}, & \text{if } n \ge N_2; \end{cases} \quad (26)$$

where

$$p_t^A(0) = [\sum_{n=0}^{N_1-1} \frac{(N_1\rho_t^A)^n}{n!} + \frac{(N_1\rho_t^A)^{N_1}}{N_1!}\frac{1}{1-\rho_t^A}]^{-1}, \quad (27)$$

[2]In a realistic queuing system, the buffer size is limited, so the queue length will be limited, while a portion of the arrivals will be blocked when the buffer is full.

$$p_t^D(0) = [\sum_{n=0}^{N_2-1} \frac{(N_2\rho_t^D)^n}{n!} + \frac{(N_2\rho_t^D)^{N_2}}{N_2!}\frac{1}{1-\rho_t^D}]^{-1}. \quad (28)$$

In this paper, we use the steady-state distribution of EVs to model the service processes of the charging station and solve the customer attrition minimization problem.

B. Service Dropping Rate of Charging Station

Since the maximal queue lengths $\bar{Q}_t^A$ and $\bar{Q}_t^D$ for EVs during time slot t are constant, the service dropping rate L_t can be given by

$$L_t = \sum_{n_1=N_1+\bar{Q}_t^A}^{\infty} \lambda_t^A p_t^A(n_1) + \sum_{n_2=N_2+\bar{Q}_t^D}^{\infty} \lambda_t^D p_t^D(n_2) \quad (29)$$

where $\sum_{n_1=N_1+\bar{Q}_t^A}^{\infty} \lambda_t^A p_t^A(n_1) = \frac{\lambda_t^A(p_t^A(N_1+\bar{Q}_t^A))^{(N_1+\bar{Q}_t^A)}}{1-\rho_t^A}$ denotes the blocking traffic for the queue of the AC mode and $\sum_{n_2=N_2+\bar{Q}_t^D}^{\infty} \lambda_t^D p_t^D(n_2) = \frac{\lambda_t^D(p_t^D(N_2+\bar{Q}_t^D))^{(N_2+\bar{Q}_t^D)}}{1-\rho_t^D}$ denotes the blocking traffic for the queue of the DC mode. According to the definitions of $p_t^A(n)$ and $p_t^D(n)$ given by (25) and (26), it can be found that $p_t^A(n)$ and $p_t^D(n)$ also depend on the values of λ_t^A and λ_t^D. Hence, the service dropping rate L_t mainly depends on the values of λ_t^A and λ_t^D.

C. Problem Transformation

According to Theorem 1 and the relationship among λ_t^A, λ_t^D, $\bar{p}_t^A$, $\bar{p}_t^D$ and C_t^D given by (3), (10) and (11), the optimal λ_t^D should satisfy

$$\lambda_t - N_1\mu^A < \lambda_t^D < N_2\mu^D. \quad (30)$$

By now, the primal Problem **P0** can be transformed to the following problem **P1**, in which the variable is λ_t^D, i.e.,

$$\textbf{P1:} \min_{\lambda_t^D} L_t, \quad (31)$$

$$s.t.\ \lambda_t - N_1\mu^A < \lambda_t^D < N_2\mu^D. \quad (32)$$

In this problem, the goal is to find the optimal λ_t^D to minimize the service dropping rate L_t, and the available range for λ_t^D is given by constraint (32). If the objective function L_t is a convex function of λ_t^D, Problem **P1** can be transformed into a typical convex optimization problem. Thus, we first establish that, if the problem is a convex optimization problem, the solution of this problem is indeed toward the global optimum [30]. To solve this problem, we first prove that convexity of the service dropping rate L_t with respect to λ_t^D, and then propose an optimal pricing scheme to obtain the optimal price $\hat{C}_t^D$.

D. Proof of Convexity

According to (29), the service dropping rate L_t is

$$L_t = \lambda_t^A p_t^A(n_1')\frac{1}{1-\rho_t^A} + \lambda_t^D p_t^D(n_2')\frac{1}{1-\rho_t^D},$$

where $n_1' = N_1 + \bar{Q}_t^A$ and $n_2' = N_2 + \bar{Q}_t^D$. Due to $\lambda_t^A = N_1\mu^A\rho_t^A$ and $\lambda_t^D = N_2\mu^D\rho_t^D$, we have

$$L_t = L_t^A + L_t^D,$$

where

$$L_t^A = N_1\mu^A\rho_t^A p_t^A(n_1')\frac{1}{1-\rho_t^A} = N_1\mu^A p_t^A(0)\frac{N_1^{N_1}(\rho_t^A)^{n_1'+1}}{N_1!}\frac{1}{1-\rho_t^A}, \quad (33)$$

$$L_t^D = N_2\mu^D\rho_t^D p_t^D(n_2')\frac{1}{1-\rho_t^D} = N_2\mu^D p_t^D(0)\frac{N_2^{N_2}(\rho_t^D)^{n_2'+1}}{N_2!}\frac{1}{1-\rho_t^D}. \quad (34)$$

It can be found that L_t^A and L_t^D have the similar structure. Thus, we first prove that the second part L_t^D is an increasing and convex function of λ_t^D, and then we can derive that the first part L_t^A is a decreasing and convex function of λ_t^D due to $\lambda_t^A = \lambda_t - \lambda_t^D$. Because ρ_t^A is a linear function of λ_t^A and such a variable substitution will not change the convexity of the objective function, we take L_t^D as the objective function and ρ_t^D as the variable in the following part. For the second part L_t^D, we have the following theorem:

Theorem 2: The service dropping rate L_t^D is an increasing and convex function of the service rate ρ_t^D.

Proof: The proof can be found in Appendix A. ■

Since L_t^A and L_t^D have the similar structure and $\lambda_t^A = \lambda_t - \lambda_t^D$, we have the following Lemma for the relationship between L_t^A and ρ_t^D:

Lemma 2: The service dropping rate L_t^A is a decreasing and convex function of the service rate ρ_t^D.

Proof: The proof can be found in Appendix B. ■

Theorem 3: The service dropping rate L_t is a convex function of λ_t^D.

Proof: Since $L_t = L_t^A + L_t^D$, $\frac{\partial^2 L_t^A}{\partial(\rho_t^D)^2} > 0$ and $\frac{\partial^2 L_t^D}{\partial(\rho_t^D)^2} > 0$, we have $\frac{\partial^2 L_t}{\partial(\rho_t^D)^2} > 0$. Since $\lambda_t^D = \rho_t^D N_2\mu^D$, $\frac{\partial\lambda_t^D}{\partial\rho_t^D} = N_2\mu^D > 0$ always holds. Hence, $\frac{\partial^2 L_t}{\partial(\lambda_t^D)^2} = (N_2\mu^D)^2\frac{\partial^2 L_t}{\partial(\rho_t^D)^2} > 0$. Thus, the service dropping rate L_t is a convex function of λ_t^D. ■

Because the relationship between λ_t^D and C_t^D is linear, the service dropping rate L_t also is a convex function of C_t^D.

E. Optimal Pricing Scheme

Since the objective function L_t is a convex function of λ_t^D and the constraint (32) is a linear constraint for λ_t^D, the transformed Problem **P1** is a convex optimization problem. Since this optimization problem will be solved by the charging station, it can be solved using existing centralized tools, such as fmincon function [31] or CVX toolbox [32] in Matlab. We omit the details of how to solve this problem.

By solving Problem **P1**, the optimal λ_t^D can be obtained, then the optimal pricing $\hat{C}_t^D$ can be calculated by

$$\hat{C}_t^D = \frac{b}{a} - \frac{\lambda_t^D}{a\lambda_t}. \quad (35)$$

The process for obtaining the optimal price $\hat{C}_t^D$ can be sketched as **Algorithm 1**.

Theorem 4: The minimal L_t can be achieved by the optimal pricing scheme shown in **Algorithm 1**.

Algorithm 1 Optimal Pricing Scheme for the Charging Station

Initialization λ_t, N_1, μ^A, N_2, μ^D, $\bar{Q}_t^A$, $\bar{Q}_t^D$

- **If** $\lambda_t \geq N_1\mu^A + N_2\mu^D$

  Chooses λ_t^D randomly in $[N_2\mu^D, \lambda_t - N_1\mu^A]$;
- **Else**

  Obtains the optimal λ_t^D by solving Problem **P1**;
- **end**

  Calculates the optimal price $\hat{C}_t^D$ by (35);

return $\hat{C}_t^D$

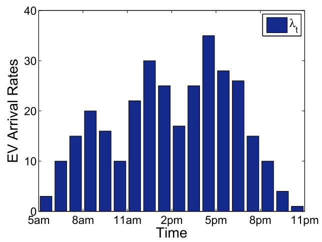


Fig. 2. The EV arrivals of the charging station.

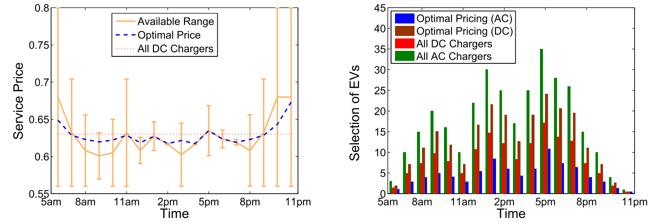
Proof: According to Theorem 1 and Lemma 1, if $\lambda_t \geq N_1\mu^A + N_2\mu^D$, the minimal service dropping rate $L_t \approx \lambda_t - N_1\mu^A - N_2\mu^D$ can be obtained by any value in the available range, i.e., $\hat{C}_t^D \in [\frac{N_1\mu^A + b\lambda_t - \lambda_t}{a\lambda_t}, \frac{b\lambda_t - N_2\mu^D}{a\lambda_t}]$. If $\lambda_t < N_1\mu^A + N_2\mu^D$, since the customer attrition minimization problem is a convex optimization problem according to Theorem 3, there exists an unique optimal solution, which can be obtained by solving Problem **P1** [30]. Thus, the proposed optimal pricing scheme in **Algorithm 1** can minimize the service dropping rate L_t. ■

IV. Performance Evaluation

In order to demonstrate the performance of the proposed algorithm, we take one charging station with dual charging modes and time-varying arrivals of EVs as a case study, and then analyze the effects of the number of chargers, the maximal waiting queue length, and the arrival rate on the optimal pricing of the charging station.

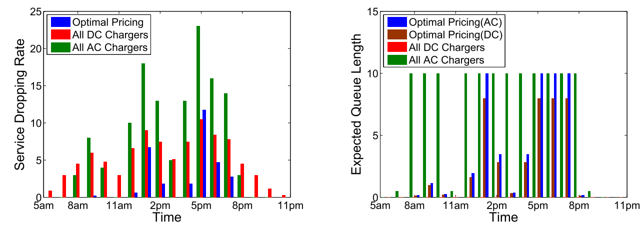
A. Case Study

Consider a charging station with $N_1 = 15$ AC chargers and $N_2 = 8$ DC chargers. The average service rates for each AC charger and DC charger are $\mu^A = \frac{2}{5}$ per hour and $\mu^D = \frac{12}{5}$ per hour, respectively. According to [33], the cost for replacing a $24kWh$ battery is about \$5500, and the lifetime with the AC mode and the DC mode are about 650 and 500 cycles, respectively. Thus, we can derived that $C_B^A \approx \$0.35/kWh$ and $C_B^D \approx \$0.46/kWh$. The maximal queue lengths are $\bar{Q}_t^A = 10$ and $\bar{Q}_t^D = 8$. We set $C_t^A = \$0.15/kWh$, $\overline{C}^D = \$0.8/kWh$, $\beta = \$0.2/kWh$, and $E = 16kWh$. The arrivals of EVs during different time slot can be found in Fig. 2. Assume that there exists another two construction plan: i) charging station with the AC mode, in which all the DC chargers (including the waiting space) are replaced by the AC chargers, and all the EVs can only select the AC mode, i.e., $\bar{N}_1 = 31$ and $\bar{Q}_t^A = 10$; ii) charging station with the DC mode, in which all the AC chargers are replaced by the DC chargers, i.e., $\bar{N}_2 = 23$ and $\bar{Q}_t^D = 18$, while the service fee of the DC mode is $\$0.63/kWh$.



(a) (b)

Fig. 3. Optimal price $\hat{C}_t^D$ where $C_t^A = \$0.15/kWh$ and selections of EVs at different time. (a) Optimal price $\hat{C}_t^D$. (b) Selection of EVs.



(a) (b)

Fig. 4. The numbers of EVs that leave the charging station without being charged and the corresponding expected queue lengths at different times. (a) L_t^A and L_t^D. (b) Q_t^A and Q_t^D.

The optimal price $\hat{C}_t^D$ of the charging station with dual charging modes and the expected selections of EVs can be found in Fig. 3. It can be found that the available range for the optimal price changes and the optimal price of the DC mode is time-varying due to time-varying EV arrival rate. With the increase of the arrival rate λ_t, the available range for optimal price will be narrowed, which means that the selections for the optimal price become less, and the optimal price $\hat{C}_t^D$ for the DC mode will be decreased, such that more EVs will select the DC mode. However, since the service fees in the charging stations with single mode are constants, all the EVs in i) charging station with the AC mode can only select the AC mode while only part of EVs in ii) charging station with the DC mode will select the DC mode due to its high battery lifetime-related cost.

The total service dropping rates for different charging stations and the expected queue lengths can be found in Fig. 4. It can be found that the optimal pricing scheme can minimize the service dropping rate since it can guide the EVs selecting the suitable charging mode to improve the utilization of chargers in the charging station. For i) the charging station with the AC mode, part of EVs leaves due to its limited service ability and the long waiting queue, while for ii) the charging station with the DC mode, part of EVs will not select this charging station due to its high battery lifetime-related cost and high charging service fee. For the charging station with dual charging modes under optimal pricing scheme, by adjusting

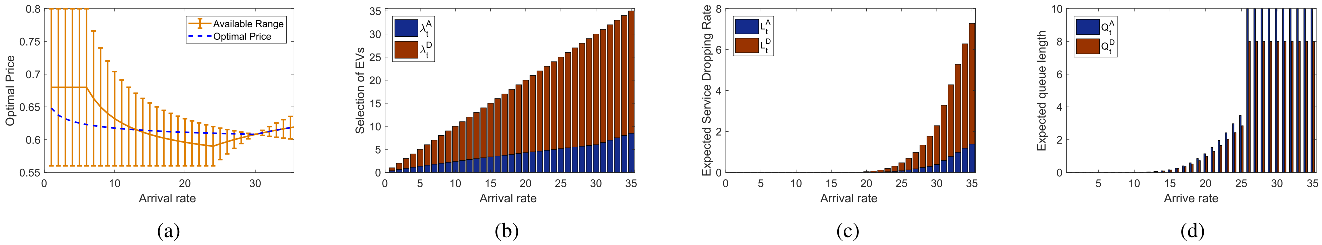


Fig. 5. Optimal solution with the increase of arrival rate λ_t^A: a) the optimal price $\hat{C}_t^D$; b) the selections of EVs λ_t^A and λ_t^D; c) the service dropping rates L_t^A and L_t^D; and d) the expected queue lengths Q_t^A and Q_t^D.

TABLE II
THE SERVICE DROPPING RATE (UNIT: EVS)

Chargers	5-6am	7-8pm	9-10am	11-12pm	1-2pm
All AC mode	0.04	11	4.04	28.00	18.00
All DC mode	3.90	11.50	7.80	15.60	12.60
Dual Modes	0	0.26	0.02	7.40	1.89
Chargers	3-4pm	5-6pm	7-8pm	9-11pm	Total
All AC mode	35.00	30.00	3.04	0	**130.10**
All DC mode	18.00	16.20	7.50	1.50	**93.60**
Dual Modes	13.6	7.51	0.01	0	**30.69**

the service fee of the DC mode, both the number of EVs that leave the charging station without being charged and the expected queue lengths can be minimized, especially when the arrival rate is high.

With same arrival rates and construction space, i) charging station with the AC mode has the smallest service ability and longest waiting queue, ii) charging station with the DC mode has the largest service ability and the smallest waiting queue, and charging station with dual charging modes under optimal pricing scheme has a middle service ability and waiting queue. In addition, i) charging station with the AC mode has the largest service dropping rate due to limited service ability, and ii) charging station with the DC mode has a higher service dropping rate due to its high battery lifetime-related cost, and the charging station with dual charging modes under optimal pricing scheme can minimize the service dropping rate, which has a higher flexibility and adaptability to deal with the time-varying arrival rate of EVs.

The expected service dropping rate for different charging stations can be found in Table II. It can be found that the charging station with dual charging modes under the proposed pricing scheme can minimize the total service dropping rate of the charging station. The charging station with the AC mode has the highest service dropping rate due to its limited charging service rate even when it has 31 chargers. The charging station with the DC mode has a middle service dropping rate due to its high service fee and high battery lifetime-related cost. As we know, since the charging station with the DC mode has enough charging service ability to service more EVs, reduce its service fee can cut down the service dropping rate. However, we found that only when the charging station with the DC mode reduce its the service fee to $\$0.59/kWh$, which is much lower than the service fee in charging station with dual charging modes, it has the similar service dropping rate of the charging station with dual charging modes.

B. Relationship Between Arrival Rate and System Performance

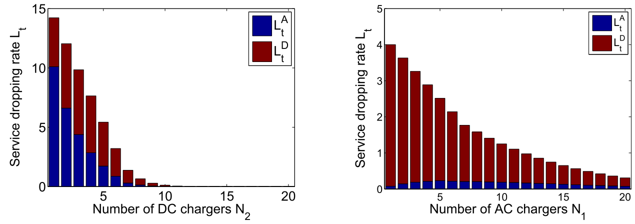
The maximal service ability of the charging station with dual charging modes is 25.2 EVs/hour. In order to explore the performance of our proposed pricing scheme, we set the arrival rate from [1, 35] and respectively show the optimal price $\hat{C}_t^D$ and its available range, the minimal service dropping rates L_t^A and L_t^D, the selections of EVs λ_t^A and λ_t^D, and the expected queue lengths for the AC mode and the DC mode in Fig. 5.

From the simulation results in Fig. 5(a), it can be found that, when the arrival rate of EVs is smaller than the maximal service ability of the charging station, with the increase of the arrival rate λ_t, the available range for the optimal price becomes narrow to ensure that both of the utilization factors ρ_t^A and ρ_t^D are smaller than 1, and the optimal price $\hat{C}_t^D$ decreases; otherwise, the available range for the optimal price becomes wider to ensure that both of the utilization factors ρ_t^A and ρ_t^D are larger than 1, and the optimal price $\hat{C}_t^D$ can be any value in the available range. Specially, when the arrival rate of EVs is smaller than the maximal service ability of the charging station, the charging station needs to select an optimal pricing by solving Problem **P1** to minimize the total service dropping rate for both the AC mode and the DC mode; and when the arrival rate of EVs is larger than the maximal service ability of the charging station, the charging station just needs to select one available value in the available range since any value in the available range will obtain the same service dropping rate.

Figs. 5(b)-5(d) show the selections of EVs, the service dropping rates and the expected queue lengths, respectively. With the increase of EV arrival rate, the selections of EVs for both the AC mode and the DC mode, the service dropping rate and the expected queue lengths will be increased. In order to minimize the service dropping rate, it can be found that the selections of EVs for both the AC mode and the DC mode increase smoothly. Also, the service dropping rate grows when the arrival rate of EVs is larger than a certain threshold and the expected queue lengths reach their upper bounds of the charging station. Since the service rate of the AC mode is much lower than the service rate of the DC mode, more EVs are assigned to select the DC mode to minimize the total service dropping rate.

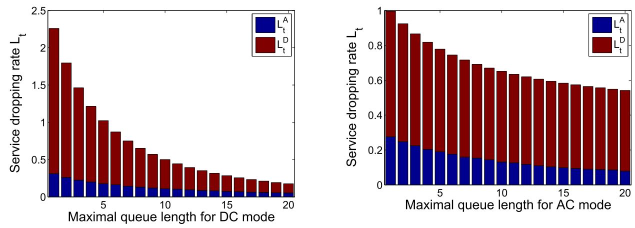
C. The Effects of N_1 and N_2 on Service Dropping Rate L_t

To explore the effect of the charging facilities in the charging station on the service dropping rate L_t, we fix the



(a) (b)

Fig. 6. The minimal service dropping rate L_t affected by the values of N_1 and N_2: (a) $N_1 = 15$ and $N_2 \in [1, 20]$; (b) $N_2 = 8$ and $N_1 \in [1, 20]$.



(a) (b)

Fig. 7. The minimal service dropping rate L_t affected by the values of $\bar{Q}_t^A$ and $\bar{Q}_t^D$: (a) $\bar{Q}_t^A = 10$ and $\bar{Q}_t^D \in [1, 20]$; (b) $\bar{Q}_t^D = 8$ and $\bar{Q}_t^A \in [1, 20]$.

EV arrival by $\lambda_t = 22$, and perform the following simulations: 1) fixing $N_1 = 15$ while adjusting N_2 from 1 to 20, the minimal service dropping rate L_t is shown in Fig. 6(a); 2) fixing $N_2 = 8$ while adjusting N_1 from 1 to 20, the minimal service dropping rate L_t is shown in Fig. 6(b). It can be found that the minimal service dropping rate L_t in the first simulation is decreasing much faster than that in the second one. That is because the service rate of each DC charger is much higher than that of each AC charger. For a charging station with limited space, more DC chargers can reduce the service dropping rate, but may decrease the number of EVs due to its high battery lifetime-related cost.

D. The Effects of $\bar{Q}_t^A$ and $\bar{Q}_t^D$ on Service Dropping Rate L_t

Since the maximal queue length affects the minimal service dropping rate L_t, we conduct several simulations to demonstrate the effects of $\bar{Q}_t^A$ and $\bar{Q}_t^D$ on the minimal service dropping rate L_t. First, we fix $\bar{Q}_t^A = 10$ and adjust $\bar{Q}_t^D$ from 1 to 20, whose minimal service dropping rate L_t is shown in Fig. 7(a). Then, we fix $\bar{Q}_t^D = 8$ and adjust $\bar{Q}_t^A$ from 1 to 20, whose minimal service dropping rate L_t is illustrated in Fig. 7(b). Obviously, the service dropping rate L_t in the first simulation will decrease more quickly than that in the second simulation. That is because the service rate for the DC chargers is much higher than that of the AC charger. However, increasing the waiting space of the charging station may not improve the service dropping rate significantly, since the service dropping rate mainly depends on the service ability of the charging station.

V. CONCLUSION

In this paper, we modeled the operation of the charging station with dual charging modes as a queuing network with the multiple servers and heterogeneous service rates, and analyzed the relationship between the service dropping rate and the selections of EVs. Then, by making use of price sensitiveness of EV owners, we designed an optimal pricing scheme to guide and coordinate the charging processes of EVs to minimize the service dropping rate of the charging station. Simulation results are provided to demonstrate the efficiency of the proposed charging scheduling scheme.

In our future work, we will consider the optimal pricing scheme for the charging station, where the arrival rate of the charging station depends on the service fee and the EVs can change their selections when the selected queue length is too long. Also, we intend to design an algorithm to determines the optimal number of chargers with dual charging modes based on the distribution of EVs, which can maximize the total profit of the charging stations and improve the service quality of the charging station.

APPENDIX A

Proof: For the special case when $\bar{Q}_t^D = 0$, according to the Erlang's C formula [34], the loss probability for $M/M/N/N$ can be given by

$$B(N_2, N_2\rho_t^D) = p_t^D(0)\frac{N_2^{N_2}(\rho_t^D)^{N_2}}{N_2!}\frac{1}{1-\rho_t^D}$$
$$= \frac{\frac{(N_2\rho_t^D)^{N_2}}{N_2!(1-\rho_t^D)}}{\sum_{n=0}^{N_2-1}\frac{(N_2\rho_t^D)^n}{n!} + \frac{(N_2\rho_t^D)^{N_2}}{N_2!(1-\rho_t^D)}}$$
$$= \Big(\sum_{n=0}^{N_2-1}\frac{N_2!(1-\rho_t^D)}{n!(N_2\rho_t^D)^{N_2-n}} + 1\Big)^{-1}. \quad (36)$$

The existing works [35], [36] have proved that $B(N_2, N_2\rho_t^D)$ is an increasing and convex function of ρ_t^D when N_2 is given. Thus, both of the first and the second derivatives $B(N_2, N_2\rho_t^D)$ with respect to ρ_t^D are larger than zero, i.e., $\partial B(N_2, N_2\rho_t^D)/\partial\rho_t^D > 0$ and $\partial^2 B(N_2, N_2\rho_t^D)/\partial(\rho_t^D)^2 > 0$.

According to (34), L_t^D can be rewritten as

$$L_t^D = N_2\mu^D(\rho_t^D)^{\bar{Q}_t^D+1}B(N_2, N_2\rho_t^D). \quad (37)$$

The first derivative L_t^D with respect to ρ_t^D is

$$\frac{\partial L_t^D}{\partial\rho_t^D} = N_2\mu^D\Big(\frac{\partial(\rho_t^D)^{\bar{Q}_t^D+1}}{\partial\rho_t^D}B(N_2, N_2\rho_t^D) + (\rho_t^D)^{\bar{Q}_t^D+1}\frac{\partial B(N_2, N_2\rho_t^D)}{\partial\rho_t^D}\Big).$$

Since $N_2\mu^D > 0$, $\frac{\partial(\rho_t^D)^{\bar{Q}_t^D+1}}{\partial\rho_t^D} = (\bar{Q}_t^D + 1)(\rho_t^D)^{\bar{Q}_t^D} > 0$, $B(N_2, N_2\rho_t^D) > 0$, $(\rho_t^D)^{\bar{Q}_t^D+1} > 0$, and $\frac{\partial B(N_2,N_2\rho_t^D)}{\partial\rho_t^D} > 0$, we have $\frac{\partial L_t^D}{\partial\rho_t^D} > 0$.

The second derivative L_t^D with respect to ρ_t^D is

$$\frac{\partial^2 L_t^D}{\partial(\rho_t^D)^2} = N_2\mu^D\Big(\frac{\partial^2(\rho_t^D)^{\bar{Q}_t^D+1}}{\partial(\rho_t^D)^2}B(N_2, N_2\rho_t^D) + 2\frac{\partial(\rho_t^D)^{\bar{Q}_t^D+1}}{\partial\rho_t^D}\frac{B(N_2, N_2\rho_t^D)}{\partial\rho_t^D} + (\rho_t^D)^{\bar{Q}_t^D+1}\frac{\partial^2 B(N_2, N_2\rho_t^D)}{\partial(\rho_t^D)^2}\Big). \quad (38)$$

Since $\frac{\partial^2(\rho_t^D)^{\bar{Q}_t^D+1}}{\partial(\rho_t^D)^2} = \bar{Q}_t^D(\bar{Q}_t^D+1)(\rho_t^D)^{\bar{Q}_t^D-1} > 0$, $\frac{\partial(\rho_t^D)^{\bar{Q}_t^D+1}}{\partial\rho_t^D} > 0$, $\frac{\partial B(N_2,N_2\rho_t^D)}{\partial\rho_t^D} > 0$, and $\frac{\partial^2\ B(N_2,N_2\rho_t^D)}{\partial(\rho_t^D)^2} > 0$, $\frac{\partial^2\ L_t^D}{\partial(\rho_t^D)^2} > 0$ always holds.

Since $\frac{\partial L_t^D}{\partial\rho_t^D} > 0$ and $\frac{\partial^2\ L_t^D}{\partial(\rho_t^D)^2} > 0$ always hold, the service dropping rate L_t^D is an increasing and convex function of the service rate ρ_t^D. ∎

Appendix B

Proof: Since $\lambda_t^A = L_t - \lambda_t^D$, $\rho_t^A = \lambda_t^A/N_1\mu^A$ and $\rho_t^D = \lambda_t^D/N_2\mu^D$, we have

$$\rho_t^A = \frac{L_t - N_2\mu^D\rho_t^D}{N_1\mu^A}, \quad (39)$$

and the first derivation of ρ_t^A with respect to ρ_t^D is

$$\frac{\partial\rho_t^A}{\partial\rho_t^D} = -\frac{N_2\mu^D}{N_1\mu^A}. \quad (40)$$

Since L_t^D has the same structure with L_t^A, it can be easily proved that $\frac{\partial L_t^D}{\partial\rho_t^D} > 0$ and $\frac{\partial^2\ L_t^D}{\partial(\rho_t^D)^2} > 0$. Due to $\frac{\partial\rho_t^D}{\partial\rho_t^A} < 0$, we have $\frac{\partial L_t^D}{\partial\rho_t^A} < 0$ and $\frac{\partial^2\ L_t^D}{\partial(\rho_t^A)^2} > 0$. Thus, the service dropping rate L_t^D is a decreasing and convex function of ρ_t^A. ∎

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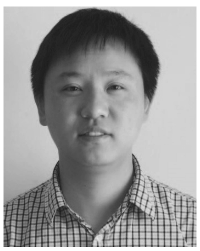

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