# Efficient Optimal Scheduling of Charging Station with Multiple Electric Vehicles via V2V

Pengcheng You and Zaiyue Yang
State Key Laboratory of Industrial Control Technology
Zhejiang University, Hangzhou, China
zjuyoupc@gmail.com, yangzy@zju.edu.cn

***Abstract*—This paper investigates the scheduling problem of an intermediary charging station with multiple electric vehicles (EV) in real-time electricity pricing environment. A charging aggregator (CA) is in charge to coordinate EVs' charging so that all EVs' requirements are met and meanwhile the total social cost is minimized. Besides, a new charging mechanism named Vehicle-to-Vehicle (V2V) is proposed to take full advantage of every EV's battery energy. Due to the binary state of EVs, i.e., charging and discharging, scheduling of the charging station is formulated as a constrained mixed-integer linear program (MILP). A distributed algorithm is applied to solve the problem by means of dual decomposition and Benders decomposition. Therefore, scheduling is carried out on each EV and coordinated by the CA. Numerical results show efficiency of the proposed approach and validate our theoretical analysis.**

## I. Introduction

Public concerns about traditional energy resources have risen rapidly in recent years. Due to the shortage of fossil fuels and emission of greenhouse gases, electricity is a perfect alternative to help address environment issues and promote sustainable development. However, the conventional power grid is facing great challenges resulting from increasing demand and aging infrastructure. Economically speaking, it is not wise to unlimitedly augment generation capacity to meet demand. With the notable advances in smart grid, demand-side management (DSM) plays a key role in effectively scheduling users' power usage [1]. Through providing various incentives and proper pricing, e.g., real-time pricing [2] and time-of-use pricing [3], power utility companies try to induce users to consume electricity in a way that helps smooth the load profile of distribution grid, i.e., reducing or shifting load from peak hours to off-peak hours.

With the rapid development of EVs and charging infrastructure, means of transportation will undergo huge changes in the near future. The electrification of vehicles is a new trend of great promise for both private cars and public transport. As EVs gradually penetrate into human's daily life, charging will become a vital part of users' electricity consumption. As a result, scheduling on EVs' charging behavior is of great significance. Many interesting related issues have been investigated. [4] proposes a distributed charging method for EVs which can adapt EVs' charging rates according to users' preferences. It shows that price information is very useful to regulate charging demand and hence balance load. [5] considers a multi-agent system and comes up with a distributed EV charging control method based on the Nash Certainty Equivalence Principle that considers network impacts. Different situations are studied where the control agents are uncoupled and weekly-coupled, respectively. Unidirectional Vehicle-to-Grid (V2G) is incorporated in [6] to help regulate EVs' charging. The use of EVs as demand response resources and benefits of unidirectional V2G are investigated. Besides, in [7], V2G market is investigated where EVs participate in to provide regulation services to the grid and a new game-theoretic model is put forward to describe the interactions among EVs and aggregators.

Typically, private EVs tend to be charged at home in the residential community or at a specific parking lot, either corporate or public [8]. In this paper, we mainly focus on the latter which can be generally regarded as a charging station coordinating a large quantity of EVs' charging behavior and propose a distributed approach to efficiently obtain the optimal charging strategy. Due to the binary state of EVs, i.e., charging and discharging, discrete variables are introduced and the charing coordination problem is formulated as a MILP with a load constraint for charing station and several charging constraints for each EV.

The contributions of this paper mainly consist of three parts, which can be summarized as follows:

1) A new charging mechanism named V2V is proposed to take full advantage of every EV's battery energy and help relieve burden on distribution gird.
2) The load constraint for charging station couples all EVs' charging behavior together. In order to tackle the coupling constraint, Lagrangian relaxation is introduced to decompose the optimization problem into a set of subproblems. Each subproblem corresponds to scheduling on single EV's charging and can be solved independently.
3) Due to the existence of discrete variables, every subproblem is still a MILP to be solved on each EV. Benders decomposition is applied to cope with the difficulty in solving MILP. Optimal solution can be obtained through iterations between master problem and slave problem.

The remainder of this paper is organized as follows. System model is described in Sec. II, followed by problem formulation and transformation in Sec. III. We focus on solving subproblems by means of Benders decomposition in Sec. IV. In Sec. V, distributed implementation of the proposed approach is introduced. Numerical results are given in Sec. VI and conclusions are drawn in Sec. VII.

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## II. SYSTEM MODEL

### A. Charging Scenario

Consider a charging station where multiple EVs can park there and be charged. The charging station is equipped with a CA which is neutral and help relieve the charging load on distribution grid. Assume a set $\mathcal{A} \triangleq \{1, 2, ..., A\}$ of EVs are parking at the charging station and each EV is embedded with a controller which optimizes EV's charging strategy through communication with the CA.

Based on the concept of V2G, a new charging mechanism named V2V is put forward. EVs arrive at the charging station and are connected to charging infrastructure. Each EV owner sets an appropriate deadline when his EV must finish charging. Upon receiving all EV owners' request, the CA and each corresponding controller have the obligation to satisfy the owners' requirements and figure out the optimal charging strategies. The CA provides EVs with electricity energy but the CA's energy is acquired from different sources. Apart from energy provided by power grid, energy in EVs' batteries can also be utilized by the CA for coordination. EV owners are paid for 'borrowing' energy to the CA. The CA's capability of transmitting energy back to power grid is not considered in this paper, thus all 'borrowed' energy is used for charging.

In our scenario, for simplicity, the CA is assumed to serve as an intermediary that only participates in the real-time balancing market to obtain enough electricity energy to meet all EV owners' demands and help clear the market. In the real-time balancing market, electricity price varies with time and is assumed to be known in advance. Thus the CA has to coordinate all EVs' charging strategy depending on time-varying electricity price.

### B. Constraints

We consider a discrete-time model with a finite horizon that covers the time period over which all EVs must finish charging. This time period consists of many time slots of equal duration $\Delta t$, indexed by $n \in \mathcal{N} \triangleq \{1, 2, ..., N\}$. Without loss of generality, $\Delta t$ is assumed to equal 1. Suppose the set $\mathcal{A}$ of EVs are all willing to participate in the V2V program. The charging deadline of EV $a$ is denoted as $N_a$ and thus $N = \max\limits_{a \in \mathcal{A}} N_a$. Let a pair of binary variables $\omega_{a,c}^n, \omega_{a,d}^n \in \{0, 1\}$ denote the charging and discharging state of EV $a$ at time slot $n$, respectively. $\omega_{a,c}^n = 1$ implies that EV $a$ is charging at time slot $n$ while $\omega_{a,d}^n = 1$ represents the opposite with the following constraints:

$$\begin{cases} \omega_{a,c}^n + \omega_{a,d}^n \leq 1, \quad \omega_{a,c}^n, \omega_{a,d}^n \in \{0, 1\}, \quad \forall n \in \mathcal{N}_a, \forall a \in \mathcal{A} \\ \omega_{a,c}^n, \omega_{a,d}^n = 0, \quad \forall n \in \mathcal{N} \backslash \mathcal{N}_a, \forall a \in \mathcal{A} \end{cases} \tag{1}$$

Let continuous variables $p_{a,c}^n$ and $p_{a,d}^n$ denote the charging rate and discharging rate of EV $a$ at time slot $n$, respectively. Apparently they can neither exceed their maximum thresholds nor fall below 0:

$$\begin{cases} 0 \leq p_{a,c}^n \leq \omega_{a,c}^n p_{a,c}^{\max}, \quad \forall n \in \mathcal{N}, \forall a \in \mathcal{A} \\ 0 \leq p_{a,d}^n \leq \omega_{a,d}^n p_{a,d}^{\max}, \quad \forall n \in \mathcal{N}, \forall a \in \mathcal{A} \end{cases} \tag{2}$$

where $p_{a,c}^{\max}$ and $p_{a,d}^{\max}$ are the maximum charging rate and discharging rate for EV $a$, respectively.

The energy stored in the battery of EV $a$ at time slot $n$ is denoted as $\pi_a^n$, which is bounded within a limited range:

$$\pi_a^{\min} \leq \pi_a^n \leq \pi_a^{\max}, \quad \forall n \in \mathcal{N}, \forall a \in \mathcal{A}$$

For ease of presentation, the dynamic change of $\pi_a^n$ is described as a linear process:

$$\pi_a^n = \pi_a^0 + \sum_{t=1}^{n} \left( \beta_a^c p_{a,c}^t - \frac{p_{a,d}^t}{\beta_a^d} \right)$$

where $\beta_a^c$ and $\beta_a^d$ are charging efficiency and discharging efficiency of EV $a$, respectively. $\pi_a^0$ denotes the initial energy stored in the battery of EV $a$. Then the energy constraint for each EV is redescribed as:

$$\pi_a^{\min} \leq \pi_a^0 + \sum_{t=1}^{n} \left( \beta_a^c p_{a,c}^t - \frac{p_{a,d}^t}{\beta_a^d} \right) \leq \pi_a^{\max}, \quad \forall n \in \mathcal{N}, \forall a \in \mathcal{A} \tag{3}$$

The deadlines and target energy to be achieved of all EVs are conveyed to the CA and each corresponding controller which function as inelastic requirements:

$$\pi_a^{N_a} = \pi_a^T, \quad \forall a \in \mathcal{A}$$

Similarly, the deadline constraint can be detailed as:

$$\pi_a^0 + \sum_{n \in \mathcal{N}_a} \left( \beta_a^c p_{a,c}^n - \frac{p_{a,d}^n}{\beta_a^d} \right) = \pi_a^T, \quad \forall a \in \mathcal{A} \tag{4}$$

For the charging station, overloading must be prevented to avoid voltage drop or even power outage in the distribution gird. Let $L^n$ denote the maximum allowable load of the charging station at time slot $n$. $L^n$ varies with time and is enforced by the power utility company according to the current and future expected operating conditions of the distribution grid. Assume $[L^n]_{\forall n \in \mathcal{N}}$ is known by the CA in advance, thus the load constraint for the charging station is formulated as:

$$0 \leq \sum_{a \in \mathcal{A}} \left( p_{a,c}^n - p_{a,d}^n \right) \leq L^n, \quad \forall n \in \mathcal{H} \tag{5}$$

### C. Battery Loss

Discharging process of EVs may cause additional harm to batteries and increases degradation effects of batteries, thus EVs' batteries cannot be used to discharge for optimal scheduling purpose unlimitedly and battery loss is taken into consideration to avoid immoderate discharging. Compared with expense/revenue incurred by charging/discharging, battery loss is relative small but unneglectable. Thus we quantify each EV's battery loss as a function of its discharging rate, $C_a(p_{a,d}^n)$, expressing EV owners' dissatisfaction due to discharging. Typically, the function $C_a(\cdot)$ is convex and non-decreasing [9]. As a special case, we assume $C_a(\cdot)$ to be a linear function, i.e., $C_a\left(p_{a,d}^n\right) \triangleq \alpha_a p_{a,d}^n$, where $\alpha_a$ is the weighting factor. Note that although linear function is taken as an example, the algorithm framework below will still apply as long as the convexity of $C_a(\cdot)$ is ensured.

## III. PROBLEM FORMULATION AND TRANSFORMATION

### A. Optimization Problem

The CA is assumed to be neutral and regulated, thus making profit through retailing electricity is not its primary concern. The objective of the CA is to induce all EVs' charging behavior in a way that achieves social fairness and minimizes social cost, i.e., the CA's cost of procuring electricity energy from the real-time balancing market plus all EVs' battery loss. Assume the electricity price of these $N$ time slots to be $\boldsymbol{\theta} \triangleq \left[\theta^1, ..., \theta^N\right]$, then combined with the constraints (1) (2) (3) (4) (5), the optimization problem that the CA aims to solve is formulated as follows:

**Primal Problem**:

$$\begin{aligned} \min_{\boldsymbol{\omega},\boldsymbol{p}} \quad & \mathcal{P}(\boldsymbol{\omega},\boldsymbol{p}) = \sum_{n\in\mathcal{N}} \theta^n \left[\sum_{a\in\mathcal{A}} \left(p_{a,c}^n - p_{a,d}^n\right)\right] \\ & + \sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}} C\left(p_{a,d}^n\right) \\ \text{s.t.} \quad & (1),(2),(3),(4),(5) \end{aligned} \tag{6}$$

*B. Primal-Dual Approach*

The load constraint (5) for the charging station couples all EVs together, which makes the primal problem (6) difficult to be separately solved on each EV. In order to accelerate computation speed, the load constraint (5) is relaxed by means of Lagrangian relaxation and then dual decomposition is introduced to decompose the primal problem (6) into a series of independent subproblems [10], each corresponding to scheduling on a single EV's charging behavior.

First of all, relax the load constraint (5) by introducing the Lagrangian multiplier vectors $\boldsymbol{\lambda} \triangleq [\lambda^n]_{n\in\mathcal{N}}$ and $\boldsymbol{\mu} \triangleq [\mu^n]_{n\in\mathcal{N}}$, where $\lambda^n \geq 0$, $\mu^n \geq 0$. Therefore, the Lagrangian is defined as:

$$\begin{aligned} & \mathcal{L}_1\left(\boldsymbol{p},\boldsymbol{\omega},\boldsymbol{\lambda},\boldsymbol{\mu}\right) \\ & = \sum_{n\in\mathcal{N}} \theta^n \left[\sum_{a\in\mathcal{A}} \left(p_{a,c}^n - p_{a,d}^n\right)\right] + \sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}} C\left(p_{a,d}^n\right) \\ & + \sum_{n\in\mathcal{N}} \lambda^n \left[\sum_{a\in\mathcal{A}} \left(p_{a,c}^n - p_{a,d}^n\right) - L^n\right] \\ & + \sum_{n\in N} \mu^n \left[-\sum_{a\in\mathcal{A}} \left(p_{a,c}^n - p_{a,d}^n\right)\right] \\ & = \sum_{a\in\mathcal{A}} \sum_{n\in\mathcal{N}} \left[\left(\theta^n + \lambda^n - \mu^n\right)\left(p_{a,c}^n - p_{a,d}^n\right) + \alpha_a p_{a,d}^n\right] \\ & - \sum_{n\in\mathcal{N}} \lambda^n L^n \end{aligned}$$

Note that the first term in the Lagrangian is separable in terms of EVs, thus the dual function which minimizes the Lagrangian over $\boldsymbol{p}$, $\boldsymbol{\omega}$ can be described as follows:

$$\mathcal{D}\left(\boldsymbol{\lambda},\boldsymbol{\mu}\right) = \min_{\boldsymbol{p},\boldsymbol{\omega}} \mathcal{L}_1\left(\boldsymbol{p},\boldsymbol{\omega},\boldsymbol{\lambda},\boldsymbol{\mu}\right) = \sum_{a\in\mathcal{A}} \mathcal{S}^a\left(\boldsymbol{\lambda},\boldsymbol{\mu}\right) - \sum_{n\in\mathcal{N}} \lambda^n L^n \tag{7}$$

where $\mathcal{S}^a\left(\boldsymbol{\lambda},\boldsymbol{\mu}\right)$ is defined as the $a^{th}$ subproblem to be solved on EV $a$. Note that each subproblem can be tackled independently which simplifies the whole solving process.

**Subproblem**:

$$\begin{aligned} \mathcal{S}^a\left(\boldsymbol{\lambda},\boldsymbol{\mu}\right) = \min_{\boldsymbol{\omega},\boldsymbol{p}} \quad & \sum_{n\in\mathcal{N}_a} \left[r^n\left(p_{a,c}^n - p_{a,d}^n\right) + \alpha_a p_{a,d}^n\right] \\ \text{s.t.} \quad & (1),(2),(3),(4) \end{aligned}$$

where $r^n \triangleq \theta^n + \lambda^n - \mu^n$. $r^n$ serves as the retail price broadcast by the CA to all controllers. The subproblem is an optimization problem solved by each controller to seek for an optimal charging strategy that minimizes the total charging cost based on the retail price information.

The corresponding dual problem is to maximize the dual function (7) over $\boldsymbol{\lambda},\boldsymbol{\mu}$:

**Dual Problem**:

$$\begin{aligned} \max_{\boldsymbol{\lambda},\boldsymbol{\mu}} \quad & \mathcal{D}\left(\boldsymbol{\lambda},\boldsymbol{\mu}\right) \\ \text{s.t.} \quad & \lambda^n \geq 0, \quad \mu^n \geq 0, \quad \forall n \in \mathcal{N}_a \end{aligned} \tag{8}$$

Note that there exist binary variables in the primal problem (6), thus only weak duality is guaranteed and there is a duality gap in between. That is to say, the maximum value of the dual problem (8) is actually a lower bound of the range of the primal problem (6).

Due to the non-differentiability of the dual function (7), the optimal Lagrangian multipliers can be achieved iteratively by means of subgradient method. The Lagrangian multipliers are adjusted in the subgradient direction of the dual function (7):

$$\begin{cases} \lambda^n(k+1) = \left[\lambda^n(k) + \gamma_\lambda g_\lambda^n(k)\right]^+ \\ \mu^n(k+1) = \left[\mu^n(k) + \gamma_\mu g_\mu^n(k)\right]^+ \end{cases}, \quad \forall n \in \mathcal{N} \tag{9}$$

where $k \in \mathbb{N}^+$ represents the index of iterations. $\boldsymbol{\gamma} \triangleq [\gamma_\lambda, \gamma_\mu]$ stands for the step size adjusting the convergence rate, where $\gamma_\lambda > 0$, $\gamma_\mu > 0$. $g_\lambda^n(k)$ and $g_\mu^n(k)$ are subgradients of the dual function with respect to $\lambda^n$ and $\mu^n$, respectively. Since the concavity of the dual function (7) is always guaranteed, subgradient method will converge to the optimal solution quickly with a proper step size.

## IV. Benders Decomposition

Every subproblem is a MILP to be solved on a single EV. Benders decomposition is introduced in this section to cope with the difficulty in solving MILP. Based on Benders decomposition, continuous variables and integer variables are solved separately and iteratively towards the optimal solution. As a result, computational complexity is greatly relieved. Here we begin with solving one subproblem which corresponds to scheduling on single EV's charging behavior. The notation $a$ is dropped for ease of presentation.

**Subproblem**:

$$\begin{aligned} \min_{\boldsymbol{\omega},\boldsymbol{p}} \quad & \Phi(\boldsymbol{p}) = \sum_{n\in\mathcal{N}_a} \left[r^n\left(p_c^n - p_d^n\right) + \alpha p_d^n\right] \\ \text{s.t.} \quad & (1),(2),(3),(4) \end{aligned} \tag{10}$$

MILP is usually hard to handle, thus we decompose the subproblem (10) into a master problem and a slave problem. Only integral constraints are considered in the master problem while other constraints are taken into account in the slave problem when integer variables are given. On the one hand, the master problem aims to find better integer variables and approach the optimal ones at every iteration through the shrink of its own feasible region. On the other hand, the slave problem is dominated by the given integer variables from the master problem and aims to search for the corresponding optimal continuous variables. Through analyzing the feasibility of the solution to the slave problem, we can add a feasibility constraint or an infeasibility constraints into the master problem at every iteration so as to diminish its feasible region [11]. As a result, the two correlative problems are formulated respectively as follows:

**Master Problem**:

$$\begin{aligned} \Phi^*_{lower} = \min_{\boldsymbol{\omega},\Phi} \quad & \Phi \\ \text{s.t.} \quad & (1) \\ & \text{feasibility constraints} \\ & \text{infeasibility constraints} \end{aligned} \tag{11}$$

**Slave Problem**:

$$\begin{aligned} \Phi^*_{upper} = \min_{\boldsymbol{p}} \quad & \Phi(\boldsymbol{p}) = \sum_{n\in\mathcal{N}_a} \left[r^n \left(p_c^n - p_d^n\right) + \alpha p_d^n\right] \\ \text{s.t.} \quad & (2), (3), (4) \\ & \text{for given } \hat{\boldsymbol{\omega}} \end{aligned}$$

where $\Phi^*_{lower}$ is a lower bound of $\Phi^*$, which is the optimal value of the subproblem (10), while $\Phi^*_{upper}$ is an upper bound of $\Phi^*$. $\hat{\boldsymbol{\omega}}$ in the slave problem is given by solving the master problem. The objective function of the master problem is a scalar of the same physical meaning as that of the subproblem (10), but the constraints are loosened. Thus through solving the master problem, we obtain a lower bound $\Phi^*_{lower}$. However, at every iteration feasibility or infeasibility constraints will be added into the master problem which help narrow the feasible region of integer variables, thus finally the optimal integer variables can be achieved. The optimal value of the slave problem $\Phi^*_{upper}$ is an upper bound because the given integer may not be optimal and all we obtain may only turn out to be a feasible solution. Consequently, $\Phi^*$ lies between $\Phi^*_{lower}$ and $\Phi^*_{upper}$, and through iterations between the master problem and the slave problem, the gap between $\Phi^*_{lower}$ and $\Phi^*_{upper}$ will quickly decrease. When the gap is small enough, $\Phi^*$ is achieved.

The whole process of Benders decomposition algorithm is detailed as follows:

**Step 1: Initialization**

Set the iteration index $k = 0$, $\Phi^*_{lower} = -\infty$ and $\Phi^*_{upper} = +\infty$. The feasibility and infeasibility constraints are set to null. Randomly choose the initial integer variables $\boldsymbol{\omega}(0)$ which is feasible, satisfying the constraint (1).

**Step 2: Solving Slave Problem (at the $k^{th}$ iteration)**

Given the integer variables $\boldsymbol{\omega}(k)$, the primal slave problem is formulated as below:

**Primal Slave Problem**:

$$\begin{aligned} \Phi^*_{upper} = \min_{\boldsymbol{p}} \quad & \Phi(\boldsymbol{p}) = \sum_{n\in\mathcal{N}_a} \left[r^n \left(p_c^n - p_d^n\right) + \alpha p_d^n\right] \\ \text{s.t.} \quad & (2), (3), (4) \\ & \text{for given } \boldsymbol{\omega}(\mathrm{k}) \end{aligned}$$

Define the Lagrangian for the primal slave problem by introducing $\boldsymbol{\delta_c} \triangleq [\delta_c^n]_{n\in\mathcal{N}_a}$, $\boldsymbol{\delta_d} \triangleq [\delta_d^n]_{n\in\mathcal{N}_a}$, $\boldsymbol{\zeta} \triangleq [\zeta^n]_{n\in\mathcal{N}_a}$, $\boldsymbol{\eta} \triangleq [\eta^n]_{n\in\mathcal{N}_a}$, and $\rho$ as Lagrangian multipliers for corresponding constraints, where $\delta_c^n, \delta_d^n, \zeta^n, \eta^n \geq 0$:

$$\begin{aligned} & \mathcal{L}_2\left(\boldsymbol{p}, \boldsymbol{\delta_c}, \boldsymbol{\delta_d}, \boldsymbol{\zeta}, \boldsymbol{\eta}, \rho\right) \\ & = \sum_{n\in\mathcal{N}_a} \left[r^n \left(p_c^n - p_d^n\right) + \alpha p_d^n\right] \\ & + \sum_{n\in\mathcal{N}_a} \left\{\delta_c^n \left[p_c^n - \omega_c^h(k)\, p_c^{\max}\right]\right\} \\ & + \sum_{n\in\mathcal{N}_a} \left\{\delta_d^n \left[p_d^n - \omega_d^n(k)\, p_d^{\max}\right]\right\} \\ & + \sum_{n\in\mathcal{N}_a} \left\{\zeta^n \left[\pi^0 + \sum_{t=1}^{n} \left(\beta^c p_c^t - \frac{p_d^t}{\beta^d}\right) - \pi^{\max}\right]\right\} \\ & + \sum_{n\in\mathcal{N}_a} \left\{\eta^n \left[\pi^{\min} - \pi^0 - \sum_{t=1}^{n} \left(\beta^c p_c^t - \frac{p_d^t}{\beta^d}\right)\right]\right\} \\ & + \rho \left[\pi^0 + \sum_{n\in\mathcal{N}_a} \left(\beta^c p_c^n - \frac{p_d^n}{\beta^d}\right) - \pi^T\right] \\ & = \sum_{n\in\mathcal{N}_a} \left[r^n + \delta_c^n + \beta^c \rho + \beta^c \sum_{t=n}^{N_a} \left(\zeta^n - \eta^n\right)\right] p_c^n \\ & + \sum_{n\in\mathcal{N}_a} \left[\alpha - r^n + \delta_d^n - \frac{\rho}{\beta^d} - \frac{1}{\beta^d} \sum_{t=n}^{N_a} \left(\zeta^n - \eta^n\right)\right] p_d^n \\ & + \sum_{n\in\mathcal{N}_a} \Big[-\omega_c^n(k)\, p_c^{\max} \delta_c^n - \omega_d^n(k)\, p_d^{\max} \delta_d^n \\ & + \left(\pi^0 - \pi^{\max}\right) \zeta^n + \left(\pi^{\min} - \pi^0\right) \eta^n\Big] + \left(\pi^0 - \pi^T\right) \rho \end{aligned}$$

Note that $p_c^n, p_d^n \geq 0$, we define the corresponding dual slave problem as follows:

**Dual Slave Problem**:

$$\begin{aligned} \max_{\substack{\boldsymbol{\delta_c}, \boldsymbol{\delta_d}, \\ \boldsymbol{\zeta}, \boldsymbol{\eta}, \rho}} \quad & \Psi\left(\boldsymbol{\delta_c}, \boldsymbol{\delta_d}, \boldsymbol{\zeta}, \boldsymbol{\eta}, \rho\right) = \sum_{n\in\mathcal{N}_a} \Big[-\omega_c^n(k)\, p_c^{\max} \delta_c^n \\ & -\omega_d^n(k)\, p_d^{\max} \delta_d^n + \left(\pi^0 - \pi^{\max}\right) \zeta^n \\ & + \left(\pi^{\min} - \pi^0\right) \eta^n\Big] + \left(\pi^0 - \pi^T\right) \rho \\ \text{s.t.} \quad & r^n + \delta_c^n + \beta^c \rho + \beta^c \sum_{t=n}^{N_a} \left(\zeta^n - \eta^n\right) \geq 0 \\ & -r^n + \delta_d^n + \alpha - \frac{\rho}{\beta^d} + \frac{1}{\beta^d} \sum_{t=n}^{N_a} \left(\zeta^n - \eta^n\right) \geq 0 \\ & \delta_c^n, \delta_d^n, \zeta^n, \eta^n \geq 0, \quad \forall n \in \mathcal{N}_a \end{aligned}$$

In the dual slave problem, $\omega_c^n(k), \omega_d^n(k)$ are given integer variables while $\delta_c^n, \delta_d^n, \zeta^n, \eta^n, \rho$ are continuous variables to be solved. Due to strong duality of linear program, there exists no duality gap which means the optimal value of the primal slave problem is identical to that of the dual slave problem, i.e., $\Phi^*_{upper} = \min_{\boldsymbol{p}} \Phi = \max_{\substack{\boldsymbol{\delta_c}, \boldsymbol{\delta_d}, \\ \boldsymbol{\zeta}, \boldsymbol{\eta}, \rho}} \Psi$. Therefore, it is equivalent to solve either problem and we will mainly focus on the dual form.

**Step 3: Solving Master Problem (at the $(k+1)^{th}$ iteration)**

According to the solution to the dual slave problem, integer variables can be improved by adding constraints to the master problem (11) as long as the given ones are not optimal.

1) If the dual slave problem is infeasible, then the subproblem (10) will have either no feasible solution or an unbounded solution. Therefore, there exists no solution with a precise physical meaning under this circumstance.
2) If the dual slave problem has a bounded solution $\boldsymbol{\delta_c}(k), \boldsymbol{\delta_d}(k), \boldsymbol{\zeta}(k), \boldsymbol{\eta}(k), \rho(k)$, then due to duality the primal slave problem is feasible. Define $\mathcal{I}$ as the set of iterations at which the solution to the dual slave problem is bounded. In this case the $k^{th}$ iteration should be added into $\mathcal{I}$, i.e., $\mathcal{I} = \mathcal{I} \cup k$. The optimal solution we obtain is a feasible solution to the subproblem (10), but better solutions may exist. As a result, the optimal value of the slave problem turns out to be an upper bound of $\Phi^*$ and thus $\Phi^*_{upper} = \min\left\{\Phi^*_{upper}, \Phi^*_{upper}(k)\right\}$. At the same time, a new feasibility constraint is generated and added into the master problem based on the bounded solution to the slave problem at the $k^{th}$ iteration, which helps raise the lower bound of $\Phi^*$. Thus the master problem is confined by a set of feasibility constraints described as below.
**feasibility constraints**:
$$\begin{aligned}\Phi \geq & \sum_{n\in\mathcal{N}_a}\left[-\omega_c^n(k+1)\,p_c^{\max}\delta_c^n(i) - \omega_d^n(k+1)\,p_d^{\max}\delta_d^n(i)\right.\\ &\left.+\left(\pi^0-\pi^{\max}\right)\zeta^n(i) + \left(\pi^{\min}-\pi^0\right)\eta^n(i)\right]\\ &+\left(\pi^0-\pi^T\right)\rho(i), \quad \forall i\in\mathcal{I}\end{aligned}$$
3) If the dual slave problem has an unbounded solution, it means the corresponding primal slave problem is infeasible with the given integer variables $\boldsymbol{\omega}(k)$. Therefore, these integer variables should be ruled out of our search region. Define $\mathcal{J}$ as the set of iterations at which the solution to the dual slave problem is unbounded. Under this circumstance the $k^{th}$ iteration should be added into $\mathcal{J}$, i.e., $\mathcal{J} = \mathcal{J} \cup k$. The direction vector of the unbounded solution to the dual slave problem $\boldsymbol{\delta_c}(k), \boldsymbol{\delta_d}(k), \boldsymbol{\zeta}(k), \boldsymbol{\eta}(k), \rho(k)$ can be easily obtained using simplex method. Based on the direction vector of the unbounded solution at the $k^{th}$ iteration, a new infeasibility constraint is generated and added into the master problem to exclude the given integer variables. Thus the master problem is confined by a set of infeasibility constraints described as below.
**infeasibility constraints**:
$$\begin{aligned}0 \geq & \sum_{n\in\mathcal{N}_a}\left[-\omega_c^n(k+1)\,p_c^{\max}\delta_c^n(j) - \omega_d^n(k+1)\,p_d^{\max}\delta_d^n(j)\right.\\ &\left.+\left(\pi^0-\pi^{\max}\right)\zeta^n(j) + \left(\pi^{\min}-\pi^0\right)\eta^n(j)\right]\\ &+\left(\pi^0-\pi^T\right)\rho(j), \quad \forall j\in\mathcal{J}\end{aligned}$$

Dynamically, there is always one new constraint added into the master problem at every iteration as long as the optimal solution to the subproblem (10) exists and hasn't been achieved. Through solving the newly modified master problem with tighter constraints, the lower bound $\Phi^*_{lower}$ is sure to be upgraded. Compare the $\Phi^*_{upper}$ and $\Phi^*_{lower}$ we have obtained from the slave problem and master problem respectively, if $\left|\Phi^*_{upper} - \Phi^*_{lower}\right| \leq \varepsilon$, where $\varepsilon$ is a sufficiently small threshold with positive value, then the upper bound and lower bound converge to achieve the optimal solution and the whole process ceases. Otherwise, the algorithm goes back to step 2 and repeat the iteration.

## V. Distributed Implementation

The V2V charging mechanism can be implemented in a distributed manner, namely through iterations between the CA level and EV level. At the CA level, Lagrangian multipliers are updated according to feedbacks from all EVs. At the EV level, every controller optimizes its EV's charging strategy based on retail prices set by the CA. In a sense, Lagrangian multipliers serve as an coordination signal which aligns individual welfare with social welfare. First of all, the CA sets the initial Lagrangian multipliers, e.g., $\lambda^n(0) = 0, \mu^n(0) = 0, \forall n \in \mathcal{N}$. During iteration, given $\boldsymbol{\lambda}(k), \boldsymbol{\mu}(k)$, the CA broadcasts the retail price $[r^n(k)]_{\forall n\in\mathcal{N}}$ to all controllers, where $r^n(k) \triangleq \theta^n(k) + \lambda^n(k) - \mu^n(k)$. Every controller solves its own MILP (10) independently based on the retail price to obtain an optimal charging strategy. Then the charging strategy $[\boldsymbol{\omega}(k), \boldsymbol{p}(k)]$ of each EV is reported back to the CA. Upon receiving all feedbacks, the CA updates the Lagrangian multipliers according to the rule (9) and makes all controllers informed. Note that given $\boldsymbol{\lambda}(k), \boldsymbol{\mu}(k)$, the update rule of the Lagrangian multipliers at every time slot can be easily derived based on the subgradients of the dual function (7):

$$\left\{\begin{array}{l}\lambda^n(k+1) = \left\{\lambda^n(k) + \gamma_\lambda\left[\sum\limits_{a\in\mathcal{A}}\left(p^n_{a,c}(k) - p^n_{a,d}(k)\right) - L^n\right]\right\}^+\\ \mu^n(k+1) = \left\{\mu^n(k) + \gamma_\mu\left[-\sum\limits_{a\in\mathcal{A}}\left(p^n_{a,c}(k) - p^n_{a,d}(k)\right)\right]\right\}^+\end{array}\right.$$

The iteration between the CA and EVs will continue until an equilibrium is reached. Then the optimal retail price $\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*$ is finally released by the CA to coordinate all EVs' charging behavior.

## VI. Numerical Results

For ease of illustration, we take a charging station with 3 EVs into consideration, which can be extended to a sufficiently large-scale charging station due to distributed implementation and parallel computation. EVs are assumed to be identical with parameters listed in TABLE I. The maximum allowable loads and hourly-based real-time prices of the corresponding time slots for the charging station are shown in TABLE II.

Fig. 1 shows the convergence of retail prices. The retail prices of time slot 2 and 4 remain the same as the original real-time prices because load of either time slot satisfies the corresponding load constraint without coordination. However, the retail prices of time slot 1 and 3 are iteratively adjusted due to conflict of the load constraint, and ultimately converge to the optimal prices.

As long as the convergence of retail prices is reached, the optimal scheduling on EVs' charging behavior can be easily obtained, which is shown in Fig. 2. With sufficient time to be charged to the target energy level, EV 1 tends to discharge and get paid at time slot 1 when the retail price is high. Then it choose to charge with the maximum charging rate at time

TABLE I
PARAMETER SETTINGS FOR EVS

| Parameter \ EV | 1 | 2 | 3 |
|---|---|---|---|
| $\alpha_a$ | 0.01 | 0.01 | 0.01 |
| $p_{a,c}^{\max}(kW)$ | 5 | 5 | 5 |
| $p_{a,d}^{\max}(kW)$ | 3 | 3 | 3 |
| $\beta_a^c$ | 0.99 | 0.99 | 0.99 |
| $\beta_a^d$ | 0.99 | 0.99 | 0.99 |
| $\pi_a^0(kWh)$ | 16 | 12 | 12 |
| $\pi_a^{\min}(kWh)$ | 2 | 2 | 2 |
| $\pi_a^{\max}(kWh)$ | 24 | 24 | 24 |
| $\pi_a^T(kWh)$ | 22.8 | 22.8 | 22.8 |
| $N_a(h)$ | 4 | 3 | 3 |

TABLE II
PARAMETER SETTINGS FOR CHARGING STATION

| Parameter \ Time slot | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $L(kW)$ | 9.9 | 10.8 | 13.5 | 14.2 |
| $\theta(cent/kWh)$ | 9.5 | 8.3 | 6.2 | 5.3 |

slot 4 because the retail price is the lowest. The retail prices at time slot 2 and 3 are almost equal, thus EV 1 will adjust its charging rate properly to avoid overloading of the charging station. Other EVs follow the similar principles to be charged, but there are a few special circumstances. For example, neither EV 2 nor EV 3 has the choice to discharge due to lack of time. Fig. 2(d) indicates the total charging load of the charging station. We can find that the load constraint for the charging station is satisfied. As a result, it's obviously observed that price information is very useful to regulate charging demand at every time slot.

## VII. CONCLUSION

In this paper, optimal scheduling on EVs' charging behavior in a charging station is investigated. Based on the new concept

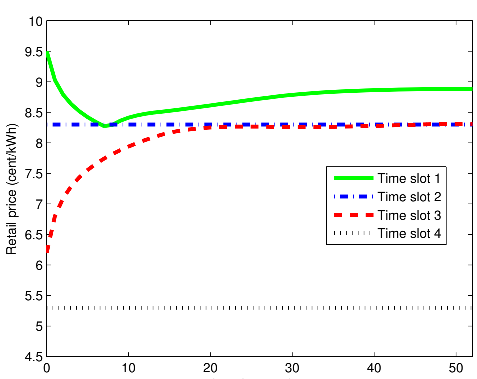


Fig. 1. Convergence of retail prices

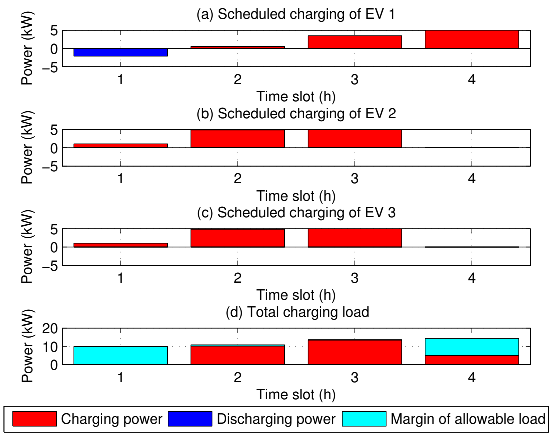


Fig. 2. Optimal scheduling on EVs' charging behavior

V2V, we formulate the charging coordination problem as a MILP with multiple constraints. Since the load constraint for the charging station couples all EVs' charging behavior together, dual decomposition is introduced to decompose the primal problem into a set of subproblems. Each subproblem is still a MILP to be solved on each EV. Benders decomposition is then applied to efficiently solve MILP. Distributed implementation of the proposed approach shows its adequacy for large-scale charging stations which can hold a great quantity of EVs. Numerical results validate our theoretical analysis.

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