2015 European Control Conference (ECC)
July 15-17, 2015. Linz, Austria

Efficient Battery Charging Schedule of Battery-Swapping Station for Electric Buses

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Abstract— **This paper investigates the battery charging schedule problem of a battery-swapping station for electric buses (EB). An EB assignment policy is proposed such that there is a one-to-one correlation between EBs and batteries. By this means, the battery charging schedule problem aiming to minimize the total cost of the battery-swapping station is formulated as a constrained convex program with both spatially and temporally coupled constraints. Based on dual decomposition and our proposed EB assignment policy, the battery charging schedule problem can be decomposed into a series of local subproblems, which can be independently tackled. Furthermore, a fast search method in combination with binary search is put forward to deal with subproblems. Therefore, the battery charging schedule problem can be solved efficiently in a distributed manner. Numerical results confirm the validity of our proposed approach.**

I. INTRODUCTION

The conventional power system is under transformation from a centralized design towards a decentralized one, evolving into an intelligent power system, known as the smart grid [1], [2]. There are many pressing issues that push the pace of such a transformation, e.g., the ever-increasing power load, the aging infrastructures, the shortage of fossil fuels and the emission of green house gases, etc. The smart grid, in combination with communication technologies, has the advantage of generation diversification and demand response [3], [4], which makes the balance between power supply and demand more stable.

Accompanied by the rapid advance of the smart grid, electric vehicles (EV), characterized to be zero-emission, are gradually penetrating into the car market. EVs like Tesla even lead the forefront of the latest vehicular technology. With the increase in the penetration of EVs, the corresponding charging load will bring great burden on the distribution grid [5]. Fortunately, batteries in EVs provide the flexibility of peak clipping and valley filling. Under different circumstances, EVs can function as either load (normal charging) or storage (V2G).

As another form of EVs, EBs are faster to be put into widespread use as they are usually under centralized control of a battery-swapping station where all batteries are charged to the full state-of-charge (SOC) and prepared to be swapped. In addition, the arrival process of EBs is more periodical compared with private EVs or electric taxies, which brings much convenience to the management of battery charging. With the scale expansion in the immediate future, a battery-swapping station is destined to be a tremendous load source. Therefore, it is of great significance to coordinatively schedule the charging of all batteries in a battery-swapping station so as to relax the stress on the distribution grid. A lot of literatures have studied the related issues on battery-swapping stations [6]–[10]. In [6], a new business model of a microgrid-based battery-swapping station is proposed. Meanwhile, a new optimal dispatching strategy of a microgrid containing battery-swapping stations, wind generators, photovoltaic systems, fuel cells, micro turbines and diesel generators is given, considering battery and charger constraints. [7] proposes the use of the batteries in a battery-swapping station as a countermeasure for surplus electricity from PVs. [8] discusses the planning of the location and sizing of a battery-swapping station, which has a great impact on the popularisation of EVs and the security of the distribution grid. Similarly, [9] describes a model for identifying the optimal geographic locations for battery-swapping stations and investigates how to best stage the roll-out of battery-swapping stations in Australia over an extended time period. Moreover, charging infrastructures are also developing fast to support the construction of battery-swapping stations, e.g., the functionality of a commercialized fast charger for a lithium-ion electric vehicle propulsion battery is presented in [10]. However, to the best of our knowledge, there are few literatures researching the schedule issues of battery-swapping stations for EBs.

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In this paper, we focus on the battery charging schedule of a battery-swapping station for EBs, which will bring economic benefits to the battery-swapping station and meanwhile maintain the stability of the distribution grid. Assume all batteries of the battery-swapping station are placed in the charging boxes to be charged. Station operators have to appropriately schedule the charging of all batteries in order to prepare for the battery swap requests from EBs and meanwhile achieve the total cost minimization, which makes each battery temporally coupled. Moreover, since there always exists a maximum threshold for the total load of the battery-swapping station, all batteries are spatially coupled. Therefore, the battery charging schedule problem aiming to minimize the total cost of the battery-swapping station is unable to be directly tackled.

The main contributions of this paper are summarized as follows:

1) An EB assignment policy is proposed such that the battery charging schedule problem can be formulated as a constrained convex program.

2) To deal with spatially coupled constraints, dual decomposition is introduced, which, in combination with our proposed EB assignment policy, can decompose the battery charging schedule problem into separable subproblems.
3) To solve each subproblem, a fast search method, which is considered more efficient, is proposed instead of standard convex optimization techniques.

The remainder of this paper is organized as follows. We describe the system model in Sec. II. Dual decomposition is introduced in Sec. III, followed by the fast search method we put forward in Sec. IV. Then the distributed implementation of our proposed approach is demonstrated in Sec. V. At last the related numerical results are presented in Sec. VI, while the conclusion is drawn in Sec. VII.

II. System model

A. Scenario

Consider a battery-swapping station which is located in a bus transportation hub. All batteries are placed in the charging boxes with three states: idle, charging or fully-charged. A group of EBs are under management with either their origin station or terminal station right in the bus transportation hub. When in operation, the bus transportation hub sends out EBs according to a time schedule to satisfy passenger demand. Simultaneously, there are EBs coming from other stations to the bus transportation hub. In order to afford a trip, an EB must keep enough electricity energy in its battery. When battery energy runs short, the EB has to move to the battery-swapping station to swap its current battery for a fully-charged one. Since every EB must be prepared for the next trip upon its arrival at the bus transportation hub, some EBs even have to immediately leave, e.g., circling buses, we assume those EBs short of battery energy must be provided with a fully-charged battery immediately when they arrive at the battery-swapping station.

When a coming EB detects that its battery energy is not sufficient for the next trip, it sends a signal to remotely notify the battery-swapping station of its request for battery swap. Through telecommunication, the battery-swapping station can easily collect messages from all those EBs short of battery energy. As every EB is always required to follow its own time schedule and take a binding route, the arrival time of EBs is predictable and can be estimated from historical data. In this sense, we suppose the arrival process of EBs at the battery-swapping station is a given deterministic process, which is known by station operators in advance. Let $\mathbb{D}$ denote the set of D EBs that are coming to the battery-swapping station for battery swap. We consider a discrete time horizon $\mathbb{T} := \{1, 2, \ldots, T\}$ which covers all the arrivals of D EBs. Set the duration of a time slot $\Delta t = 1$ and thus it will be omitted in the discussion below. Assume for simplicity that the arrival of every EB removes a fully-charged battery and returns a low-SOC battery which will take the place of the removed battery in the charging box, thus the number of batteries in the battery-swapping station keeps constant and is assumed to be B. Typically, the low-SOC batteries EBs return are proximately exhausted, thus we suppose that the SOC of every returned battery is below a sufficiently small threshold, such that the return of a high-SOC battery will not happen. For ease of illustration, we assume that the SOC upon arrival of every EB is given through communication and estimation. All the B batteries compose the battery set $\mathbb{B}$. Note that the battery in charging box b is denoted as battery b. Therefore, we do not refer to a certain battery, but a series of batteries charged in charging box b in turn when we talk about battery b.

To be more practical, all batteries and EBs are assumed to be unified and identical. This makes sense in that standardized infrastructures can bring much convenience to battery-charging management. In order to keep the whole bus transportation system running, all EBs' demands of battery swap ought to be fulfilled. In this paper, we suppose that a fully-charged battery refers to a battery with its SOC no lower than a threshold, i.e., π_{ful}.

B. EB assignment policy

Based on the assumption that the arrival process of EBs at the battery-swapping station is a given deterministic process, the sequence of EBs' arrivals is known, thus we propose an EB assignment policy to assign every EB to a certain battery in the battery-swapping station, such that for every EB, there is always a battery preparing to fulfill its battery swap request. The key point of the EB assignment policy lies in that the first coming EB is assigned to the battery with the highest SOC, labeled as $b = 1$, the second coming EB is assigned to the battery with the second highest SOC, labeled as $b = 2$, and so on. After a cycle, the $(B+1)^{th}$ coming EB is again assigned to battery 1, and so are the rest of the coming EBs. The same operations will be carried out in the following cycles until no EB is expected to come. For those EBs that arrive at the battery-swapping station simultaneously, we rank them based on random sort. The assignment ensures that there is a one-to-one correlation between EBs and batteries.

C. Problem formulation

Denote the set of the EBs assigned to battery b as $\mathbb{D}_b$, then $\mathbb{D} = \bigsqcup_{b\in\mathbb{B}} \mathbb{D}_b$. For every EB $d \in \mathbb{D}_b$, its assigned battery b has a charging cycle $\mathbb{T}_{b,d}$ from $T^s_{b,d}$ to $T^e_{b,d}$, where $T^s_{b,d}$ denotes the first time slot after EB $(d-1) \in \mathbb{D}_b$ comes for battery swap and $T^e_{b,d}$ denotes the time slot when EB d comes for battery swap. Correspondingly, the SOC of battery b at any time slot $t \in \mathbb{T}_{b,d}$ is denoted as $\pi^t_{b,d}$.

Let p^t_b denote the normalized charging power of battery b at time slot t with respect to the unified battery capacity, which is the key control variable. Meanwhile, define $\boldsymbol{p} \triangleq [p^t_b]_{t\in\mathbb{T},b\in\mathbb{B}}$, $\boldsymbol{p}_b \triangleq [p^t_b]_{t\in\mathbb{T}}$ and $\boldsymbol{p}_{b,d} \triangleq [p^t_b]_{t\in\mathbb{T}_{b,d}}$. For now, we ignore the possibility of vehicle-to-grid (V2G) service, then for each battery $b \in \mathbb{B}$, its charging power is bounded as

$$0 \le p^t_b \le p_{\max}, \quad \forall t \in \mathbb{T}, \forall b \in \mathbb{B} \tag{1}$$

where $p_{\max}$ is the maximum charging power of a battery.

In addition, considering the limits of transmission line capacity and transformer capacity, overloading must be averted. For the battery-swapping station, its total load cannot exceed a maximum threshold, otherwise damage will be caused to electric equipment. Thus, a load constraint is introduced:

$$\sum_{b\in\mathbb{B}} p_b^t \leq L, \quad \forall t\in\mathbb{T} \tag{2}$$

where L is the maximum allowable load of the battery-swapping station to ensure safety of the power system.

As far as the charging process of batteries is concerned, a linear state equation is applied to depict the dynamic course of a battery's SOC:

$$\pi_{b,d}^{t+1} = \pi_{b,d}^t + \beta_b p_b^t, \quad \forall d\in\mathbb{D}_b, \forall b\in\mathbb{B}$$

where $\beta_b \in (0,1]$ denotes the charging efficiency of battery b. Then the final SOC of a battery when it is handed over to the corresponding EB can be expressed as

$$\pi_{b,d}^e = \pi_{b,d}^s + \sum_{t\in\mathbb{T}_{b,d}} \beta_b p_b^t, \quad \forall d\in\mathbb{D}_b, \forall b\in\mathbb{B}$$

where $\pi_{b,d}^e$ and $\pi_{b,d}^s$ represent the final SOC and the initial SOC of battery b corresponding to EB d, respectively. Since each battery must be able to sustain the running of an EB for a sufficiently long distance to avoid frequent battery swaps, only fully-charged batteries are permitted to be provided to EBs, which requires $\pi_{b,d}^e \geq \pi_{ful}$, i.e.,

$$\pi_{b,d}^s + \sum_{t\in\mathbb{T}_{b,d}} \beta_b p_b^t \geq \pi_{ful}, \quad \forall d\in\mathbb{D}_b, \forall b\in\mathbb{B} \tag{3}$$

Based on the knowledge that charging a battery with different charging power causes varying degrees of degradation effect to the battery, typically the larger the charging power, the more serious the degradation effect caused, battery loss is taken into consideration as an impact factor for battery charging schedule to avert overlarge charging power. In this paper, battery loss is quantified as a convex and non-decreasing function of charging power, i.e., $C\left(p_b^t\right)$. The convexity of $C(\cdot)$ implies non-decreasing marginal battery loss, i.e., $C''(\cdot) \geq 0$, which will offset the economic benefit brought by overlarge charging power.

Suppose the battery-swapping station purchases electricity energy from the real-time market. Since expected real-time prices can be estimated from historical data, we assume the real-time prices are known in advance for simplicity. Let θ^t denote the real-time price at time slot t. Therefore, the cost of battery b at time slot t comprises of the charging expense plus the resulting battery loss: $F_b^t\left(p_b^t\right) = p_b^t\theta^t + \alpha C\left(p_b^t\right)$, where $\alpha \geq 0$ is a weight.

The battery charging schedule problem for the battery-swapping station is to control the charging power of all batteries over all time slots, with the aim of minimizing the total cost, meanwhile subject to (1), (2) and (3):

Primal problem:

$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \sum_{b\in\mathbb{B}}\sum_{t\in\mathbb{T}} F_b^t\left(p_b^t\right) \\ \text{s.t.} \quad & (1)\ (2)\ (3) \end{aligned} \tag{4}$$

III. Dual decomposition

Since (2) couples all batteries together, which makes it unable to separately schedule each battery's charging behavior, dual decomposition is applied to decompose (4) into a series of interim problems, each corresponding to a single-battery charging schedule.

First of all, introduce the Lagrangian multiplier vector $\boldsymbol{\lambda} \triangleq \left[\lambda^t\right]_{t\in\mathbb{T}}$ with $\lambda^t \geq 0$ to relax (2), thereby forming the Lagrangian for (4):

$$\begin{aligned} &\mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{p}) \\ &= \sum_{b\in\mathbb{B}}\sum_{t\in\mathbb{T}} F_b^t\left(p_b^t\right) + \sum_{t\in\mathbb{T}}\left(\sum_{b\in\mathbb{B}} p_b^t - L\right)\lambda^t \\ &= \sum_{b\in\mathbb{B}}\sum_{t\in\mathbb{T}}\left[p_b^t\left(\theta^t+\lambda^t\right) + \alpha C\left(p_b^t\right)\right] - \Gamma(\boldsymbol{\lambda}) \end{aligned}$$

where $\Gamma(\boldsymbol{\lambda}) \triangleq \sum_{t\in\mathbb{T}} L\lambda^t$.

The corresponding dual function is given as

$$\begin{aligned} D(\boldsymbol{\lambda}) &= \min_{\boldsymbol{p}} \mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{p}) \\ &= \sum_{b\in\mathbb{B}} S_b(\boldsymbol{\lambda}) - \Gamma(\boldsymbol{\lambda}) \end{aligned} \tag{5}$$

where $S_b(\boldsymbol{\lambda})$ is an interim problem and can be further decomposed as

$$\begin{aligned} S_b(\boldsymbol{\lambda}) &= \min_{\boldsymbol{p}_b} \sum_{t\in\mathbb{T}}\left[p_b^t\left(\theta^t+\lambda^t\right) + \alpha C\left(p_b^t\right)\right] \\ &= \min_{\boldsymbol{p}_b} \sum_{d\in\mathbb{D}_b}\sum_{t\in\mathbb{T}_{b,d}}\left[p_b^t\left(\theta^t+\lambda^t\right) + \alpha C\left(p_b^t\right)\right] \\ &= \sum_{d\in\mathbb{D}_b}\left\{\min_{\boldsymbol{p}_{b,d}} \sum_{t\in\mathbb{T}_{b,d}}\left[p_b^t\left(\theta^t+\lambda^t\right) + \alpha C\left(p_b^t\right)\right]\right\} \\ &= \sum_{d\in\mathbb{D}_b} R_{b,d}\left(\boldsymbol{\lambda}_{b,d}\right) \end{aligned}$$

where $\boldsymbol{\lambda}_{b,d} \triangleq \left[\lambda^t\right]_{t\in\mathbb{T}_{b,d}}$ and $R_{b,d}\left(\boldsymbol{\lambda}_{b,d}\right)$ is a local subproblem corresponding to the schedule of charging battery b to prepare for the arrival of EB d. Note that both $\sum_{b\in\mathbb{B}} S_b\left(\boldsymbol{\lambda}_b\right)$ and $\sum_{d\in\mathbb{D}_b} R_{b,d}\left(\boldsymbol{\lambda}_{b,d}\right)$ is separable since batteries attached to one charging box are charged in sequence independently.

For ease of presentation, we drop the notations b and d, thus every single subproblem $R(\boldsymbol{\lambda})$ is given as

$$\begin{aligned} R(\boldsymbol{\lambda}) \triangleq \min_{\boldsymbol{p}} \quad & \sum_{t\in\mathbb{T}}\left[p^t\left(\theta^t+\lambda^t\right) + \alpha C\left(p^t\right)\right] \\ \text{s.t.} \quad & \pi^s + \sum_{t\in\mathbb{T}} \beta p^t \geq \pi_{ful} \\ & 0 \leq p^t \leq p_{\max}, \quad \forall t\in\mathbb{T} \end{aligned} \tag{6}$$

The dual problem is to maximize the dual function (5) over the Lagrangian multiplier vector $\boldsymbol{\lambda}$:

Dual problem:

$$\begin{aligned} \max_{\boldsymbol{\lambda}} \quad & D(\boldsymbol{\lambda}) \\ \text{s.t.} \quad & \lambda^t \geq 0, \quad \forall t\in\mathbb{T} \end{aligned} \tag{7}$$

Note that we mainly focus on the occasions when the primal problem (4) is feasible (An infeasible primal problem means that its corresponding dual problem has an unbounded optimal solution). Hence, according to dual theory, the maximum value of the dual problem (7) gives a lower bound on the range of the primal problem (4) and coincides with its minimum value since there is no duality gap, considering the concavity of the primal problem (4). Therefore, as long as $\boldsymbol{\lambda}^*$, the optimal solution to the dual problem (7), is given, we can obtain $\boldsymbol{p}^*$, the optimal solution to the primal problem (4), through solving every local subproblem (6).

Due to strong duality, solving the primal problem (4) is equivalent to solving the dual problem (7). Hence, we focus on the latter. On account of the differentiability of the dual function (5), the optimal solution to the dual problem (7) can be obtained iteratively by means of gradient descent method. The Lagrangian multiplier vector $\boldsymbol{\lambda}$ is updated in the gradient direction of the dual function (5):

$$\lambda_{k+1}^t = \left[\lambda_k^t + \nu \frac{\partial D(\boldsymbol{\lambda}_k)}{\partial \lambda_k^t}\right]^+, \quad \forall t \in \mathbb{T} \tag{8}$$

where $\nu > 0$ stands for the step size which modulates the convergence rate and $k \in \mathbb{N}^+$ denotes the iteration index. Since the concavity of the dual function (5) always holds, convergence towards the optimal solution will be guaranteed if a sufficiently small step size is chosen, such that the gradient of the dual function (5) satisfies the Lipchitz continuity condition.

IV. Fast Search Method to Subproblem

The subproblem (6) is a constrained convex program with only temporally coupled constraints, which can be tackled by means of some standard convex optimization techniques, e.g., interior point methods. However, conventional approaches suffer the same weakness of high computational complexity. In order to accelerate computation speed, we propose a fast search method based on binary search to efficiently solve the subproblem (6).

Firstly, the Karush-Kuhn-Tucker (KKT) conditions for the subproblem (6) are given as

Primal feasibility

$$(\sigma) \quad \pi_{ful} - \pi^s - \sum_{t\in\mathbb{T}} \beta p^t \le 0 \tag{9}$$

$$(\xi^t) \quad p^t - p_{\max} \le 0, \quad \forall t \in \mathbb{T} \tag{10}$$

$$(\eta^t) \quad 0 - p^t \le 0, \quad \forall t \in \mathbb{T} \tag{11}$$

Stationarity

$$\theta^t + \lambda^t + \alpha C'(p^t) - \beta\sigma + \xi^t - \eta^t = 0, \quad \forall t \in \mathbb{T} \tag{12}$$

Dual feasibility

$$\sigma \ge 0 \tag{13}$$

$$\xi^t, \eta^t \ge 0, \quad \forall t \in \mathbb{T} \tag{14}$$

Complementary slackness

$$\sigma\left(\pi_{ful} - \pi^s - \sum_{t\in\mathbb{T}} \beta p^t\right) = 0 \tag{15}$$

$$\xi^t(p^t - p_{\max}) = 0, \quad \forall t \in \mathbb{T} \tag{16}$$

$$\eta^t(0 - p^t) = 0, \quad \forall t \in \mathbb{T} \tag{17}$$

where σ, ξ^t, η^t are the KKT multipliers corresponding to the four constraints (9)-(11).

Consider the following form of solution which is a function of σ:

$$\text{for } \forall t \in \mathbb{T},\quad \begin{cases} p^t(\sigma) = \left[(C')^{-1}\left(\frac{\beta\sigma - \theta^t - \lambda^t}{\alpha}\right)\right]_0^{p_{\max}} \\ \xi^t(\sigma) = \left[\beta\sigma - \theta^t - \lambda^t - \alpha C'(p_{\max})\right]_0 \\ \eta^t(\sigma) = \left[\theta^t - \beta\sigma + \lambda^t + \alpha C'(0)\right]_0 \end{cases} \tag{18}$$

where $[x]_0^y = \max\{\min\{x, y\}, 0\}$ and $[x]_0 = \max\{x, 0\}$. Obviously, for any $\sigma \ge 0$, the solution (18) satisfies (10), (11), (13) and (14).

Theorem 1: For any $\sigma \ge 0$, the solution (18) satisfies (12), (16) and (17).

The detailed proof is omitted due to page limit. Synthetically analyzing all cases easily leads to Theorem 1.

Hence, in order to make the solution (18) meet the KKT conditions, thereby solving the subproblem (6), we only need to search for the proper $\sigma \ge 0$, such that (9) and (15) are both satisfied.

Define

$$h(\sigma) \triangleq \sum_{t\in\mathbb{T}} \beta p^t(\sigma) = \sum_{t\in\mathbb{T}} \beta \left[(C')^{-1}\left(\frac{\beta\sigma - \theta^t - \lambda^t}{\alpha}\right)\right]_0^{p_{\max}}$$

Then we need to find $\sigma^* \ge 0$ such that

$$\begin{cases} h(\sigma) \ge \pi_{ful} - \pi^s \\ \sigma\left[\pi_{ful} - \pi^s - h(\sigma)\right] = 0 \end{cases} \tag{19}$$

Note that from (18), $p^t(\sigma)$ can be expressed as follows:

$$p^t(\sigma) = \begin{cases} 0, \quad \sigma \le \frac{\theta^t + \lambda^t + \alpha C'(0)}{\beta} \\ (C')^{-1}\left(\frac{\beta\sigma - \theta^t - \lambda^t}{\alpha}\right), \quad \text{otherwise} \\ p_{\max}, \quad \sigma \ge \frac{\theta^t + \lambda^t + \alpha C'(p_{\max})}{\beta} \end{cases}$$

Obviously, each $p^t(\sigma)$ is piecewise and monotonically non-decreasing with σ. Considering that the sum of such functions is linear combination, $h(\sigma)$ will still preserve the same property.

Take the three cases below into account:

1) If $\sum_{t\in\mathbb{T}} \beta p_{\max} < \pi_{ful} - \pi^s$, which means (19) cannot be satisfied, then there exists no feasible solution to the subproblem (6).
2) If $h(0) \ge \pi_{ful} - \pi^s$, then $\sigma^* = 0$, thereby leading to the optimal solution as
$$p^{t*} = \left[(C')^{-1}\left(-\frac{\theta^t + \lambda^t}{\alpha}\right)\right]_0^{p_{\max}}$$
3) If $h(0) < \pi_{ful} - \pi^s \le h(\sigma^*)$, then $\sigma^* > 0$, due to the monotonicity of $h(\sigma)$. Hence, σ^* must satisfy
$$h(\sigma^*) = \pi_{ful} - \pi^s, \quad \sigma^* > 0$$

In allusion to the third case, we search for σ^* in the following way:

Since there are a number T of $p^t(\sigma)$, $t = 1, 2, \ldots, T$, and each $p^t(\sigma)$ has two breakpoints, there are $2T$ breakpoints in total, $\frac{\theta^t + \lambda^t + \alpha C'(0)}{\beta}$ and $\frac{\theta^t + \lambda^t + \alpha C'(p_{\max})}{\beta}$, $t = 1, 2, \ldots, T$, for the piecewise function $h(\sigma)$. Let $\sigma_1, \sigma_2, \ldots, \sigma_N$ denote all the positive and non-repeated breakpoints among the $2T$ breakpoints, where $N \leq 2T$ and $\sigma_0 = 0 < \sigma_1 < \sigma_2 < \cdots < \sigma_N$. Then for $\sigma = \sigma_0$, $h(\sigma) = h(0)$, and for $\sigma \geq \sigma_N$, $h(\sigma) = \sum_{t\in\mathbb{T}} \beta p_{\max}$. We propose **Algorithm 1** to locate σ^* between two breakpoints. Then since the search range of σ^* is greatly reduced within two breakpoints, we can apply binary search to obtain σ^*. Once σ^* is obtained, the optimal solution to the subproblem (6), i.e., $[p^{t*}]_{t\in\mathbb{T}}$, will be achieved. The algorithm is considered efficient in that its number of elementary computational steps is bounded by a polynomial in the size of the time horizon T.

Algorithm 1: Fast Search Method to Subproblem

if $\sum_{t\in\mathbb{T}} \beta p_{\max} < \pi_{ful} - \pi^s$ **then**
  return no feasible solution;
else if $h(0) \geq \pi_{ful} - \pi^s$ **then**
  return $\sigma^* = 0$;
else
  $l \leftarrow 0$, $r \leftarrow N$;
  while $r - l > 1$ **do**
    $n \leftarrow \left\lfloor \frac{1}{2}(l+r) \right\rfloor$, where $\lfloor\cdot\rfloor$ is the floor function;
    $R \leftarrow h(\sigma_n)$;
    if $R == \pi_{ful} - \pi^s$ **then**
      return $\sigma^* = \sigma_n$;
    else if $R < \pi_{ful} - \pi^s$ **then**
      $l \leftarrow n$;
    else
      $r \leftarrow n$;
    end if
  end while
  $\mathbb{T}_1 \leftarrow \left\{ t \middle| \frac{\theta^t + \lambda^t + \alpha C'(0)}{\beta} \geq \sigma_r \right\}$, $\mathbb{T}_2 \leftarrow \left\{ t \middle| \frac{\theta^t + \lambda^t + \alpha C'(p_{\max})}{\beta} \leq \sigma_l \right\}$, $\mathbb{T}_3 \leftarrow \mathbb{T} - \mathbb{T}_1 - \mathbb{T}_2$;
  return $\sigma^* = \arg\left\{ \sum_{t\in\mathbb{T}_2} \beta p_{\max} + \sum_{t\in\mathbb{T}_3} \beta (C')^{-1}\left(\frac{\beta\sigma - \theta^t - \lambda^t}{\alpha}\right) = \pi_{ful} - \pi^s \right\}$;
end if
$p^{t*} \leftarrow p^t(\sigma^*), \quad \forall t \in \mathbb{T}$

V. Distributed implementation

The battery charging schedule problem can be addressed in a distributed manner, i.e., a two-level iterative approach. The two level refers to the station level and battery level, respectively.

At the battery level, each battery corresponds to several sequent subproblems. Given $\boldsymbol{\lambda}_k$, the local optimal solution to each subproblem can be individually obtained, thereby forming the whole local optimal solution to the battery charging schedule problem. However, local optimality may not equal global optimality. According to duality theory, there exists an optimal Lagrangian multiplier vector $\boldsymbol{\lambda}^*$, of which the corresponding local optimal solution is indeed global optimal. In this sense, $\boldsymbol{\lambda}$ functions as a coordination signal, aiming to align local optimality with global optimality. Therefore, at the station level, the Lagrangian multiplier vector $\boldsymbol{\lambda}$ is updated towards $\boldsymbol{\lambda}^*$ step by step.

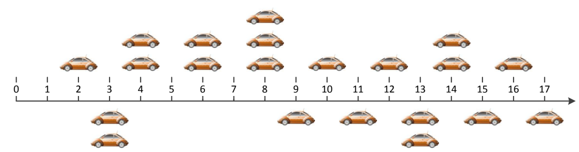


Fig. 1. Arrival process of EBs

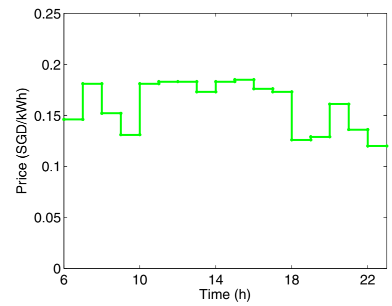


Fig. 2. Real-time prices on September 18, 2014

Note that given $\boldsymbol{\lambda}_k$, the corresponding local optimal solution to the battery charging schedule problem, i.e., $\boldsymbol{p}_k$, can be obtained. Therefore, the dual function (5) is simplified as

$$D(\boldsymbol{\lambda}_k) = \sum_{b\in\mathbb{B}} \sum_{t\in\mathbb{T}} \left[p_{b,k}^t \left(\theta^t + \lambda_k^t\right) + \alpha C\left(p_{b,k}^t\right) \right] - \sum_{t\in\mathbb{T}} L\lambda_k^t \quad (20)$$

Substituting (20) into (8), we obtain the update rule of the Lagrangian multiplier vector $\boldsymbol{\lambda}$ to iteratively solve the dual problem (7):

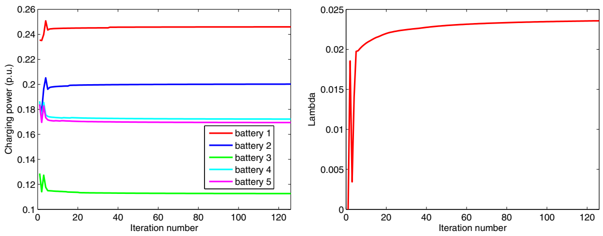
$$\lambda_{k+1}^t = \left[\lambda_k^t + \nu \left(\sum_{b\in\mathbb{B}} p_{b,k}^t - L \right) \right]^+, \quad \forall t \in \mathbb{T} \quad (21)$$

The iterations between the station level and battery level will continue until a state of equilibrium is reached, thereby obtaining the global optimal solution $\boldsymbol{p}^*$.

VI. Numerical results

For ease of illustration, we take into consideration a battery-swapping station with 5 charging boxes, which can be extended to more with similar simulation results. There are 5 original batteries with their initial SOCs in a decreasing order: 0.7, 0.68, 0.66, 0.64 and 0.62. We consider a time horizon that covers 17 time slots. The arrival process of EBs is shown in Fig. 1, which gives the number of EBs that will arrive at each time slot. The SOC upon arrival of every EB is randomly generated, subject to $\mathcal{N}(0.2, 0.025)$. The real-time prices shown in Fig. 2 are taken from electricity prices in Singapore from 6 a.m. to 11 p.m. on September 18, 2014. Other parameters are set as follows: $\pi_{ful} = 0.9$, $p_{\max} = 0.3$, $L = 0.9$, $\alpha = 1$ and $\beta_b = 1$, $\forall b \in \mathbb{B}$.

Respectively, Fig. 3(a) and Fig. 3(b) demonstrate that at time slot 8, the corresponding charging power of all the



(a) (b)

Fig. 3. Convergence of (a) Charging power at time slot 8 (b) λ^8

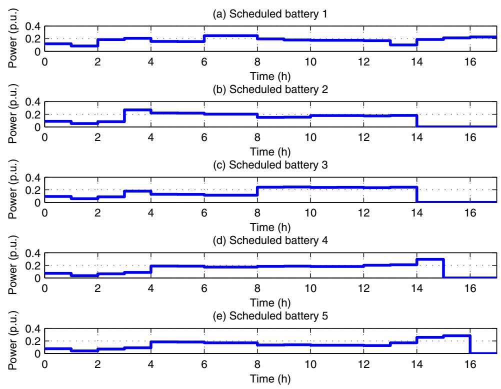


Fig. 4. Scheduled battery charging process

batteries and λ^8 are iteratively obtained. They converge quickly within a small number of iterations. Time slot 8 is taken as an example to show the fast convergence of our proposed approach and all the other time slots share the same convergence speed, which is desirable in real-time operations.

The scheduled battery charging process is shown in Fig. 4. It can be seen that if possible, all the batteries tend to be charged when the real-time price is low. Meanwhile, the coordination of the 5 batteries is achieved through the adjustment of the Lagrangian multiplier vector $\boldsymbol{\lambda}$. In this way, those batteries with more deadline pressure are charged with priority. Fig. 5 shows the SOC dynamics of the 5 batteries. The sudden fall of the SOC during the charging process of each battery indicates that a fully-charged battery is replaced by a low-SOC battery. It can be observed that by means of our proposed approach, all the battery swap requests are fulfilled, and at the same time, the total cost of the battery-swapping station is minimized through efficient battery charging schedule.

VII. Conclusion

In this paper, the battery charging schedule problem of a battery-swapping station for EBs is investigated. Based on our proposed EB assignment policy, all coming EBs are assigned to a specific battery, such that the battery-swapping station can schedule the charging of all batteries to prepare for the battery swap requests from EBs in advance. Taking into consideration some spatially and temporally coupled physical constraints, the battery charging schedule problem is formulated as a constrained convex program, which can be decomposed into a series of local subproblems by means of dual decomposition and our proposed EB assignment policy. In allusion to each subproblem, an efficient fast search method is put forward, thereby ensuring that the battery charging schedule problem can be addressed in a distributed manner. Numerical results validate our theoretical analysis.

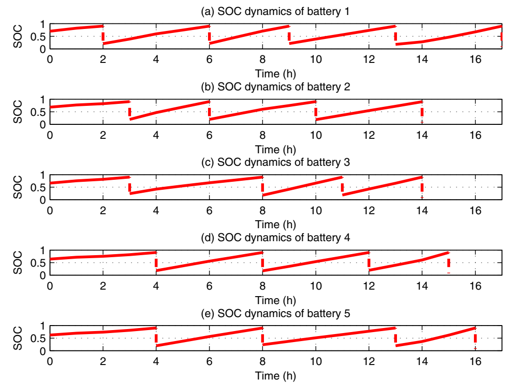


Fig. 5. SOC dynamics

Acknowledgement

This work was supported by Zhejiang International Collaboration Project under grant 2013C24008.

References

[1] B. Chai and Z. Yang, "Impacts of unreliable communication and modified regret matching based anti-jamming approach in smart microgrid," *Ad Hoc Networks*, vol. 22, pp. 69–82, 2014.

[2] R. Deng, J. Chen, X. Cao, Y. Zhang, S. Maharjan, and S. Gjessing, "Sensing-performance tradeoff in cognitive radio enabled smart grid," *Smart Grid, IEEE Transactions on*, vol. 4, no. 1, pp. 302–310, 2013.

[3] B. Chai, J. Chen, Z. Yang, and Y. Zhang, "Demand response management with multiple utility companies: A two-level game approach," *Smart Grid, IEEE Transactions on*, vol. 5, no. 2, pp. 722–731, 2014.

[4] R. Deng, Z. Yang, J. Chen, N. Asr, and M.-Y. Chow, "Residential energy consumption scheduling: A coupled-constraint game approach," *Smart Grid, IEEE Transactions on*, vol. 5, no. 3, pp. 1340–1350, 2014.

[5] P. You and Z. Yang, "Efficient optimal scheduling of charging station with multiple electric vehicles via V2V," in *Smart Grid Communications (SmartGridComm), 2014 IEEE International Conference on*, pp. 716–721, IEEE, 2014.

[6] Y. Miao, Q. Jiang, and Y. Cao, "Battery switch station modeling and its economic evaluation in microgrid," in *Power and Energy Society General Meeting, 2012 IEEE*, pp. 1–7, IEEE, 2012.

[7] M. Takagi, Y. Iwafune, K. Yamaji, H. Yamamoto, K. Okano, R. Hiwatari, and T. Ikeya, "Economic value of PV energy storage using batteries of battery-switch stations," *Sustainable Energy, IEEE Transactions on*, vol. 4, no. 1, pp. 164–173, 2013.

[8] C. Wang, J. Yang, N. Liu, and Y. Mao, "Study on siting and sizing of battery-switch station," in *Electric Utility Deregulation and Restructuring and Power Technologies (DRPT), 2011 4th International Conference on*, pp. 657–662, IEEE, 2011.

[9] C. McPherson, J. Richardson, O. McLennan, and G. Zippel, "Planning an electric vehicle battery-switch network for Australia," in *2011 Proceedings of the Australasian Transport Research Forum*, 2011.

[10] A. Kuperman, U. Levy, J. Goren, A. Zafransky, and A. Savernin, "Battery charger for electric vehicle traction battery switch station," *Industrial Electronics, IEEE Transactions on*, vol. 60, no. 12, pp. 5391–5399, 2013.