Residential Load Reduction Scheduling with Optimal Power Flow and Segmented Incentives

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Abstract—**In this paper, we formulate an optimal scheduling problem for demand response (DR) that schedules residential electrical appliances load to shave peak load. The schedule aims to minimize the total incentive cost and power loss over the power flow variables, subject to load capacity of users, grid operational constraints and the AC power flow equations. To deal with the nonconvexity of power flow equations, we present a solution based on second-order cone programming (SOCP) relaxation of optimal power flow. Furthermore, if the SOCP relaxation is exact, our algorithm yields a globally optimal solution. Finally, we evaluate the performance of our method by numerical simulations, which indicate that it is efficient and flexible.**

Index Terms—**Demand Response (DR), second-order cone programming (SOCP), *DistFlow* equations, convex relaxation**

I. Introduction

THE electricity demand has been rapidly increasing, especially in residential sectors. The seasonal variation of loads has brought a lot of challenges in system security and operation of the grid, for example, residential air conditioning accounts for more than a third of the peak load, even 40% in summer, while in winter, the air conditioning and water heater load also take a high proportion [1]. Hence, it is necessary to flat the load curves of residential users by load curtailment in peak period. It is widely believed that well controlled residential appliances could help stabilize the power system [2]. There are already many works on direct load control (DLC) [3]-[5].

Great efforts have been made to reduce peak loads by demand response (DR). Intelligent control devices, such as smart plugs, have been widely connected to the smart gird, which could monitor and control electrical appliances by energy management service (EMS) [6],[7]. Recently, residential demand has played an important role in shaving peak electricity demand, since residential loads response are faster-responsive and more flexible. Residential appliances could be operated centrally by a home energy management systems (HEMS) [8]-[11]. Aggregated load modeling and control have also been studied extensively for load shifting purpose in the literature, especially for thermostatically controlled loads (TCLs). [12] presents a concept of thermal comfort level within comfort zone for aggregate load modeling. However, residents have to bear some degree of discomforts for changing their consumption behavior, since it is difficult to model dynamic behavior of users in response to the incentives from the EMS [13]. As for Non-thermostatically control load, EMS refers to shift load from the on-peak hours to the off-peak hours, which will bring great benefit to not only the user but also the grid. Therefore, the main challenge is how to attract more users to participate in load reduction scheduling.

In the power system, the optimal power flow (OPF) is a fundamental problem. OPF determines a minimum cost operating point for an electric power system. Therefore, there has been a great deal of research on OPF [14]-[20]. OPF seeks to optimize a certain objective function, for example power loss, generation cost, and user utilities, subject to Kirchhoff's laws, and operational constraints on the voltages and line flows [21]. OPF underlies the whole power network operation with load reduction. However, generally the AC power flow equations are nonlinear and even nonconvex. Thus, it is challenging to solve a joint load reduction and OPF problem [22].

In this paper, an optimal load reduction allocation is proposed. Our main contributions are as follows:

1) A step-increase incentive mechanism is designed to encourage load reduction from residents.
2) Load reduction is implemented in a radial distribution network with consideration of line power losses. A joint load reduction and OPF problem is formulated to determine an optimal allocation among all nodes.

The rest of this paper is organized as follows. Section II, presents the load reduction model as well as incentive mechanism. Section III, solves a SOCP relaxation of the OPF. This allows us to get an optimal solution. In Section IV, we run a simulation with a 56 buses radial network. Finally, the conclusion is drawn in section V.

II. Problem Formulation

We mainly consider a scenario where all household electrical appliances, e.g., heating, ventilation and air conditioning (HVAC), are connected by a smart plug. Residents are voluntary to participate in DR programs hosted by the utility company for better operation of the distribution network. Therefore, it is assumed EMS has direct load control over the state of residential electrical appliances. By this mechanism, end users in the distribution grid are active in the sense that their controllable loads contribute to improving the system operation by reducing or shifting in a minimum-

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interruption way. In each scheduling slot, when power generation becomes economically inefficient to meet residential demand, the EMS could adjust the status of electrical appliances to achieve another balance with higher efficiency. Usually, the EMS could be structured as follows. In the system, each node is connected to a home gateway and all electrical appliances with smart plugs via a wireless communication network, gather power consumption on each of the monitored household appliances. Therefore, smart plugs could collect data and then save in the database, which is the hardware foundation.

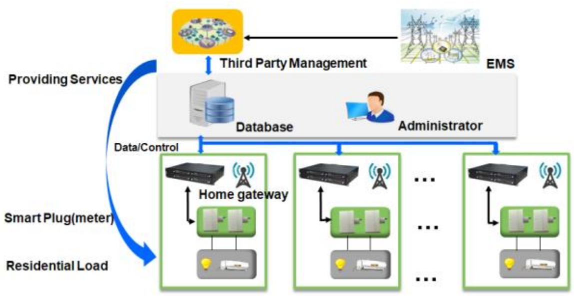

Fig. 1. Demonstration of the EMS Structure

Our goal is to design a scheduling scheme that minimizes a weighted sum of the incentive cost for demand response and power loss in the distribution network, while respecting the grid operational constraints and AC power flow equations.

We make one simplifying assumption that load control is implemented at the same timescale of optimal power flow, thus we will only focus on one slot scheduling.

A. Network Model

We consider a radial distribution network with a connected directed graph $G=(N,E)$, where $G=(N,E)$ is the set of nodes (demand response users), and $E \subseteq N \times N$ is the set of edges (lines). The network has a tree topology with slack bus 0 representing a substation as illustrated in Fig.2.

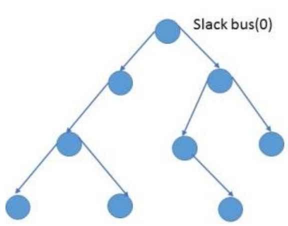

Fig. 2. A Distribution Radial Network

Assume the power flow always points away from the slack bus to leaf nodes. Denote a line by (*j*,*k*) or $j \to k$ if it points from node *j* to node *k*. Let z_{jk} be the complex impedance on the line (*j*,*k*). Let i_{jk} be the complex current from node *j* to node k. Let v_j, v_k be the complex voltage at node *j* and *k*, respectively. Let $S_{jk}=P_{jk}+iQ_{jk}$ be the sending-end complex power from node *j* to node *k*, where P_{jk} and Q_{jk} denote the active and reactive power flow respectively.

At each node *j* has the base load $S_j^l = p_j^l + iq_j^l$, including the HVAC load, where p_j^l and q_j^l denote the active and reactive base loads. Assume each node (user) may also has its own distributed generation $S_j^g = p_j^g + iq_j^g$, e.g. rooftop solar panel, small wind turbine. Let S_j denote the nodal complex power injection, which follows:

$$S_j = p_j + iq_j \tag{1a}$$

$$p_j = p_j^g - (p_j^l - P_j) \tag{1b}$$

$$q_j = q_j^g - q_j^l \tag{1c}$$

where P_j denotes the scheduled load reduction at bus *j*. p_j, q_j denote the active and reactive power injection respectively.

We use the *DistFlow* equations to characterize the distribution network flows.

$$\sum_{k:j\to k} S_{jk} = \sum_{i:i\to k} (S_{ij} - z_{ij} l_{ij}) + S_j,\ j \in N \tag{2a}$$

$$v_j - v_k = 2\,\mathrm{Re}(z_{jk}^H S_{jk}) - \left|z_{jk}\right|^2 l_{jk},\ j \to k \in E \tag{2b}$$

$$v_j l_{jk} = P_{\mathrm{jk}}^2 + Q_{jk}^2,\ j \to k \in E \tag{2c}$$

where $v_j =: \left|V_j\right|^2$ and $l_{jk} := \left|I_{jk}\right|^2$. The equations (2a) imposes power balance at each node, the equations(2b) is the Ohm law, and (2c) defines the branch power. The quantity $z_{ij}\left|I_{ij}\right|^2$ represents loss on line (*j*,*k*), thus $S_{ij} - z_{ij}\left|I_{ij}\right|^2$ is the receiving-end complex power at bus *j* from bus *i*.

Let the set of the branch flow equations vector variables $(s,v,i,S) = (p_j, q_j, v_j, i_{jk}, P_{jk}, Q_{jk}, j,k \in N, (j,k) \in E)$, and the equation (2c) are quadratic.

The operation of the distribution network must meet certain specifications, the squared voltage magnitudes must satisfy:

$$\underline{v}_j \le v_j \le \overline{v}_j,\ j \in N \tag{3a}$$

where $\underline{v}_j$ and $\overline{v}_j$ are given lower and upper bounds on the squared voltage magnitude at bus *j*. The active and reactive generations must satisfy:

$$\underline{p}_j^g \le p_j^g \le \overline{p}_j^g,\ j \in N_1 \tag{3b}$$

$$\underline{q}_j^g \le q_j^g \le \overline{q}_j^g,\ j \in N_1 \tag{3c}$$

where $\underline{p}_j^g, \overline{p}_j^g, \underline{q}_j^g$ and $\overline{q}_j^g$ are given lower and upper bounds on the real and reactive power generation at bus *j* respectively. N_1 is the number of generator. The power flow on line (*j*,*k*) must satisfy:

$$\underline{P}_{jk}^l \le \mathrm{P}_{jk}^l \le \overline{P}_{jk}^l,\ j \to k \in E \tag{3d}$$

where $\underline{P}_{jk}^l$ and $\overline{P}_{jk}^l$ denote the capacity of line (*j*,*k*).

The *DistFlow* model is quite general. If a quantity is known and fixed, then we set both its upper and lower bounds to the given quantity. e.g., if there is no distributed generation at bus

j , the $\overline{p}_j^g = \underline{p}_j^g = \overline{q}_j^g = \underline{q}_j^g = 0$.

B. Response Demand Incentive

Traditionally, peak demand is supplied by capacity reserves from generators, which are idle most of the time. Demand response is widely regarded as an economically efficient resource that can actively participate in the peak load shaving in power system. By providing incentives, the EMS can induce end users to reduce their demand.

With incentives from the EMS, each customer has the different adjustable capacity. Let *M* denote the different levels of scheduling capabilities. Their locations are fixed and known. Before scheduling the load of user, Suppose the utility company and users have a protocol that according to the historical electricity profile of user, different scheduling capabilities have different incentives. The energy management service offers a different incentive package, represented by

$$c_i = \begin{cases} c_1, p_1 \le p_{ref}^1 \\ \vdots \quad \vdots \quad \vdots \\ c_m, p_m \le p_{ref}^m \end{cases} \tag{4}$$

where c_i denotes the incentive factor, and p_{ref}^m denotes different adjustable capacity levels, it is the package adjustable power upper bounds.

It is legitimate to reduce load by controlling the state of residential appliances without affecting the normal life of user. However, different electrical characteristics lead to different scheduling methods.

Load type: The residential loads could be categorized into three types [23],[24].

1) Thermostatically controlled load such as the air conditioning, water heaters and ventilation of which the power consumption is directly related to the temperature. For example, when user initially sets the temperature to 23 Celsius degrees, EMS could adjust it to 26 Celsius degrees if needed.
2) Non-thermostatically controlled load like the PHEVs, washing machines, and cloth dryers, which can flexibly shift their power consumption as long as the tasks are finished within the given period.
3) Critical Loads like the refrigerator which must be turned on all the time.

In order to achieve the purpose of reducing power, EMS needs to schedule these three types of residential loads.

We make an assumption that the target total load reduction is always achievable by all adjustable loads. Suppose the grid has in total *hMW* power deficit. This assignment satisfies the following condition:

$$\sum_{m=1}^{M} \sum_{i \in \Omega m} p_i^m = h \tag{5}$$

$$p_i^m \le p_{ref}^m, \forall i \in \Omega_m, m = 1, 2, .., M \tag{6}$$

$$p_i^m \le p_i^l, \forall i \in \Omega_m, m = 1, 2, .., M \tag{7}$$

where p_i^m denotes the reduction load at node *i*. i.e., the total power deficit is made up through the reduction of all loads.

The load reduction demand response couples with the OPF. We are interested in the following optimization problem:

$$\begin{aligned} \min \quad & \alpha \sum_{m=1}^{M} \sum_{i \in \Omega m} c_m p_i^m + \sum_i \sum_j r_{ij} l_{ij} \\ s.t. \quad & (1)(2)(3)(4)(5)(6)(7) \end{aligned} \tag{8}$$

where $\sum_i \sum_j r_{ij} l_{ij}$ is the total real power loss of the lines and $\alpha > 0$, is a weight that makes the incentive cost and the power loss comparable.

III. SOLUTION

The joint load reduction and OPF problem (8) are generally difficult to solve because (2c) is nonconvex, and the incentive is segmented. The following is our solution strategy.

SOCP relaxation. We first relax the nonconvex constraint (2c) into a second-order cone, i.e., replace the *DistFlow* equations (2) by

$$\sum_{k: j \to k} S_{jk} = \sum_{i: i \to k} (S_{ij} - z_{ij} l_{ij}) + S_j, j \in N \tag{9a}$$

$$v_j - v_k = 2\,\mathrm{Re}(z_{jk}^H S_{jk}) - |z_{jk}|^2 l_{jk}, j \to k \in E \tag{9b}$$

$$v_j l_{jk} \ge P_{\mathrm{jk}}^2 + Q_{jk}^2, j \to k \in E \tag{9c}$$

In order to verify the accuracy of equation (9c) at the optimum after relaxation, check if D is less than a small enough predetermined constant ε .

$$D = \left| (P_{jk}{}^2 + Q_{jk}{}^2) - (i_{jk} v_j) \right| \tag{9d}$$

Then the SOCP relaxation of the problem (8) is:

$$\begin{aligned} \min \quad & \alpha \sum_{m=1}^{M} \sum_{i \in \Omega m} c_m p_i^m + \sum_i \sum_j r_{ij} l_{ij} \\ s.t. \quad & (1)(3)(4)(5)(6)(7)(9) \end{aligned} \tag{10}$$

The problem (10) is a convex problem, which is a relaxation of the problem (8). Given c_m , in the sense that the optimal objective of the relaxation (10) lower bounds that of the original problem (8). If an optimal solution to the relaxation (10) attains equality in (9c) then the solution is also feasible, and therefore optimal, for the original problem (8). In this case, we say that the SOCP relaxation is exact. Sufficient conditions are known that guarantee the exactness of the SOCP relaxation. Hence, we will solve (10) instead of (8) by turning to the off-the-shelf solvers, e.g., CVX.

IV. NUMERICAL RESULTS

In this section, we evaluate the proposed method through numerical simulations using a 56-bus Southern California Edison (SCE) distribution grid case with a radial structure. More details about the feeder can be found in TABLE I. Suppose in a certain control interval, the constant incentive vector is $c = [0.8, 1.0, 1.3]$. Each node could respond to scheduling quickly. We set the total power deficit $h = 1.0MW$. The unit of power, the unit of voltage and the unit of resistance, are respectively MVA, KV, and Ω. All numerical tests are tested on a laptop with Inter(R) Core(TM) i7-6700HQ CPU @ 2.60GHz 8GB RAM, and 64-bit Windows 10.

TABLE I
Distributed generator data

bus	$Q_{\max}$	$Q_{\min}$	$P_{\max}$	$P_{\min}$
1	2.0	-2	4.0	0
4	1.5	-1.5	2.5	0
26	1.5	-1.5	2.5	0
34	1.5	-1.5	2.5	0

Without DCL policy. Without direct control load, the default policy is that all generators raise their power generations uneconomically. Moreover, it can cause voltage instability, the voltage magnitudes of some buses may drop below a pre-specified threshold.

Optimal allocation. Fig. 3 shows the optimal allocation computed using the proposed method for the two cases ($h = 1.0MW$ and $h = 3.0MW$). At the peak period, we need to adjust the load at each node. Grid operational constraints such as voltage stability are taken into account. The tradeoff between the incentive cost and the power loss is optimized as follow:

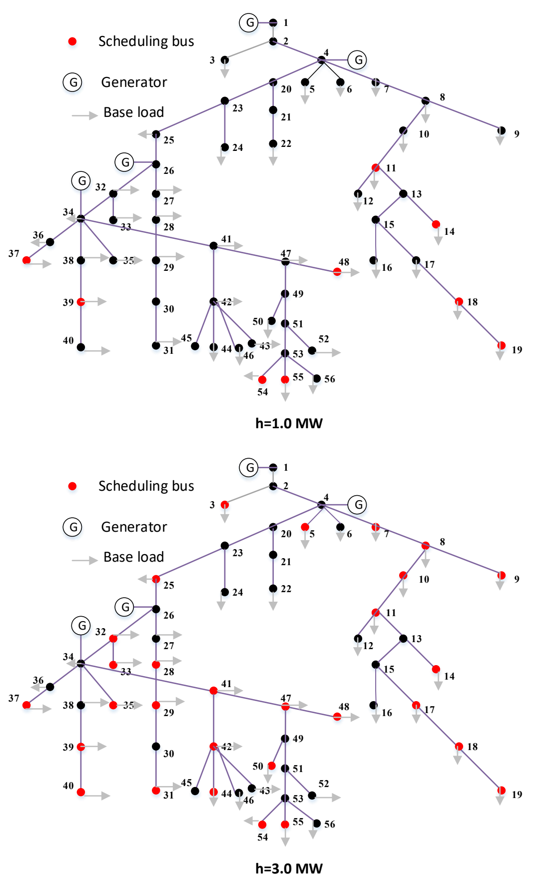

Fig. 3. Scheduling of Distribution grid Load Reduction

As we can see from the Fig. 3, reducing loads at nodes electrically far away from the slack bus, can reduce more line loss. The corresponding partial OPF results of the $h = 1.0MW$ case is listed in TABLE II.

TABLE II
Partial Power Scheduling Bus under Optimal Allocation

bus	$p_1(k)$	$\lvert v_j \rvert (p.u)$	bus	$p_1(k)$	$\lvert v_j \rvert (p.u)$
11	0.0844	1.0249	39	0.0638	1.0411
14	0.0718.	1.0219	48	0.1715	1.0338
18	0.1264	1.0167	54	0.0769	1.0311
19	0.1096	1.0160	55	0.0693	1.0315
37	0.2262	1.0393	/		

Exactness of SOCP relaxation. Finally, the exactness of SOCP relaxation can be guaranteed at the optimum as shown in the equation (11). We tried many tests, and in most cases, the relaxation is exact. In general, SOCP relaxation is able to help solve load reduction scheduling during peak period.

$$\begin{cases} D_{h=1.0} = 3.5941\text{e-}07 \\ D_{h=3.0} = 1.8763\text{e-}07 \end{cases} \tag{11}$$

Calculation work. Our optimal scheduling problem, requires fast speed to demonstrate the appliance potential of this method. The time required for calculation of different levels of load reduction is calculated. Essentially, this is a nonlinear second-order cone programming. We use the CVX to solve this problem on MATLAB R2015a platform. Fig. 4 shows the calculation time for different levels of load reduction, which validates computational efficiency of this method. When we schedule the load reduction of $1.0MW$, the average calculation time is about 2.1 seconds (The total load $=4.83MW$). Fig. 5 shows the average scheduling bus number with different reduction load. The more load to be scheduled, the more buses are needed.

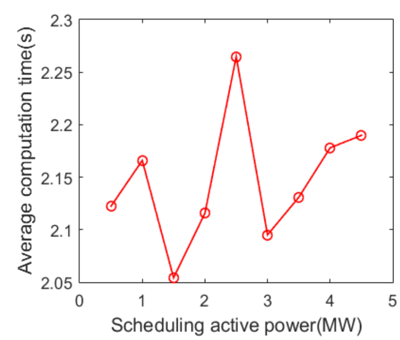

Fig. 4. Average computation time

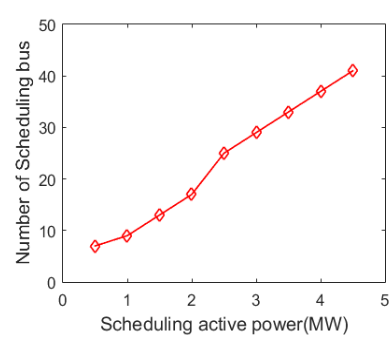

Fig. 5. Scheduling bus number

Benefit. Fig. 6 shows the reduction of power loss with respect to total load reduction under different α's. With the increase of load reduction levels, the corresponding power loss becomes smaller. Besides, smaller the weight α is, the smaller power loss is.

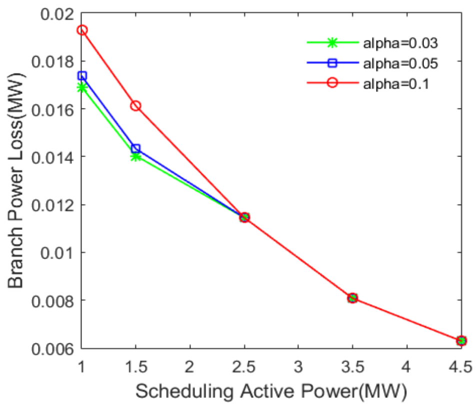

Fig. 6. Relative Reduction Branch Active Power Loss

V. CONCLUSION

We formulate an optimal scheduling problem for demand response (peak shaving) that allocates each node a best power reduction consumption based on its electrical appliances load status by EMS adjustment. The schedule aims to minimize the total user incentive cost and power loss on lines over power flow variables, subject to grid operational constraints and AC power flow equations. We propose a centralized solution that relaxes the nonconvex constraint of the OPF into a second-order cone to handle allocation. Numerical case studies on the 56 buses distribution network show the SOCP relaxation is mostly exact and computes an optimal solution efficiently.

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