# Optimal Cooperative Charging Strategy for a Smart Charging Station of Electric Vehicles

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***Abstract*—This paper proposes a novel cooperative charging strategy for a smart charging station in the dynamic electricity pricing environment, which helps electric vehicles (EVs) to economically accomplish the charging task by the given deadlines. This strategy allows EVs to share their battery-stored energy with each other under the coordination of an aggregator, so that more flexibility is given to the aggregator for better scheduling. Mathematically, the scheduling problem is formulated as a constrained mixed-integer linear program (MILP) to capture the discrete nature of the battery states, i.e., charging, idle and discharging. Then, an efficient algorithm is proposed to solve the MILP by means of dual decomposition and Benders decomposition. At last, the algorithm can be implemented in a distributed fashion, which makes it scalable and thus suitable for large-scale scheduling problems. Numerical results validate our theoretical analysis.**

***Index Terms*—Benders decomposition, distributed optimization, dual decomposition, pricing.**

## I. INTRODUCTION

THE conventional power grid is facing great challenges caused by increasing demand and aging infrastructure. Economically, it is not wise to unlimitedly augment generation capacity to meet demand. Therefore, smart grid emerges to enhance the tolerance of the power grid for future potential demand, e.g., electric vehicles (EVs') charging load, and intermittent renewables, e.g., wind and solar energy. Smart grid allows active participation of users via demand response (DR), which plays a key role in load scheduling [2]–[6].

With the rapid development of EVs, means of transportation will undergo great changes in the near future [7]. As EVs gradually penetrate into our daily life, they will consume a tremendous amount of electricity energy. According to [8], if 30% of conventional vehicles in the US were substituted by EVs, the total charging load would reach around 18% of the US summer peak load. Moreover, uncoordinated charging of EVs easily leads to a load peak which coincides with the original base load peak [9], and may crash the distribution grid. Therefore, scheduling on EVs' charging behavior is essential to relieve the burden on the distribution grid. Charging stations are among the most common places for EVs to be charged, where it is convenient to coordinate numerous EVs' charging behavior [10]–[13]. Typically, it is unrealistic to directly control each EV's individual charging behavior, but it is possible to incentivize EVs through financial benefits to make the optimal charging decisions. The PowerMatcher [14], an agent-based software technology, can function as a charging decision-maker for each EV. However, individual optimality, achieved by PowerMatcher, in most cases cannot lead to global optimality. To mitigate the gap between local optimality and global optimality, the coordination of numerous EVs' charging behavior in a charging station is needed.

An important feature of EVs is their capability of storing energy, which provides great flexibility for scheduling in energy management and demand response. For example, vehicle-to-grid (V2G) is a widely studied paradigm that enables EVs to transfer energy back to the power grid. By this means, EVs can provide various ancillary services to the power grid, which benefits both sides. Moreover, [15] investigates the possibility of EVs' transferring energy to homes, named vehicle-to-home (V2H), such that the residential power scheduling can be more flexible and enhanced. The mechanism of vehicle-to-building (V2B) is utilized in [16] to reduce a building's peak demand and daily electricity cost. Conceptually, EVs' capability of storing energy also contributes to better scheduling of a charging station via a similar mechanism of transferring energy among EVs, named vehicle-to-vehicle (V2V). With this mechanism, in the charging station some EVs can function as energy storage and share their stored energy with other more urgent EVs.

Many literatures have studied the scheduling issues of EV charging [17]–[23]. [17] augments the scheduling problem for PHEV charging into an optimal power flow (OPF) problem to obtain joint optimization of OPF-charging. [18] proposes a decentralized algorithm to optimally schedule EV charging. The elasticity of EV loads is exploited to fill the valleys in electric load profiles. Multi-agent systems (MASs) are utilized in [19], [20], [21] to coordinate the battery charging of EVs in the distribution network, considering the technical constraints and impacts from the distribution network. However, the feedback ability of EVs is not investigated in these literatures. Some other

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studies, like [22], [23], show potential benefits of establishing a V2G market. However, to the best of our knowledge the concept of V2V has not been considered in previous research.

In this paper we mainly focus on the scheduling problem of EV charging in a smart charging station which operates under the mechanism of V2V. In order to guarantee the proper function of this mechanism in a charging station, there is an aggregator which coordinates the charging/discharging behavior of all EVs. When discharging, the EV is transferring its energy to other EVs through the aggregator. The objective of the aggregator is to minimize the social cost of all EVs in the charging station. Due to discrete battery states of charging, idle and discharging, the scheduling problem is formulated as a constrained MILP. By means of dual decomposition and Benders decomposition, a distributed algorithm is proposed to efficiently obtain the optimal scheduling strategy. The main contributions of this paper consist of 3 parts as follows:

1) The cooperative scheduling problem of a smart charging station, which utilizes the V2V energy transfer mechanism, is analyzed and formulated as an MILP.
2) As the load constraint of a charging station couples all EVs' charging/discharging behavior together, dual decomposition is introduced to decompose the scheduling problem into a series of separable subproblems. Each subproblem corresponds to the scheduling on a single EV's charging/discharging behavior and can be solved independently.
3) Due to the existence of binary variables, each subproblem is still an MILP that cannot be directly solved by conventional methods. Thus, Benders decomposition is applied to efficiently solve each subproblem.

The remainder of this paper is organized as follows. System model is described in Section II, followed by problem formulation and transformation in Section III. We focus on solving subproblems by means of Benders decomposition in Section IV. Then in Section V, distributed implementation of the proposed approach is introduced. Numerical results are given in Section VI, and conclusions are drawn at last in Section VII.

## II. System Model

### *A. V2V and Cooperative Charging*

Consider a charging station shown in Fig. 1, where a set $\mathcal{A} \triangleq \{1, 2, \ldots, A\}$ of EVs are parking and being charged. All EVs in the charging station are connected to an aggregator, which is a coordinator. Let each EV be embedded with a controller, e.g., the PowerMatcher, as its decision-maker to optimize its own charging strategy through information exchange with the aggregator. After parking his EV in the charging station, each EV owner will set the charging task to be accomplished, i.e., the deadline for charging and the terminal energy level.

Assume that the charging station allows energy transfer among EVs, i.e., V2V. With this mechanism, some EVs can function as energy storage and transfer their stored energy to other EVs with more urgent deadlines, on the premise that each EV's charging task must be fulfilled. Meanwhile, the aggregator, assumed to be in the dynamic electricity pricing environment, also procures electricity energy from the power grid for charging. Since this paper will mainly focus on investigating the benefit of V2V, the aggregator's capability of transmitting electricity energy back the power grid, i.e., V2G, is not considered here.

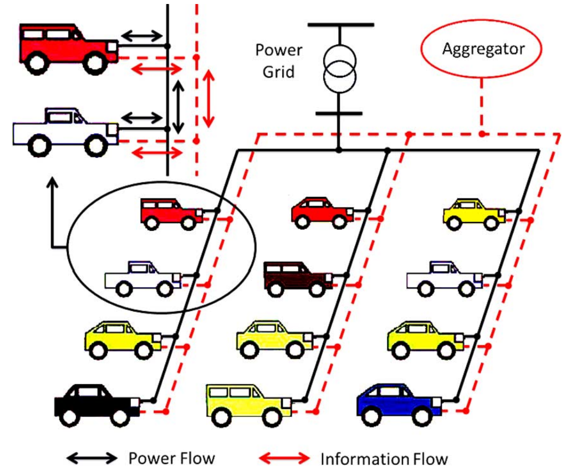

Fig. 1. Charging station.

In order to encourage EVs to participate the V2V program, each EV will make its own charging/discharging decision in an autonomous way under the coordination of the aggregator. That is, the aggregator will set virtual prices for EVs, which function as a coordination signal; based on virtual prices, every controller will optimize its EV's charging strategy to accomplish the charging task in the most economical way. In particular, if an EV chooses to discharge at a certain time slot, it will transfer (sell) its battery-stored energy to other EVs at the corresponding virtual price through the aggregator. Ideally, we assume each controller has full knowledge of its EV's battery states and coordination signals from the aggregator.

### *B. Constraints*

We consider a discrete-time model with a finite time horizon within which all EVs must finish charging. The time slots of the finite time horizon are indexed by $n \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$. The deadline of EV $a$ is denoted as $N_a$ and thus its charging period is $\mathcal{N}_a \triangleq \{1, 2, \ldots, N_a\}$. Note that $N = \max_{a \in \mathcal{A}} N_a$. Let a pair of binary variables $\omega_{a,c}^n, \omega_{a,d}^n \in \{0, 1\}$ denote the charging and discharging states of EV $a$ at time slot $n$, respectively. $\omega_{a,c}^n = 1$ implies charging and $\omega_{a,d}^n = 1$ implies discharging. Note that $\omega_{a,c}^n, \omega_{a,d}^n = 0$ represents the idle state accordingly. Thus, we have the following constraints:

$$\begin{cases} \omega_{a,c}^n + \omega_{a,d}^n \leq 1, & \forall n \in \mathcal{N}_a, \ \forall a \in \mathcal{A} \\ \omega_{a,c}^n, \omega_{a,d}^n = 0, & \forall n \in \mathcal{N} \setminus \mathcal{N}_a, \ \forall a \in \mathcal{A} \\ \omega_{a,c}^n, \omega_{a,d}^n \in \{0, 1\}, & \forall n \in \mathcal{N}, \ \forall a \in \mathcal{A}. \end{cases} \tag{1}$$

Let continuous variables $p_{a,c}^n$ and $p_{a,d}^n$ denote the charging and discharging power, respectively, which are bounded as follows:

$$\begin{cases} 0 \leq p_{a,c}^n \leq \omega_{a,c}^n p_{a,c}^{\max}, & \forall n \in \mathcal{N}, \ \forall a \in \mathcal{A} \\ 0 \leq p_{a,d}^n \leq \omega_{a,d}^n p_{a,d}^{\max}, & \forall n \in \mathcal{N}, \ \forall a \in \mathcal{A} \end{cases} \tag{2}$$

where $p_{a,c}^{\max}$ and $p_{a,d}^{\max}$ are the maximum charging and discharging power, respectively. Note that both $p_{a,c}^n$ and $p_{a,d}^n$ refer to the power input/output between EV $a$ and the aggregator.

The energy stored in the battery of EV $a$ at time slot $n$ is denoted as $\pi_a^n$, which is bounded as $\pi_a^{\min} \le \pi_a^n \le \pi_a^{\max}$. $\pi_a^{\min}$ and $\pi_a^{\max}$ serve as the lower bound and the upper bound of $\pi_a^n$, respectively, which are given according to battery characteristics. Meanwhile, the dynamic change of $\pi_a^n$ is simplified as a linear process:

$$\pi_a^n = \pi_a^0 + \sum_{t=1}^{n} \left( \beta_a^c p_{a,c}^t - \frac{p_{a,d}^t}{\beta_a^d} \right), \quad \forall n \in \mathcal{N}_a,\ \forall a \in \mathcal{A}$$

where $\pi_a^0$ is the initial energy stored in the battery, and $\beta_a^c, \beta_a^d \in (0,1]$ are the charging and discharging efficiencies of EV $a$, respectively. Then the energy constraint for each EV is redescribed as

$$\pi_a^{\min} \le \pi_a^0 + \sum_{t=1}^{n} \left( \beta_a^c p_{a,c}^t - \frac{p_{a,d}^t}{\beta_a^d} \right) \le \pi_a^{\max},\ \forall n \in \mathcal{N}_a, \forall a \in \mathcal{A}. \tag{3}$$

In particular, the terminal energy level $\pi_a^T$ at the deadline is an inelastic requirement, i.e.,

$$\pi_a^0 + \sum_{n \in \mathcal{N}_a} \left( \beta_a^c p_{a,c}^n - \frac{p_{a,d}^n}{\beta_a^d} \right) = \pi_a^T, \quad \forall a \in \mathcal{A}. \tag{4}$$

For the charging station, let $L^n$ denote the maximum allowable load at time slot $n$, which is a conservative load upper bound. $L^n$ is given by the utility company according to the overall load information of the distribution grid and may vary over time to help realize peak clipping and valley filling. Assume $[L^n]_{\forall n \in \mathcal{N}}$ is known by the aggregator in advance, thus the load constraint for the charging station is formulated as

$$0 \le \sum_{a \in \mathcal{A}} \left( p_{a,c}^n - p_{a,d}^n \right) \le L^n, \quad \forall n \in \mathcal{N}. \tag{5}$$

### C. Battery Loss

Discharging processes of EVs may accelerate battery degradation, causing additional financial costs to EV owners. As a result, EVs' batteries cannot be discharged for optimal scheduling purpose too frequently. In this paper each EV's battery loss is included in its owner's total cost, which we quantify as a convex and non-decreasing function of discharging power, $C_a(p_{a,d}^n)$ [24]. For simplicity, we assume it to be a linear function, i.e., $C_a(p_{a,d}^n) \triangleq \alpha_a p_{a,d}^n$, where $\alpha_a$ is the weighting factor. Note that the approach proposed below will still apply as long as the convexity of $C_a(\cdot)$ holds.

## III. Problem Formulation and Transformation

### A. Optimization Problem of Aggregator

The aggregator aims to induce all EVs' charging/discharging behavior in a way that minimizes the total social cost, i.e., the aggregator's cost of procuring electricity energy from the power grid plus all EVs' battery losses. Assume the real-time prices of the $N$ slots to be $\boldsymbol{\theta} \triangleq [\theta^1, \ldots, \theta^N]$, and recall the constraints (1)–(5), the scheduling problem for the aggregator is formulated as an MILP:

**Primal Problem**:

$$\begin{aligned} \min_{\boldsymbol{\omega}, \boldsymbol{p}} \quad & \sum_{n \in \mathcal{N}} \sum_{a \in \mathcal{A}} \left[ \left( p_{a,c}^n - p_{a,d}^n \right) \theta^n + C_a \left( p_{a,d}^n \right) \right] \\ \text{s.t.} \quad & (1), (2), (3), (4), (5) \end{aligned} \tag{6}$$

where $\boldsymbol{\omega}$ and $\boldsymbol{p}$ are the vectors comprising all binary variables and all continuous variables, respectively.

### B. Primal-Dual Approach and Subproblem of EV

The load constraint (5) couples all EVs together, which makes the primal problem (6) difficult to solve. Thus, dual decomposition is introduced to decompose (6) into a series of separable subproblems, each corresponding to the scheduling on the charging/discharging behavior of a single EV.

Firstly, (5) is relaxed by introducing the Lagrangian multipliers $\boldsymbol{\lambda} \triangleq [\lambda^n]_{n \in \mathcal{N}}$ and $\boldsymbol{\mu} \triangleq [\mu^n]_{n \in \mathcal{N}}$, where $\lambda^n, \mu^n \ge 0$. Then we define the Lagrangian for the primal problem (6) as

$$\begin{aligned} &\mathcal{L}_1(\boldsymbol{\omega}, \boldsymbol{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ &\quad = \sum_{n \in \mathcal{N}} \sum_{a \in \mathcal{A}} \left[ \left( p_{a,c}^n - p_{a,d}^n \right) \theta^n + C_a \left( p_{a,d}^n \right) \right] \\ &\qquad + \sum_{n \in \mathcal{N}} \lambda^n \left[ \sum_{a \in \mathcal{A}} \left( p_{a,c}^n - p_{a,d}^n \right) - L^n \right] \\ &\qquad + \sum_{n \in N} \mu^n \left[ - \sum_{a \in \mathcal{A}} \left( p_{a,c}^n - p_{a,d}^n \right) \right] \\ &\quad = \sum_{a \in \mathcal{A}} \sum_{n \in \mathcal{N}} \left[ \left( \theta^n + \lambda^n - \mu^n \right) \left( p_{a,c}^n - p_{a,d}^n \right) + \alpha_a p_{a,d}^n \right] \\ &\qquad - \sum_{n \in \mathcal{N}} \lambda^n L^n. \end{aligned} \tag{7}$$

Note that the first term in the Lagrangian (7) is separable in terms of EVs, thus the dual function which minimizes the Lagrangian (7) over $\boldsymbol{\omega}$ and $\boldsymbol{p}$ can be described as follows:

$$\begin{aligned} \mathcal{D}(\boldsymbol{\lambda}, \boldsymbol{\mu}) &= \min_{\boldsymbol{\omega}, \boldsymbol{p}: (1)(2)(3)(4)} \mathcal{L}_1(\boldsymbol{\omega}, \boldsymbol{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ &= \sum_{a \in \mathcal{A}} \mathcal{S}^a(\boldsymbol{\lambda}, \boldsymbol{\mu}) - \sum_{n \in \mathcal{N}} \lambda^n L^n \end{aligned} \tag{8}$$

where $\mathcal{S}^a(\boldsymbol{\lambda}, \boldsymbol{\mu})$ is a subproblem corresponding to the scheduling on the charging/discharging behavior of EV $a$. Note that each subproblem can be tackled independently, which simplifies the whole solving process. For ease of presentation, we drop the notation $a$, thus each subproblem $\mathcal{S}(\boldsymbol{\lambda}, \boldsymbol{\mu})$ is detailed as follows:

**Subproblem**:

$$\begin{aligned} \min_{\boldsymbol{\omega}, \boldsymbol{p}} \quad & \Phi(\boldsymbol{p}) = \sum_{n \in \mathcal{N}} \left[ r^n \left( p_c^n - p_d^n \right) + \alpha p_d^n \right] \\ \text{s.t.} \quad & \omega_c^n + \omega_d^n \le 1, \quad \omega_c^n, \omega_d^n \in \{0,1\}, \quad \forall n \in \mathcal{N} \\ & 0 \le p_c^n \le \omega_c^n p_c^{\max}, \quad 0 \le p_d^n \le \omega_d^n p_d^{\max}, \quad \forall n \in \mathcal{N} \\ & \pi^{\min} \le \pi^0 + \sum_{t=1}^{n} \left( \beta^c p_c^t - \frac{p_d^t}{\beta^d} \right) \le \pi^{\max}, \quad \forall n \in \mathcal{N} \\ & \pi^0 + \sum_{n \in \mathcal{N}} \left( \beta^c p_c^n - \frac{p_d^n}{\beta^d} \right) = \pi^T \end{aligned} \tag{9}$$

where $r^n \triangleq \theta^n + \lambda^n - \mu^n$. $r^n$ serves as the virtual price and also the coordination signal of time slot $n$, released by the aggregator to all EVs. That is, all EVs will charge (buy) electricity energy from or discharge (sell) electricity energy to the aggregator at virtual prices. The subproblem (9) is a scheduling problem solved by each controller to seek the optimal charging strategy for its EV that minimizes the total individual cost based on virtual prices.

Virtual prices, mainly decided by Lagrangian multipliers, are set on the basis of real-time prices. Lagrangian multipliers $\lambda^n$ and $\mu^n$ are the coordination parameters to avoid the conflict of the load constraint (5), which aligns local optimality with global optimality. If (5) is satisfied without coordination, virtual prices are consistent with real-time prices, i.e., $\lambda^n, \mu^n = 0, \forall n \in \mathcal{N}$. If overloading is to occur at time slot $n$, the corresponding virtual price will rise to induce EVs to shift their charging demand, i.e., $\lambda^n > 0$, $\mu^n = 0$. On the contrary, the virtual price falls to encourage EVs' charging in case of unbalanced discharging, i.e., $\lambda^n = 0$, $\mu^n > 0$. Through the adjustment of virtual prices, coordination among EVs can be realized to ensure the load constraint (5) is satisfied. The main advantage of this mechanism is that all EVs can accomplish their charging tasks in the most economical way. That is, for less urgent EVs, they can reduce their individual costs by sharing their stored energy; and for more urgent EVs, they can charge faster by using other EVs' energy and meet their deadlines.

The corresponding dual problem of the primal problem (6) is to maximize the dual function (8) over $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$:

**Dual Problem**:

$$\begin{aligned} \max_{\boldsymbol{\lambda},\boldsymbol{\mu}} \quad & \mathcal{D}(\boldsymbol{\lambda},\boldsymbol{\mu}) \\ \text{s.t.} \quad & \lambda^n, \mu^n \geq 0, \quad \forall n \in \mathcal{N}. \end{aligned} \tag{10}$$

The dual problem (10) is an optimization problem solved by the aggregator to seek the optimal virtual prices, which can induce all EVs' charging/discharging behavior in a way that minimizes the total social cost. Note that there exist binary variables in (6), thus only weak duality is guaranteed and there exists a duality gap between (6) and (10). That is, the maximum value of (10) is a lower bound of (6).

According to primal-dual theory [25], even though the primal problem (6) is not convex, the concavity of the dual function (8) is guaranteed, which makes the dual problem a typical convex program. Due to the non-differentiability of the dual function (8), the optimal Lagrangian multipliers, which may not be unique, can be achieved iteratively by the subgradient method. Considering the correspondence between the primal and dual problems, the total social cost is minimized and the load constraint (5) is satisfied at the meantime with the optimal Lagrangian multipliers. The Lagrangian multipliers are adjusted in the subgradient direction of the dual function (8):

$$\begin{cases} \lambda^n(k+1) = \left[\lambda^n(k) + \gamma_\lambda g_\lambda^n(k)\right]^+ \\ \mu^n(k+1) = \left[\mu^n(k) + \gamma_\mu g_\mu^n(k)\right]^+ \end{cases}, \quad \forall n \in \mathcal{N} \tag{11}$$

where $[x]^+ = \max\{x, 0\}$, and $k \in \mathbb{N}^+$ represents the iteration index of the subgradient method. $\boldsymbol{\gamma} \triangleq [\gamma_\lambda, \gamma_\mu]$ stands for the step size adjusting the convergence rate, where $\gamma_\lambda, \gamma_\mu > 0$. $g_\lambda^n(k)$ and $g_\mu^n(k)$ are the subgradients of the dual function at the $k$th iteration with respect to $\lambda^n$ and $\mu^n$, respectively. With a sufficiently small step size $\boldsymbol{\gamma}$, the subgradient of the dual function (8) satisfies the Lipchitz continuity condition [26], which guarantees the convergence towards the optimal solution to the dual problem (10).

## IV. Subproblem Solution via Benders Decomposition

The subproblem (9) is still an MILP to be solved by each controller, and Benders decomposition is introduced for efficiency. Based on Benders decomposition, continuous variables and integer variables are solved separately and iteratively towards the optimal solution [27], [28]. As a result, computational complexity can be greatly relieved.

According to Benders decomposition, (9) shall be decomposed into a master problem and a slave problem. Constraints that only comprise integer variables are considered in the master problem, while other constraints are taken into account in the slave problem when integer variables are given. On the one hand, the master problem aims to find better integer variables and approach the optimal ones at every iteration through the shrink of its own feasible region. On the other hand, the slave problem is dominated by the given integer variables from the master problem and aims to search for the corresponding optimal continuous variables. At every iteration, through analyzing the feasibility of the solution to the slave problem, we can add a feasibility constraint or an infeasibility constraint into the master problem to diminish its feasible region [29].

The two correlative problems are formulated respectively as follows:

**Master Problem**:

$$\begin{aligned} \Phi_{lower}^* = \min_{\boldsymbol{\omega},\Phi} \quad & \Phi \\ \text{s.t.} \quad & \omega_c^n + \omega_d^n \leq 1, \\ & \omega_c^n, \omega_d^n \in \{0,1\}, \quad \forall n \in \mathcal{N} \\ & \text{feasibility constraints} \\ & \text{infeasibility constraints} \end{aligned} \tag{12}$$

**Slave Problem**:

$$\begin{aligned} \Phi_{upper}^* = \min_{\boldsymbol{p}} \quad & \Phi(\boldsymbol{p}) = \sum_{n\in\mathcal{N}} \left[r^n\left(p_c^n - p_d^n\right) + \alpha p_d^n\right] \\ \text{s.t.} \quad & p_c^n \leq \omega_c^n(m) p_c^{\max}, \quad \forall n \in \mathcal{N} \\ & p_d^n \leq \omega_d^n(m) p_d^{\max}, \quad \forall n \in \mathcal{N} \\ & \pi^0 + \sum_{t=1}^{n}\left(\beta^c p_c^t - \frac{p_d^t}{\beta^d}\right) \leq \pi^{\max}, \ \forall n \in \mathcal{N} \\ & \pi^0 + \sum_{t=1}^{n}\left(\beta^c p_c^t - \frac{p_d^t}{\beta^d}\right) \geq \pi^{\min}, \ \forall n \in \mathcal{N} \\ & \pi^0 + \sum_{n\in\mathcal{N}}\left(\beta^c p_c^n - \frac{p_d^n}{\beta^d}\right) = \pi^T \\ & p_c^n, p_d^n \geq 0, \quad \forall n \in \mathcal{N} \end{aligned} \tag{13}$$

where $\Phi^*_{lower}$ and $\Phi^*_{upper}$ are the lower and upper bounds of $\Phi^*$, which is the optimal value of the subproblem (9). $\Phi$ is a scalar variable, whose relation with $\boldsymbol{\omega}$ is given in the feasibility constraints. $m \in \mathbb{N}^+$ denotes the iteration index of Benders decomposition, and $\boldsymbol{\omega}(m)$ in the slave problem is given by solving the master problem at the $m$th iteration.

The objective function of the master problem has the same physical meaning as that of the subproblem (9), but its constraints are loosened, thus solving the master problem yields a lower bound $\Phi^*_{lower}$. However, at every iteration a feasibility constraint or an infeasibility constraint will be added into the master problem which narrows the search region of integer variables, thus finally the optimal integer variables can be achieved. The optimal value of the slave problem $\Phi^*_{upper}$ is an upper bound because the given integer variables $\boldsymbol{\omega}(m)$ may not be optimal and what we obtain could just be a feasible solution. Consequently, $\Phi^*$ lies between $\Phi^*_{lower}$ and $\Phi^*_{upper}$, and through the iterations between the two problems (12) and (13), the gap between $\Phi^*_{lower}$ and $\Phi^*_{upper}$ will gradually decrease. When the gap is small enough, $\Phi^*$ is achieved.

The whole process of Benders decomposition algorithm is detailed as follows:

**Step 1: Initialization**

Set the iteration index $m = 1$, $\Phi^*_{lower} = -\infty$ and $\Phi^*_{upper} = +\infty$. The feasibility and infeasibility constraints are set to null. Randomly choose the initial integer variables $\boldsymbol{\omega}(1)$, as long as it satisfies the constraint (1).

**Step 2: Solving Slave Problem (at the $\boldsymbol{m}$th iteration)**

Given the integer variables $\boldsymbol{\omega}(m)$, the primal slave problem is formulated as (13).

By introducing the Lagrangian multipliers $\boldsymbol{\delta_c} \triangleq [\delta_c^n]_{n\in\mathcal{N}}$, $\boldsymbol{\delta_d} \triangleq [\delta_d^n]_{n\in\mathcal{N}}$, $\boldsymbol{\zeta} \triangleq [\zeta^n]_{n\in\mathcal{N}}$, $\boldsymbol{\eta} \triangleq [\eta^n]_{n\in\mathcal{N}}$ and $\rho$ for the corresponding constraints, where $\delta_c^n, \delta_d^n, \zeta^n, \eta^n \geq 0$, we define the Lagrangian for the primal slave problem (13) as

$$\begin{aligned}
&\mathcal{L}_2(\boldsymbol{p}, \boldsymbol{\delta_c}, \boldsymbol{\delta_d}, \boldsymbol{\zeta}, \boldsymbol{\eta}, \rho)\\
&= \sum_{n\in\mathcal{N}} \left[r^n \left(p_c^n - p_d^n\right) + \alpha p_d^n\right] + \sum_{n\in\mathcal{N}} \left\{\delta_c^n \left[p_c^n - \omega_c^n(m) p_c^{\max}\right]\right\}\\
&+ \sum_{n\in\mathcal{N}} \left\{\delta_d^n \left[p_d^n - \omega_d^n(m) p_d^{\max}\right]\right\}\\
&+ \sum_{n\in\mathcal{N}} \left\{\zeta^n \left[\pi^0 + \sum_{t=1}^{n} \left(\beta^c p_c^t - \frac{p_d^t}{\beta^d}\right) - \pi^{\max}\right]\right\}\\
&+ \sum_{n\in\mathcal{N}} \left\{\eta^n \left[\pi^{\min} - \pi^0 - \sum_{t=1}^{n} \left(\beta^c p_c^t - \frac{p_d^t}{\beta^d}\right)\right]\right\}\\
&+ \rho \left[\pi^0 + \sum_{n\in\mathcal{N}} \left(\beta^c p_c^n - \frac{p_d^n}{\beta^d}\right) - \pi^T\right]. \qquad (14)
\end{aligned}$$

By applying the following transformation:

$$\begin{aligned}
&\sum_{n\in\mathcal{N}} \left[(\zeta^n - \eta^n) \sum_{t=1}^{n} \left(\beta^c p_c^t - \frac{p_d^t}{\beta^d}\right)\right]\\
&\qquad = \sum_{n\in\mathcal{N}} \left[\left(\beta^c p_c^n - \frac{p_d^n}{\beta^d}\right) \sum_{t=n}^{N} (\zeta^t - \eta^t)\right] \qquad (15)
\end{aligned}$$

we can rewrite the Lagrangian (14) as

$$\begin{aligned}
&\mathcal{L}_2(\boldsymbol{p}, \boldsymbol{\delta_c}, \boldsymbol{\delta_d}, \boldsymbol{\zeta}, \boldsymbol{\eta}, \rho)\\
&= \sum_{n\in\mathcal{N}} \left[r^n + \delta_c^n + \beta^c \rho + \beta^c \sum_{t=n}^{N} (\zeta^n - \eta^n)\right] p_c^n\\
&+ \sum_{n\in\mathcal{N}} \left[\alpha - r^n + \delta_d^n - \frac{\rho}{\beta^d} - \frac{1}{\beta^d} \sum_{t=n}^{N} (\zeta^n - \eta^n)\right] p_d^n\\
&+ \sum_{n\in\mathcal{N}} \left[-\omega_c^n(m) p_c^{\max} \delta_c^n - \omega_d^n(m) p_d^{\max} \delta_d^n\right.\\
&\left. + (\pi^0 - \pi^{\max}) \zeta^n + (\pi^{\min} - \pi^0) \eta^n\right] + (\pi^0 - \pi^T) \rho. \qquad (16)
\end{aligned}$$

Note that $p_c^n, p_d^n \geq 0$, we define the corresponding dual slave problem as follows:

**Dual Slave Problem**:

$$\begin{aligned}
\max_{\substack{\boldsymbol{\delta_c}, \boldsymbol{\delta_d},\\ \boldsymbol{\zeta}, \boldsymbol{\eta}, \rho}} \quad & \Psi(\boldsymbol{\delta_c}, \boldsymbol{\delta_d}, \boldsymbol{\zeta}, \boldsymbol{\eta}, \rho) = \sum_{n\in\mathcal{N}} \left[-\omega_c^n(m) p_c^{\max} \delta_c^n\right.\\
& - \omega_d^n(m) p_d^{\max} \delta_d^n + (\pi^0 - \pi^{\max}) \zeta^n\\
& \left. + (\pi^{\min} - \pi^0) \eta^n\right] + (\pi^0 - \pi^T) \rho\\
\text{s.t.} \quad & r^n + \delta_c^n + \beta^c \rho + \beta^c \sum_{t=n}^{N} (\zeta^n - \eta^n) \geq 0, \ \forall n \in \mathcal{N}\\
& \alpha - r^n + \delta_d^n - \frac{\rho}{\beta^d} + \frac{1}{\beta^d} \sum_{t=n}^{N} (\zeta^n - \eta^n) \geq 0, \ \forall n \in \mathcal{N}\\
& \delta_c^n, \delta_d^n, \zeta^n, \eta^n \geq 0, \ \forall n \in \mathcal{N}. \qquad (17)
\end{aligned}$$

In the dual slave problem (17), $\omega_c^n(m)$ and $\omega_d^n(m)$ are given integer variables, while $\delta_c^n, \delta_d^n, \zeta^n, \eta^n$ and $\rho$ are continuous variables to be solved. Since (13) is a linear program, strong duality is guaranteed and there exists no duality gap, which means the optimal value of the primal slave problem (13) is identical to that of the dual slave problem (17), i.e., $\Phi^*_{upper} = \min_{\boldsymbol{p}} \Phi = \max_{\substack{\boldsymbol{\delta_c}, \boldsymbol{\delta_d},\\ \boldsymbol{\zeta}, \boldsymbol{\eta}, \rho}} \Psi$. Therefore, it is equivalent to solve either problem.

**Step 3: Solving Master Problem (at the $(\boldsymbol{m}+1)$ th iteration)**

According to the solution to the dual slave problem (17), integer variables can be improved by adding a constraint into the master problem (12) as long as the given ones are not optimal.

1) If the dual slave problem (17) is infeasible, the subproblem (9) will have an unbounded solution which is impractical. Therefore, there exists no solution with a precise physical meaning under this circumstance.
2) If the dual slave problem (17) has a bounded solution $\boldsymbol{\delta_c}(m)$, $\boldsymbol{\delta_d}(m)$, $\boldsymbol{\zeta}(m)$, $\boldsymbol{\eta}(m)$ and $\rho(m)$, due to duality the primal slave problem (13) is feasible. Let $\mathcal{I}$ denote the set of iterations at which the solution to the dual slave problem (17) is bounded. In this case the $m$th iteration should be added into $\mathcal{I}$, i.e., $\mathcal{I} = \mathcal{I} \cup m$. The optimal solution obtained from the primal slave problem (13) with the given integer variables is a feasible solution to the subproblem (9), but better solutions may exist. As a result, the optimal value of the primal slave problem (13) turns out to be an upper bound of $\Phi^*$ and $\Phi^*_{upper} = \min\{\Phi^*_{upper}, \Phi^*_{upper}(m)\}$. At the same time, a new feasibility constraint is generated and added into

the master problem (12) based on the bounded solution to the dual slave problem (17) at the $m^{th}$ iteration, which raises the lower bound of $\Phi^*$. Therefore, at the $(m+1)^{th}$ iteration the master problem (12) is confined by a set of feasibility constraints described as follows:

**Feasibility Constraints**:

$$\Phi \geq \sum_{n\in\mathcal{N}} [-\omega_c^n(m+1)p_c^{\max}\delta_c^n(i) - \omega_d^n(m+1)p_d^{\max}\delta_d^n(i) + (\pi^0 - \pi^{\max})\zeta^n(i) + (\pi^{\min} - \pi^0)\eta^n(i)] + (\pi^0 - \pi^T)\rho(i), \quad \forall i \in \mathcal{I}. \tag{18}$$

3) If the dual slave problem (17) has an unbounded solution, due to duality the primal slave problem (13) is infeasible with the given integer variables $\boldsymbol{\omega}(m)$. Therefore, these integer variables should be ruled out of our search region. Let $\mathcal{J}$ denote the set of iterations at which the solution to the dual slave problem (17) is unbounded. Under this circumstance the $m$th iteration should be added into $\mathcal{J}$, i.e., $\mathcal{J} = \mathcal{J} \cup m$. The direction of the unbounded solution to the dual slave problem (17) can be described as $\boldsymbol{\delta_c}(m)$, $\boldsymbol{\delta_d}(m)$, $\boldsymbol{\zeta}(m)$, $\boldsymbol{\eta}(m)$ and $\rho(m)$, which can be obtained by solving the feasibility check problem (FCP) and its corresponding dual problem. The FCP is an optimization problem aiming to check the feasibility of the primal slave problem (13) and figure out the direction of the unbounded solution to the dual slave problem (17). Then a new infeasibility constraint is generated and added into the master problem (12) to exclude the given integer variables based on the direction of the unbounded solution to the dual slave problem (17) at the $m$th iteration. Therefore, at the $(m+1)$th iteration the master problem (12) is confined by a set of infeasibility constraints described as follows:

**Infeasibility Constraints**:

$$0 \geq \sum_{n\in\mathcal{N}} [-\omega_c^n(m+1)p_c^{\max}\delta_c^n(j) - \omega_d^n(m+1)p_d^{\max}\delta_d^n(j) + (\pi^0 - \pi^{\max})\zeta^n(j) + (\pi^{\min} - \pi^0)\eta^n(j)] + (\pi^0 - \pi^T)\rho(j), \quad \forall j \in \mathcal{J}. \tag{19}$$

The detailed derivation of feasibility and infeasibility constraints is given in the Appendix.

Dynamically, at every iteration there is always one new constraint added into the master problem (12). Through solving the newly modified master problem with tighter constraints, the lower bound $\Phi^*_{lower}$ is certainly lifted.

$\Phi^*_{upper}$ is compared with $\Phi^*_{lower}$ at the end of every iteration. Let $\varepsilon$ denote a sufficiently small positive threshold. If $|\Phi^*_{upper} - \Phi^*_{lower}| \leq \varepsilon$, which means the upper and lower bounds converge to the optimal solution, the whole process ceases. Otherwise, the algorithm goes to step 2 and repeats the iteration.

## V. Distributed Implementation

The scheduling problem (6) can be addressed in a distributed manner, namely through the iterations between the aggregator level and EV level. As shown in Fig. 2, at the aggregator level Lagrangian multipliers are updated according to feedbacks from all EVs, while at the EV level every controller optimizes its EV's charging strategy based on virtual prices released by the aggregator. In a sense, Lagrangian multipliers serve as a coordination signal which aligns individual welfare with social welfare.

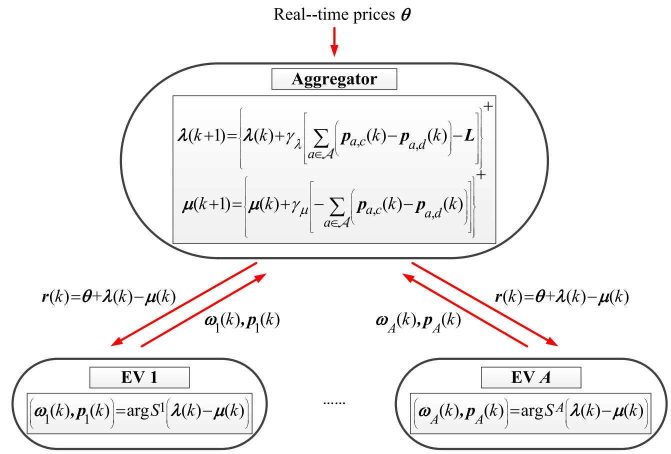


Fig. 2. Distributed implementation.

First, the aggregator sets the initial Lagrangian multipliers, e.g., $\lambda^n(0), \mu^n(0) = 0, \forall n \in \mathcal{N}$. During iteration $k$, given $\boldsymbol{\lambda}(k)$ and $\boldsymbol{\mu}(k)$, the aggregator releases the virtual prices $[r^n(k)]_{\forall n\in\mathcal{N}}$ to all controllers, where $r^n(k) \triangleq \theta^n + \lambda^n(k) - \mu^n(k)$. Every controller solves its own subproblem (9) independently based on the virtual prices to obtain an optimal charging strategy. Then the charging strategy of each EV is reported back to the aggregator. Upon receiving all feedbacks $[\boldsymbol{\omega}(k), \boldsymbol{p}(k)]$, the aggregator updates the Lagrangian multipliers according to (11) and then releases the new virtual prices $[r^n(k+1)]_{\forall n\in\mathcal{N}}$ to all controllers. Note that given $\boldsymbol{\lambda}(k)$ and $\boldsymbol{\mu}(k)$, the dual function (8) is simplified and the update rule of the Lagrangian multipliers for any time slot $n \in \mathcal{N}$ can be easily derived:

$$\begin{cases} \lambda^n(k+1) = \left\{\lambda^n(k) + \gamma_\lambda\left[\sum_{a\in\mathcal{A}}\left(p^n_{a,c}(k) - p^n_{a,d}(k)\right) - L^n\right]\right\}^+ \\ \mu^n(k+1) = \left\{\mu^n(k) + \gamma_\mu\left[-\sum_{a\in\mathcal{A}}\left(p^n_{a,c}(k) - p^n_{a,d}(k)\right)\right]\right\}^+ \end{cases}. \tag{20}$$

The iterations between the aggregator and EVs will continue until an equilibrium is reached. Then the optimal virtual prices $[\theta^n + \lambda^{n*} - \mu^{n*}]_{n\in\mathcal{N}}$ are finally released by the aggregator, which ensures the total social cost is minimized.

## VI. Numerical Results

For ease of illustration, we first take a charging station with 3 EVs into consideration. Then the proposed approach is applied to large-scale charging stations to demonstrate its advantages of distributed implementation and parallel computation. EVs are assumed to be identical with parameters listed in Table I. The hourly-based real-time prices of the four time slots are given as 9.5, 8.3, 6.2, 5.3 (cent/kWh), and the corresponding maximum allowable loads for the charging station are assumed to be 9.9, 10.8, 13.5, 14.2 (kW). These parameter settings are used, mainly for the purpose of revealing how the mechanism works under different load and price conditions. All the following simulation results are obtained by MATLAB R2012a running a laptop PC

TABLE I
PARAMETER SETTINGS FOR EVs

| Parameter \ EV | 1 | 2 | 3 |
|---|---|---|---|
| $\alpha_a$ | 0.01 | 0.01 | 0.01 |
| $p_{a,c}^{\max}(kW)$ | 5 | 5 | 5 |
| $p_{a,d}^{\max}(kW)$ | 3 | 3 | 3 |
| $\beta_a^c$ | 0.99 | 0.99 | 0.99 |
| $\beta_a^d$ | 0.99 | 0.99 | 0.99 |
| $\pi_a^0(kWh)$ | 16 | 12 | 12 |
| $\pi_a^{\min}(kWh)$ | 2 | 2 | 2 |
| $\pi_a^{\max}(kWh)$ | 24 | 24 | 24 |
| $\pi_a^T(kWh)$ | 22.8 | 22.8 | 22.8 |
| $N_a(h)$ | 4 | 3 | 3 |

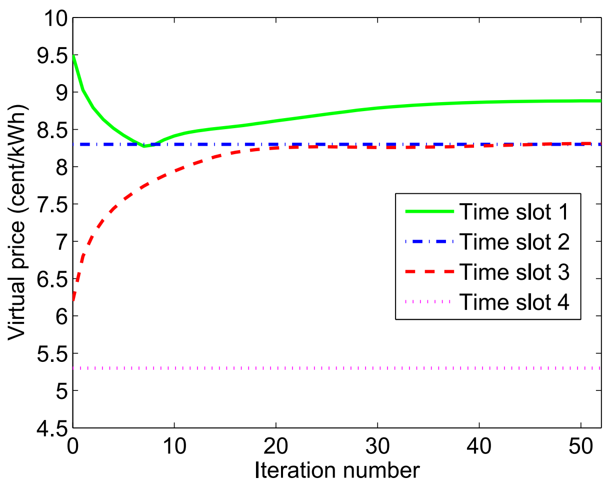


Fig. 3. Convergence of virtual prices.

with Intel Core i7-3632QM CPU @ 2.20 GHz, 8 GB RAM, and 64-bit Windows 8.1 OS.

Fig. 3 shows the convergence of the virtual prices. The virtual prices of time slot 2 and 4 remain the same as the original real-time prices, since the load of either time slot satisfies the corresponding load constraint without the aggregator's coordination. At these two time slots, only the aggregator sells electricity energy and all the 3 EVs are charging. Under this circumstance, the aggregator is a non-profit intermediary. However, the situation is different at time slot 1 and 3, where their virtual prices are iteratively adjusted and ultimately converge to the optimal prices. As the original real-time price of time slot 1 is quite high, the EVs would rather not charge, and may even discharge if possible. In order to avoid unbalanced discharging, the aggregator has to lower the virtual price. Under this circumstance, the aggregator just sets the proper virtual price such that the energy transfer among EVs is balanced inside the charging station without procuring additional electricity energy from the power grid. Hence the aggregator is still non-profit under this circumstance. On the contrary, the original real-time price of time slot 3 is relatively low, thus the EVs would prefer charging with the maximum charging power, which makes the total load of the charging station exceed the corresponding maximum allowable load. Then the aggregator has to raise the virtual price, so that some EVs would shift part of their charging power to other time slots to ensure that the maximum allowable load would not be exceeded. Since there is a price rise, the aggregator is profitable under this circumstance. It can be observed that the aggregator aims to coordinate EVs' charging/discharging behavior by pricing to satisfy the load constraint of the charging station.

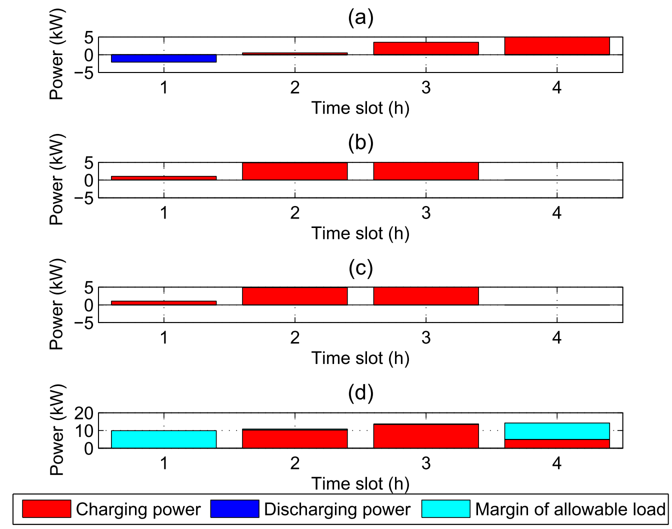


Fig. 4. Optimal scheduling on EVs' charging/discharging behavior.

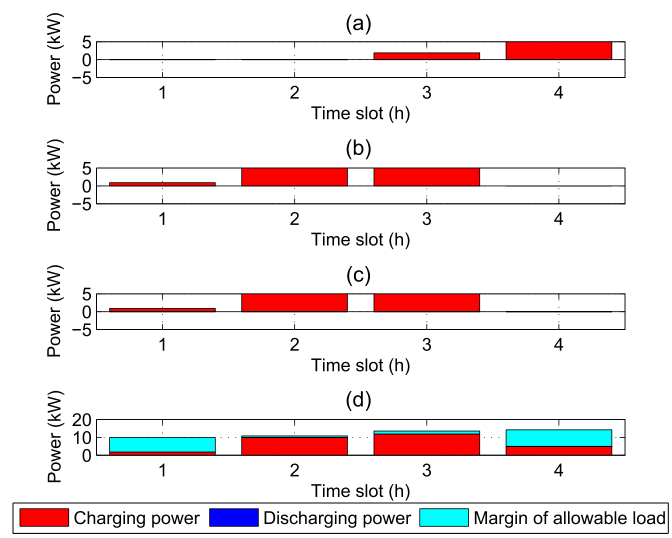


Fig. 5. Greedy scheduling on EVs' charging/discharging behavior.

As long as the convergence of the virtual prices is reached, the optimal scheduling on the EVs' charging/discharging behavior can be easily obtained, which is shown in Fig. 4. With sufficient time to be charged to the terminal energy level, EV 1 tends to discharge and get paid at time slot 1 when the virtual price is high. Then it chooses to charge with the maximum charging power at time slot 4 because the virtual price is the lowest. The virtual prices at time slot 2 and 3 are almost equal, thus EV 1 will adjust its charging power properly to avoid overloading of the charging station. Other EVs follow the similar principles, but there are a few special circumstances. For example, neither EV 2 nor EV 3 has the option to discharge due to lack of time. Fig. 4(d) shows the total charging load of the charging station. It is obvious that the load constraint for the charging station is satisfied. By the proposed approach, the total social cost is 195.17 cents.

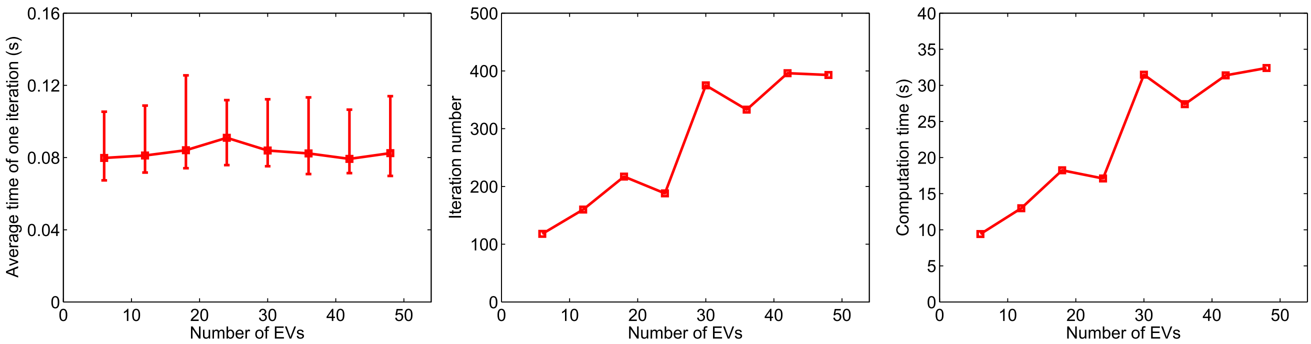


Fig. 6. Impact of EV number on (a) average time of one iteration, (b) iteration number, and (c) computation time.

For comparison, Fig. 5 shows the greedy scheduling on the EV's charging/discharging behavior with a heuristic strategy, which charges the EVs at the possible lowest-price time slots, with the possible largest charging power in priority order of deadline pressure. Note that this strategy requires the aggregator to have centralized control over all the EVs, which is not practical. Moreover, the aggregator loses much scheduling flexibility by this strategy to avoid overloading of the charging station. Under this circumstance, the total social cost is 200.36 cents. Therefore, we can observe that it is very effective to regulate the charging load of EVs by pricing. That is, by the proposed approach there is a 2.6% reduction in the total social cost, and in addition, much more scheduling flexibility is provided.

In order to show the scalability of our proposed approach, we increase the number of EVs to evaluate its impact on the average time of one iteration, the iteration number and the total computation time, respectively.[1] The deadlines of EVs are extended to 8 time slots at most to cover a more general case. The optimal scheduling with different numbers of EVs, starting with 6 and going up to 48, is investigated and the preceding three types of data are recorded, presented in Fig. 6. Fig. 6(a) shows that the minimum, average and maximum time of one iteration all vary little with the increase of EVs. However, as the EV number increases, the iteration number increases in a linear trend with slight oscillation, which is shown in Fig. 6(b). Since the average time of one iteration is roughly constant, the total computation time is mainly dependent on the iteration number. As a result, the profile of the total computation time, as shown in Fig. 6(c), is similar to that of the iteration number. It can be observed that the total computation time required for convergence increases almost linearly instead of exponentially. Approximately tens of seconds are needed to solve the optimization problem for a charging station with tens of EVs. Since it usually takes hours to charge an EV in daily life, our proposed approach is adequate for the scheduling of a large-scale charging station. Furthermore, as a comparison we directly solve the complete scheduling problem via Benders decomposition in a centralized manner. The computation time of centralized Benders decomposition, as shown in Table II, grows explosively as a charging station scales up. It takes more than 5 h, around 1000 times of the computation time of the proposed approach, to work out the optimal strategy for a 9-EV schedule, which apparently fails to meet the requirement of real-time applications.

[1]All the computation time in this paper is obtained by our laptop PC to show the relative computational efficiency.

TABLE II
CENTRALIZED BENDERS DECOMPOSITION

| Number of EV | 3 | 6 | 9 |
|---|---|---|---|
| Number of integer variables | 48 | 96 | 144 |
| Number of continuous variables | 48 | 96 | 144 |
| Number of constraints | 123 | 238 | 353 |
| Computation time (s) | 9.3297 | $1.3848\times10^3$ | $1.9276\times10^4$ |
| Time multiple of proposed approach | 1.1381 | $1.4707\times10^2$ | $1.7892\times10^3$ |

## VII. CONCLUSION AND FUTURE WORK

In this paper, the optimal scheduling on EVs' charging/discharging behavior in a charging station is investigated. Based on the newly proposed energy transfer mechanism V2V, we formulate the scheduling problem as an MILP with a load constraint for the charging station and several charging constraints for each EV. Since the load constraint couples all EVs' charging/discharging behavior together, dual decomposition is introduced to decompose the primal problem into a series of subproblems. Each subproblem is still an MILP, but can be solved on each EV. Benders decomposition is then applied to efficiently solve each subproblem. The proposed approach can be implemented in a distributed manner, thus it is suitable for the scheduling of large-scale charging stations which can hold a great quantity of EVs. Numerical results validate our theoretical analysis.

We consider a static scheduling problem with a fixed optimization horizon and a constant number of EVs in this paper. Such a simplification avoids the uncertainties due to stochastic arrivals of EVs, prediction deviation of real-time prices, etc. In practical applications, a receding-horizon framework can be applied to deal with all these uncertainties, which contains two steps, i.e., first, focus on the existing EVs, assume all future parameters are known (e.g., forecasts from historical data), and solve a deterministic scheduling problem; second, consider the arrivals/departures of EVs, update the parameters (forecasts) in real time as more information becomes available, and re-solve the scheduling problem. By this means, the proposed approach can be implemented online and the resulting charging schedule is updated in real time [30], [31].

In future work some relevant extension directions, e.g., EV demand forecasting, dynamic price forecasting, as well as the fail-safe operation, emergency operation and reliability evaluation of a charging station which operates under V2V mechanism, might be considered. Also, a charging station, as an economic entity, is important to have a profitable business model and the end-to-end economic analysis could be included. Moveover, we would like to study the interactions and mutual effects of multiple charging stations, which involves a higher aggregation level.

## Appendix
## Normalized Classical Benders Decomposition

Consider a normalized form of a mixed-integer programming problem as follows:

$$\begin{aligned} Z^* = \min_{x,y} \quad & Z(x,y) = c^T x + f(y) \\ \text{s.t.} \quad & Ax \le b_1 \\ & F(y) \le b_2 \\ & Ex + G(y) \le b_3 \\ & x \ge 0,\ y \in \mathcal{S} \end{aligned} \tag{21}$$

where $\mathcal{S}$ is a subset of $\mathbb{Z}^q$ with integer-valued components. $x$ is a continuous vector and $y$ is an integer vector. Note that $c$ and $x$ are $v$-dimensioned vectors, $y$ is a $q$-dimensioned vector, $b_1$ is a $u$-dimensioned vector, $b_2$ is a $p$-dimensioned vector, $b_3$ is a $w$-dimensioned vector, $A$ is a $u \times v$ matrix, $E$ is a $w \times v$ matrix. $Z(x,y)$ and $f(y)$ are scalar functions, $F(y)$ is a $p$-dimensioned vector function and $G(y)$ is a $w$-dimensioned vector function. All the three functions are defined on $\mathcal{S}$. We mainly focus on the derivation of the feasibility constraints and infeasibility constraints, thus $Z^*$, the optimal value of the mixed-integer programming problem (21), is assumed to be bounded.

If $y$ is given, the mixed-integer programming problem (21) is linear in $x$, and it can be rewritten as

$$\min\left\{f(y)|y \in \mathcal{G} + \min\left\{c^T x | x \in \mathcal{H}(y)\right\}\right\} \tag{22}$$

where

$$\begin{aligned} \mathcal{G} = & \{y|\ there\ exists\ x \ge 0\ such\ that\ Ex \le b_3 - G(y), \\ & F(y) \le b_2,\ y \in \mathcal{S}\} \\ \mathcal{H}(y) = & \{x | Ax \le b_1,\ Ex \le b_3 - G(y),\ x \ge 0\}. \end{aligned}$$

Therefore, the mixed-integer programming problem (21) can be decomposed into a master problem and a slave problem, and the optimal solution can be obtained by an iterative approach.

At every iteration, e.g., the $m^{th}$ iteration, we begin with solving the following master problem to obtain $y(m)$:

$$\begin{aligned} Z^*_{lower} = \min_{y,Z} \quad & Z \\ \text{s.t.} \quad & F(y) \le b_2 \\ & feasibility\ constraints \\ & infeasibility\ constraints. \end{aligned} \tag{23}$$

At the same time, a lower bound of $Z^*$ is procured, since the constraints of the master problem (23) are loosened compared with those of the mixed-integer programming problem (21). In the master problem the objective function is a scalar variable, whose relation with $y$ is given in the feasibility constraints. Note that the feasibility constraints and infeasibility constraints are set to null initially, thus at the first iteration $y(1)$ is given arbitrarily, as long as it satisfies $F(y(1)) \le b_2$.

The inner part of (22) is the primal slave problem, which can be rewritten as

$$\begin{aligned} \min_x \quad & c^T x \\ \text{s.t.} \quad & Ax \le b_1 \\ & Ex \le b_3 - G(y(m)) \\ & x \ge 0 \end{aligned} \tag{24}$$

and the dual slave problem is given as

$$\begin{aligned} \max_{\lambda,\mu} \quad & (G(y(m)) - b_3)^T \mu - b_1^T \lambda \\ \text{s.t.} \quad & c + A^T\lambda + E^T \mu \ge 0 \\ & \lambda \ge 0,\ \mu \ge 0. \end{aligned} \tag{25}$$

Due to strong duality, it's equivalent to solve either problem. In terms of the dual slave problem (25), according to its solution, two different types of constraints will be added into the master problem.

If the dual slave problem (25) has a bounded solution $\lambda(m)$ and $\mu(m)$, an upper bound of $Z^*$ is obtained:

$$Z^*_{upper} = (G(y(m)) - b_3)^T \mu(m) - b_1^T \lambda(m) + f(y(m)). \tag{26}$$

Note that the feasible region of the dual slave problem (25) is a fixed polyhedron, which is characterized by its vertices ($[\lambda(h), \mu(h)]_{h\in\mathcal{H}}$, where $\mathcal{H}$ represents the finite set of all vertices of the polyhedron). The polyhedron is independent of the given $y(m)$. As the bounded optimal value of the dual slave problem (25) must be achieved on a vertex of this polyhedron, we can make use of this property to approach the optimal solution to the mixed-integer programming problem (21), i.e., $(x^*, y^*)$.

Note that

$$\begin{aligned} \min_y\, & (G(y) - b_3)^T \mu(m) - b_1^T\lambda(m) + f(y) \\ & \le (G(y^*) - b_3)^T \mu(m) - b_1^T \lambda(m) + f(y^*) \\ & \le \max_{\lambda,\mu} (G(y^*) - b_3)^T \mu - b_1^T\lambda + f(y^*) \\ & = (G(y^*) - b_3)^T \mu^* - b_1^T \lambda^* + f(y^*) \\ & = Z^*. \end{aligned} \tag{27}$$

Therefore, $\min_y (G(y) - b_3)^T \mu(m) - b_1^T\lambda(m) + f(y)$ is actually a lower bound of $Z^*$. A corresponding feasibility constraint is then generated for the master problem (23):

$$Z \ge (G(y) - b_3)^T \mu(m) - b_1^T \lambda(m) + f(y). \tag{28}$$

If the dual slave problem (25) has an unbounded solution, i.e., the primal slave problem (24) is infeasible, the FCP is introduced to calculate the direction of the unbounded solution and generate an infeasibility constraint. The FCP is an optimization problem aiming to check the feasibility of the primal slave problem (24), which is formulated as follows:

$$\begin{aligned} \min_{\kappa_1,\kappa_2,x} \quad & 1^T\kappa_1 + 1^T\kappa_2 \\ \text{s.t.} \quad & Ax \le b_1 + \kappa_1 \\ & Ex \le b_3 - G(y(m)) + \kappa_2 \\ & \kappa_1 \ge 0,\ \kappa_2 \ge 0,\ x \ge 0 \end{aligned} \tag{29}$$

where $\kappa_1$ is a $u$-dimensioned vector and $\kappa_2$ is a $w$-dimensioned vector.

The dual problem of the FCP is given as

$$\begin{aligned} \max_{\lambda,\mu} \quad & (G(y(m)) - b_3)^T \mu - b_1^T\lambda \\ \text{s.t.} \quad & A^T\lambda + E^T\mu \ge 0 \\ & 1 \ge \lambda \ge 0,\ 1 \ge \mu \ge 0. \end{aligned} \tag{30}$$

Through solving the dual problem of the FCP (30), we can obtain the direction of the unbounded solution to the dual slave problem (25), i.e., $\lambda(m)$ and $\mu(m)$. Let $(\kappa_1^*, \kappa_2^*, x^*)$ denote the optimal solution to the FCP. Note that if the primal slave problem is infeasible, we have $1^T\kappa_1^* + 1^T\kappa_2^* > 0$. Due to strong duality, $(G(y(m)) - b_3)^T\mu(m) - b_1^T\lambda(m) = 1^T\kappa_1^* + 1^T\kappa_2^* > 0$. In order to avoid finding $y(m)$ again, i.e., to exclude $y(m)$ from the feasible region of $y$, a corresponding infeasibility constraint is generated for the master problem (23):

$$0 \ge (G(y) - b_3)^T \mu(m) - b_1^T\lambda(m). \tag{31}$$

To conclude, let $\mathcal{I}$ and $\mathcal{J}$ denote the two set of iterations at which the solution to the dual slave problem is bounded and unbounded, respectively, then the master problem is detailed as follows:

$$\begin{aligned} Z^*_{lower} = \min_{y,Z} \quad & Z \\ \text{s.t.} \quad & F(y) \le b_2 \\ & Z \ge (G(y) - b_3)^T \mu(i) - b_1^T\lambda(i) + f(y), \\ & \forall i \in \mathcal{I} \\ & 0 \ge (G(y) - b_3)^T \mu(j) - b_1^T\lambda(j), \\ & \forall j \in \mathcal{J}. \end{aligned} \tag{32}$$

## ACKNOWLEDGMENT

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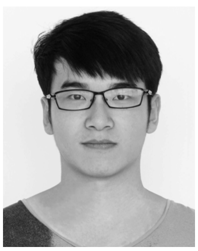

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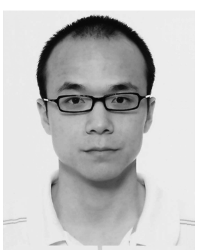

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