Battery Swapping Assignment for Electric Vehicles: A Bipartite Matching Approach

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ABSTRACT

This paper formulates a multi-period optimal station assignment problem for electric vehicle (EV) battery swapping that takes into account both temporal and spatial couplings. The goal is to reduce the total EV cost and station congestion due to temporary shortage in supply of available batteries. We show that the problem is reducible to the minimum weight perfect bipartite matching problem. This leads to an efficient solution based on the Hungarian algorithm. Numerical results suggest that the proposed solution provides a significant improvement over a greedy heuristic that assigns EVs to nearest stations.

1 INTRODUCTION

EVs are large loads that can add significant stress to electricity grids, but they are also flexible loads that can help mitigate the volatility of renewable generation through smart charging. EV charging however takes a long time. It is not suitable for commercial vehicles, such as taxis, buses, and ride-sharing cars, that are on the road most of the time, the opposite of most private cars. An alternative EV refueling method is battery swapping where an EV swaps its depleted battery for a fully-charged battery at a service station[1]. This can be done in a few minutes. Several such electric taxi programs are in pilot in China [2].

1.1 Literature review

The literature on scheduling of EV battery swapping is small. In [4] the operation of a battery charging and swapping station is modeled as a mixed queuing network, consisting of an interior closed queue of batteries going through charging and swapping, and an exterior open queue of EV arrivals. Using this model, [3] proposes an optimal charging policy. An optimal assignment problem is formulated in [5, 6] that assigns to a given set of EVs best stations to swap their batteries. The assignment aims to minimize a weighted sum of generation cost and EVs' travel distance by jointly optimizing power flow variables. The problem focuses on spatial optimization over power grid operation for a single time slot during which the set of EVs is fixed.

1.2 Summary

This paper investigates the battery swapping assignment for EVs and takes into account temporal optimization where EVs arrive over several time slots. Specifically, we adopt a discrete time model. In each time slot, a centralized operator optimally assigns stations to a set of EVs that need battery swapping. Consider the optimal station assignment problem at time slot 1 where stations are assigned in a way that minimizes both the total EVs' cost to travel to their assigned stations and the total congestion (battery shortage) levels at these stations. The problem is a binary program with strong temporal and spatial couplings. We show that it is polynomial-time solvable by reducing it to the standard minimum weight perfect bipartite matching problem. This leads to a solution based on the Hungarian algorithm for bipartite matching problems.

2 PROBLEM FORMULATION

Consider a group of EVs that swap their depleted batteries for fully-charged ones at stations assigned by a central operator. Time is slotted with a constant length. Fix the current time slot as time slot 1 of the time horizon $\mathbb{T} := \{-T_m + 1, \ldots, 0, 1, \ldots, T_m\}$, and let $\mathbb{T}^+ := \{1, \ldots, T_m\}$. T_m is a constant which we will interpret later. Suppose there is a set $\mathbb{J} := \{1, \ldots, J\}$ of stations that provide battery swapping service for EVs. At the current time slot 1, let $\mathbb{I} := \{1, \ldots, I\}$ be the set of EVs that require battery swapping. Our goal is to optimally assign a station $j \in \mathbb{J}$ to each EV $i \in \mathbb{I}$, such that a weighted sum of aggregate EV cost and station congestion is minimized.

2.1 Variables, states, and constraints

Let $M := (M_{ij}, i \in \mathbb{I}, j \in \mathbb{J})$ represent the current station assignment to EVs, where $M_{ij} = 1$ if station j is assigned to EV i and $M_{ij} = 0$ otherwise. We require that only one station be assigned to each EV, i.e.,

$$\begin{cases} \sum_{j \in \mathbb{J}} M_{ij} = 1, & i \in \mathbb{I} \\ M_{ij} \in \{0, 1\}, & i \in \mathbb{I}, j \in \mathbb{J} \end{cases} \tag{1}$$

Note that we also use $M_{ij}(t)$, $t = -T_m + 1, -T_m + 2, \ldots, 0$, to represent past assignments, which are given.

Let $\tau_{ij}(t)$ estimate the arrival time of EV i if it starts to travel at time slot t from its location to station j. It captures the time-dependent traffic conditions and also corresponds to an optimal routing, thus we can readily obtain the associated travel distance, denoted by $d_{ij}(t)$. We also define $\tau_{ij}^{-1}(t)$ as the inverse function of $\tau_{ij}(t)$, i.e., $\tau_{ij}^{-1}(t)$ is the time slot when station j was assigned to EV i that arrives at time slot t. For brevity, let $\tau_{ij} := \tau_{ij}(1)$ and $d_{ij} := d_{ij}(1)$, which are available by resorting to, say, Google Maps, and their explicit modeling goes beyond the scope of this paper.

Now we interpret T_m as the maximum travel time of an EV to reach a station, i.e., $T_m := \max_{i,j,t}\{\tau_{ij}(t) - t + 1\}$. The assignments before $-T_m + 1$ are summarized in n_j^0, and the states of stations after T_m will not be directly affected by the current assignment.

Let $n_j(t)$ denote the number of available (fully-charged) batteries at station j at the end of time slot t, which is the station state. In

[1]In this paper, stations refer to battery (swapping) stations.

particular, $n_j(0)$, i.e., the current number of available batteries at station j, is observed and given. Hence $n_j(t)$ increases by 1 when a battery at station j becomes fully-charged, and decreases by 1 when a fully-charged battery is removed by an EV (battery swapping time is ignored):

$$n_j(t) = n_j(t-1) + c_j(t) - \sum_{i \in \mathbb{I}_p} M_{ij}(\tau_{ij}^{-1}(t)) - \sum_{i \in \mathbb{I}} M_{ij} \cdot 1(t = \tau_{ij}), \qquad t \in \mathbb{T}^+ \tag{2}$$

where $c_j(t)$ is the number of batteries that become fully-charged at station j in time slot t (which is known a priori), and $\mathbb{I}_p$ is the set of all past EVs that were assigned stations during the time interval $[-T_m + 1, 0]$. $1(x)$ is an indicator function for the predicate x. The third and fourth terms on the right-hand-side of (2) summarize the impacts of past assignments and the current one, respectively. The second and third terms are both given while the fourth one is to be decided. Note that $n_j(t)$ can be negative. For instance, $n_j(t) = -3$ means there will be no available battery at the end of time slot t, but 3 waiting EVs.

An EV can only be assigned a station within its driving range, i.e.,

$$d_{ij} M_{ij} \le r s_i, \quad i \in \mathbb{I}, j \in \mathbb{J} \tag{3}$$

where r is the driving range per unit state of charge and s_i denotes the state of charge of EV i.

2.2 Optimal station assignment problem

The system cost has two components. First, a cost α_{ij} is incurred if station j is assigned to EV i, thus the cost of EV i is $\sum_{j \in \mathbb{J}} \alpha_{ij} M_{ij}$. For example, α_{ij} can be a weighted sum of EV i's travel distance and time from its current location to station j. Second, as explained above, $\langle -n_j(t) \rangle^+$ is the number of waiting EVs at the end of time slot t, where $\langle x \rangle^+ := \max\{x, 0\}$.

Let $n := (n_j(t), j \in \mathbb{J}, t \in \mathbb{T}^+)$ be the vector of station states. We are interested in the following *optimal station assignment* problem:

$$\begin{aligned} \min_{M, n} \quad & \sum_{i \in \mathbb{I}} \sum_{j \in \mathbb{J}} \alpha_{ij} M_{ij} + \sum_{j \in \mathbb{J}} \sum_{t \in \mathbb{T}^+} \langle -n_j(t) \rangle^+ \\ \text{s.t.} \quad & (1), (2), (3) \end{aligned} \tag{4}$$

which minimizes the weighted sum of aggregate EV cost and station congestion, subject to EVs' driving ranges.

3 POLYNOMIAL-TIME SOLUTION

The optimal station assignment problem (4) is a binary program with temporal coupling (2) and spatial coupling implied in station congestion. It can however be solved efficiently.

THEOREM 3.1. *The optimal station assignment problem* (4) *is solvable in polynomial time.*

We prove the theorem in the following two steps.

3.1 Reformulation as MILP

Note that all the constraints in (4) are linear in the variables (M, n). The only nonlinearity is $\langle -n_j(t) \rangle^+$, which can be removed by introducing auxiliary variables $v := (v_j(t), j \in \mathbb{J}, t \in \mathbb{T}^+)$ to replace aggregate station congestion by $\sum_j \sum_t v_j(t)$ and requiring $v_j(t)$ to satisfy the linear constraints $v_j(t) \ge 0$ and $v_j(t) \ge -n_j(t)$. Hence (4) is an MILP.

To reformulate it into a more convenient form, denote the number of available batteries at station j over $\mathbb{T}^+$ observed at time slot 1 before the current decision M is made by:

$$\tilde{n}_j(t) := n_j(0) + \sum_{\kappa=1}^{t} (c_j(\kappa) - \sum_{i \in \mathbb{I}_p} M_{ij}(\tau_{ij}^{-1}(\kappa))), \quad t \in \mathbb{T}^+$$

It is a known constant determined by past assignments. The evolution of $n_j(t)$ in (2) then reduces to

$$n_j(t) = \tilde{n}_j(t) - \sum_{i \in \mathbb{I}} M_{ij} \cdot 1(t \ge \tau_{ij}), \quad t \in \mathbb{T}^+ \tag{5}$$

which is decoupled across time slots, because $\tilde{n}_j(t)$ and the indicator function in (5) remove the dependency of $n_j(t)$ on $n_j(t-1)$.

The interpretation of $M_{ij} \cdot 1(t \ge \tau_{ij})$ in (5) is as follows. If station j is assigned to EV i at time slot 1, then it will arrive at time slot τ_{ij}, thus removing one available battery from station j for time slot τ_{ij} and every time slot afterwards. For each station $j \in \mathbb{J}$, define an arrival matrix $A_j \in \{0,1\}^{T_m \times I}$ such that its (t, i) entry is

$$A_j(t, i) := 1(t \ge \tau_{ij})$$

Finally, let Π denote the set of M with $M_{ij} = 0$ if station j is outside EV i's driving range, i.e., $d_{ij} > r s_i$, and put the above together, then (4) is equivalent to the following MILP:

$$\begin{aligned} \min_{M \in \Pi, v \ge 0} \quad & \sum_{j \in \mathbb{J}} \sum_{i \in \mathbb{I}} \alpha_{ij} M_{ij} + \sum_{j \in \mathbb{J}} \sum_{t \in \mathbb{T}^+} v_j(t) \\ \text{s.t.} \quad & \sum_{j \in \mathbb{J}} M_{ij} = 1, \quad i \in \mathbb{I} \\ & v_j(t) \ge -\tilde{n}_j(t) + \sum_{i \in \mathbb{I}} A_j(t, i) M_{ij}, \quad j \in \mathbb{J}, t \in \mathbb{T}^+ \end{aligned} \tag{6}$$

3.2 Reduction to Bipartite Matching

We now show that the MILP (6) can be further reduced to the minimum weight perfect bipartite matching problem, which is well known to be polynomial-time solvable.

Define a bipartite graph $\mathbb{G} = (\mathbb{A} \cup \mathbb{B}, \mathbb{E})$, where $\mathbb{A}$ and $\mathbb{B}$ are the bipartition of the vertex set and $\mathbb{E} \subseteq \mathbb{A} \times \mathbb{B}$ is the set of edges that are endowed with given weights $\omega := (\omega_{ab}, a \in \mathbb{A}, b \in \mathbb{B}, (a, b) \in \mathbb{E})$. Without loss of generality, we assume $\mathbb{G}$ is *complete* and *balanced* as we can add infinite-weight edges and dummy vertices as necessary. Let $N := |\mathbb{A}| = |\mathbb{B}|$. The standard minimum weight perfect matching problem defined on $\mathbb{G}$ is

$$\begin{aligned} \min_{x} \quad & \sum_{(a,b) \in \mathbb{E}} \omega_{ab} \, x_{ab} \\ \text{s.t.} \quad & \sum_{b} x_{ab} = 1, \quad a \in \mathbb{A} \\ & \sum_{a} x_{ab} = 1, \quad b \in \mathbb{B} \\ & x_{ab} \in \{0, 1\}, \quad a \in \mathbb{A}, b \in \mathbb{B} \end{aligned} \tag{7}$$

where $x := (x_{ab}, a \in \mathbb{A}, b \in \mathbb{B})$. Hence an instance of the bipartite matching problem (7) is specified by the nodes $\mathbb{A}, \mathbb{B}$ and the weights ω.

Given an instance of the MILP (6), we now construct an instance of the bipartite matching problem (7) such that an optimal solution to (7) yields an optimal solution to (6).

Let $\mathbb{A} := \mathbb{I} \cup \tilde{\mathbb{I}} \cup \mathbb{I}^d$. $\tilde{\mathbb{I}}$ is the set of EVs that were previously assigned stations, but have yet to have their batteries swapped (either on the way or waiting at stations). We restrict the matchings of EVs in $\tilde{\mathbb{I}}$

only with batteries at their originally assigned stations, as captured in (6). $\mathbb{I}^d$ is the set of dummy EVs if necessary to make $\mathbb{A}$ and $\mathbb{B}$ *balanced*.

Let $\mathbb{B} := \bigcup_{j\in\mathbb{J}} \mathbb{B}_j \cup \mathbb{B}^d$. $\mathbb{B}_j$ is the set of available batteries at station j, including not only the currently available batteries, but also those that will become available in $\mathbb{T}^+$. The time slot when battery $b \in \mathbb{B}_j$ becomes available is denoted as ρ_b, and $\rho_b = 0$ for the currently available batteries. $\mathbb{B}^d$ is the set of dummy batteries if necessary to make $\mathbb{A}$ and $\mathbb{B}$ *balanced*.

Note that $n_j(t)$ can be negative in (6). We have to make up the shortfall when $|\mathbb{I}\cup\tilde{\mathbb{I}}| > |\mathbb{B}_j|$ for station j by adding dummy batteries. More precisely, $\mathbb{B}^d := \bigcup_{j\in\mathbb{J}} \mathbb{B}^d_j$, where $|\mathbb{B}^d_j| = \max\{|\mathbb{I}\cup\tilde{\mathbb{I}}| - |\mathbb{B}_j|, 0\}$. Then $\mathbb{I}^d$ with $|\mathbb{I}^d| = |\bigcup_{j\in\mathbb{J}}(\mathbb{B}_j \cup \mathbb{B}^d_j)| - |\mathbb{I}\cup\tilde{\mathbb{I}}|$ is added to maintain balance between $\mathbb{A}$ and $\mathbb{B}$.

The nonnegative weight ω_{ab} of the match (a, b) corresponds to the incremental cost added to the objective of (6) if station j which battery b belongs to is assigned to EV a. Note that $\alpha_{ab} = \alpha_{aj}$, $d_{ab} = d_{aj}$ and $\tau_{ab} = \tau_{aj}$ when $a \in \mathbb{I}\cup\tilde{\mathbb{I}}$, $b \in \mathbb{B}_j \cup \mathbb{B}^d_j$. To determine ω_{ab}, the main idea is to translate the congestion of stations to the waiting time each EV suffers.

Case 1: $a \in \mathbb{I}, b \in \mathbb{B}_j$. Set $\omega_{ab} := \alpha_{ab} + \max\{\rho_b - \tau_{ab}, 0\}$. Here $\max\{\rho_b - \tau_{ab}, 0\}$ is the time length for which EV a has to wait until battery b becomes available. If $d_{ab} > rs_a$, $\omega_{ab} := \infty$.

Case 2: $a \in \mathbb{I}, b \in \mathbb{B}^d_j$. EVs matched with dummy batteries will wait until the end of $\mathbb{T}^+$ after their arrivals. Hence $\omega_{ab} := \alpha_{ab} + (T_m + 1 - \tau_{ab})$. If $d_{ab} > rs_a$, $\omega_{ab} := \infty$.

Case 3: $a \in \tilde{\mathbb{I}}, b \in \mathbb{B}_j$. EVs $a \in \tilde{\mathbb{I}}$ stick to their originally assigned stations. If station j is originally assigned to EV a, $\omega_{ab} := \max\{\rho_b - \tau_{ab}, 0\}$; otherwise, $\omega_{ab} := \infty$. No EV cost is included.

Case 4: $a \in \tilde{\mathbb{I}}, b \in \mathbb{B}^d_j$. Likewise, if station j is originally assigned to EV a, $\omega_{ab} := T_m + 1 - \tau_{ab}$; otherwise, $\omega_{ab} := \infty$.

Case 5: $a \in \mathbb{I}^d, b \in \mathbb{B}_j$. Dummy EVs do not really exist, and have no impact on the match result. Thus we have $\omega_{ab} := 0$.

Case 6: $a \in \mathbb{I}^d, b \in \mathbb{B}^d_j$. Likewise, $\omega_{ab} := 0$.

From above, the parameters of (7) including N and $(\omega_{ab}, a \in \mathbb{A}, b \in \mathbb{B})$ can be computed in time of $O(N^2)$ given an instance of (6). On the other hand, if we have an optimal matching x^* for (7), an optimal assignment is straightforward:

$$M^*_{ij} = \sum_{b\in\mathbb{B}_j\cup\mathbb{B}^d_j} x^*_{ib}, \quad i \in \mathbb{I}, j \in \mathbb{J} \tag{8}$$

which is obtainable in time of $O(N)$.

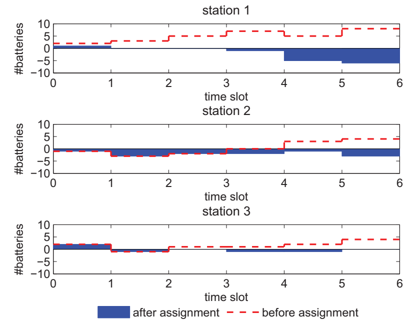
Hence the optimal station assignment problem (4) is reduced to the minimum weight perfect bipartite matching problem (7), which is solvable in polynomial time of $O(N^3)$ by the well-known Hungarian algorithm [1]. This proves Theorem 3.1.

4 NUMERICAL RESULTS

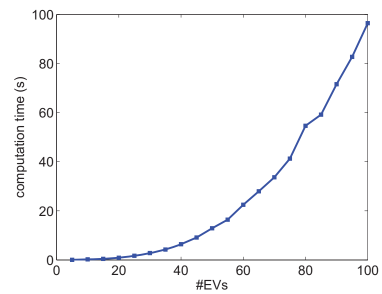
We illustrate with a case study of $I = 25$ EVs and $J = 3$ stations. Fix $T_m = 6$, and other parameters are randomly generated, given which $(\tilde{n}_j(t), j = 1, 2, 3, t = 1, 2, \ldots, 6)$ is attainable, as the red dash lines show in Fig. 1(a).

The proposed approach efficiently computes an optimal assignment; see Fig. 1(a) for how the number of available batteries at each station evolves after the assignment. Batteries at station 1 in the first half of the time horizon are almost fully utilized to avoid unduly congesting stations 2 and 3. In the second half, all stations run out of batteries. Then the optimal assignment strikes a compromise between the least EV cost and the latest time of arrival. For this test case, a 49.20% improvement is achieved by the proposed approach compared with a heuristic that assigns each EV to its nearest station.

We check the computational efficiency of the proposed approach by scaling up the number of EVs that require battery swapping while fixing other parameters with $J = 10$. The computation time required to run our algorithm on a normal laptop PC is shown in Fig. 1(b).



(a)



(b)

Figure 1: (a) Optimal assignment of our test case. (b) Scalability (#stations=10).

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