

Efficient Online Station Assignment for EV Battery Swapping

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ABSTRACT

This paper investigates the online station assignment for (commercial) electric vehicles (EVs) that make battery swapping requests to a central operator, with the aim of minimizing cost to EVs and congestion at service stations. Inspired by a polynomial-time solvable offline solution via a bipartite matching approach, we develop an efficient online station assignment algorithm that provably achieves a tight (optimal) competitive ratio under mild conditions.

1 INTRODUCTION

One way to drastically reduce carbon footprints is by electrifying transportation and increasing renewable generation of electricity. There are rising sales of commercial EVs, mainly driven by governmental promotion. For instance, over 150,000 commercial EVs were delivered in China in 2016, and most of them are fully electric taxis and buses.

Unlike private cars, these commercial EVs are on the road most of the time. They consume more energy, making them good candidates for electrification, but are less flexible in refueling. Battery swapping, where an EV swaps a depleted battery at a service station for a fully-charged one, is more suitable for commercial EVs. This model has been demonstrated in several pilot programs in China. The State Grid of China, for instance, operates several city-size battery swapping based electric taxi programs, which exemplify a vertically integrated system consisting of EVs, batteries and service stations.

In this paper we consider such a centralized system where EVs send their battery swapping requests to an operator when their batteries are running low. The operator assigns service stations in response to these requests based on locations of the requesting EVs and the availability of fully-charged batteries at service stations in the system. We design an efficient online algorithm for station assignment that aims to minimize the cost to EVs and congestion at service stations. To the best of our knowledge, this paper is the first to schedule EV battery swapping in a practical online setting.

2 SYSTEM MODEL

Suppose a central operator that also manages service stations takes charge of station assignment for battery swapping when EVs that are running out of energy make requests. We focus on real-time station assignments over a multi-hour continuous time horizon

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$\mathbb{T} := [0, T]$ where the supply of fully-charged batteries at each station is taken as a deterministic process by assuming battery charging is exogenous and given. An EV a that requests battery swapping at time r_a will also reveal its current position σ_a and state of charge s_a to the operator. Without knowing future battery swapping demand, the operator has to determine an irrevocable station assignment for EV a immediately based on the EV state, station state as well as exogenous real-time traffic flows.

Let $\mathbb{A} := \{1, 2, \dots, A\}$ denote the sequential set of EVs that request battery swapping within \mathbb{T} . The whole set \mathbb{A} is available only in hindsight, while in practice EVs $a \in \mathbb{A}$, along with their states including request times r_a , positions σ_a and states of charge s_a , are observed sequentially. Let $\mathbb{I} := \{1, 2, \dots, I\}$ be the set of service stations managed by the operator. Each station i at location σ_i maintains a certain number of available (fully-charged) batteries to serve forthcoming battery swapping requests. Define $n_i(t) > 0$ as the number of available batteries at station i at time t . Let $n := (n_i(t), i \in \mathbb{I}, t \in \mathbb{T})$. Note that in the case of heavy demand when $n_i(t) < 0$, i.e., there is a deficit in available batteries to serve EVs, $-n_i(t)$ is also interpreted as the number of EVs waiting for batteries at station i at time t , which we shall term as *station congestion* in this context.

Let an external function $\tau(\sigma, \sigma', t)$, where σ and σ' are arbitrary positions, estimate the arrival time if a vehicle sets off from σ to σ' at time t , given a city transportation network and real-time traffic conditions. We assume this function is handy for the operator. Every time an EV a requests battery swapping, the operator gathers $\tau_{ai} := \tau(\sigma_a, \sigma_i, r_a)$, $i \in \mathbb{I}$. Let $\theta_{ai} := \tau_{ai} - r_a$ be the corresponding travel time. Here we use r_a to approximate the time when EV a sets off, which further requires the response of station assignment to be quickly made. Meanwhile, τ_{ai} corresponds to a planned route with the travel distance denoted by d_{ai} . Suppose station i is assigned to EV a , it will arrive at time τ_{ai} and reduce $n_i(\tau_{ai})$ by 1.

The key decision variable is the instant station assignment to be determined upon each battery swapping request. Let $M := (M_a, a \in \mathbb{A})$ represent the station assignment variables for all EVs, where $M_a := (M_{ai}, i \in \mathbb{I})$ is the individual station assignment for EV a with

$$M_{ai} = \begin{cases} 1 & \text{if station } i \text{ is assigned to EV } a \\ 0 & \text{otherwise} \end{cases}$$

We require that only one station be assigned to each EV, i.e.,

$$M_{ai} \in \{0, 1\}, \quad a \in \mathbb{A}, i \in \mathbb{I} \quad (1a)$$

$$\sum_{i \in \mathbb{I}} M_{ai} = 1, \quad a \in \mathbb{A} \quad (1b)$$

At station i at time t , the change in $n_i(t)$ is captured by

$$\Delta n_i(t) = c_i(t) - \sum_{a \in \mathbb{A}} M_{ai} \cdot 1(t = \tau_{ai}), \quad i \in \mathbb{I}, t \in \mathbb{T} \quad (2)$$

where $c_i(t)$ is the number of batteries that turn available at station i at time t (which is known a priori), and $1(z)$ is an indicator function

for the predicate z . Therefore, $n_i(t)$ evolves according to

$$n_i(t)^+ = n_i(t)^- + \Delta n_i(t), \quad i \in \mathbb{I}, t \in \mathbb{T} \quad (3)$$

where $n_i(t)^+ := \lim_{y \rightarrow t^+} n_i(y)$ and $n_i(t)^- := \lim_{y \rightarrow t^-} n_i(y)$.

The global system cost over the whole time horizon \mathbb{T} has two components. First, certain EV cost α_{ai} will be incurred for EV a to reach the assigned station i . Let $\alpha_{ai} := \alpha_1 \theta_{ai} + \alpha_2 d_{ai}$, where α_1 and α_2 are proper nonnegative weights. Note that we require the planned route of EV a to station i should minimize α_{ai} . The cost of EV a is then $\sum_{i \in \mathbb{I}} \alpha_{ai} M_{ai}$. Second, the congestion of station i at time t can be represented by $\langle -n_i(t) \rangle^+$, where $\langle y \rangle^+ := \max\{y, 0\}$. Its integral congestion over \mathbb{T} is $\int_{t \in \mathbb{T}} \langle -n_i(t) \rangle^+ dt$. The goal of station assignment is to minimize both costs over all EVs and stations. However, the challenge is that they have to be determined in an online fashion:

Online station assignment problem

online decision variables: $M_a, a \in \mathbb{A}$

constraints: (1)

station state equations: (2)(3)

cost: $C(M) := \sum_{a \in \mathbb{A}} \sum_{i \in \mathbb{I}} \alpha_{ai} M_{ai} + \sum_{i \in \mathbb{I}} \int_{t \in \mathbb{T}} \langle -n_i(t) \rangle^+ dt$

3 ONLINE STATION ASSIGNMENT

The offline optimal station assignment problem is formalized as

$$\begin{aligned} \min_{M, n} \quad & \sum_{a \in \mathbb{A}} \sum_{i \in \mathbb{I}} \alpha_{ai} M_{ai} + \sum_{i \in \mathbb{I}} \int_{t \in \mathbb{T}} \langle -n_i(t) \rangle^+ dt \quad (4) \\ \text{s.t.} \quad & (1)(2)(3) \end{aligned}$$

In [2], a bipartite matching approach is proposed that can efficiently solve the above problem. Specifically, (4) is reducible to a problem of minimum weight maximum bipartite matching defined between EVs and individual batteries on a bipartite graph $\mathbb{G} = (\mathbb{A} \cup \mathbb{B}, \mathbb{E})$:

$$\min_x \quad \sum_{(a,b) \in \mathbb{E}} \omega_{ab} x_{ab} \quad (5a)$$

$$\text{s.t.} \quad \sum_b x_{ab} = 1, \quad a \in \mathbb{A} \quad (5b)$$

$$\sum_a x_{ab} \leq 1, \quad b \in \mathbb{B} \quad (5c)$$

$$x_{ab} \in \{0, 1\}, \quad (a, b) \in \mathbb{E} \quad (5d)$$

where \mathbb{A} and \mathbb{B} are the bipartition of the vertex set, representing respectively the EV set and the battery set, and $\mathbb{E} = \mathbb{A} \times \mathbb{B}$ is the set of all possible edges between \mathbb{A} and \mathbb{B} with endowed weights $\omega := (\omega_{ab}, (a, b) \in \mathbb{E})$. $x := (x_{ab}, (a, b) \in \mathbb{E})$ is the matching to be decided: $x_{ab} = 1$ if vertex a is matched with vertex b , and 0 otherwise. The standard form of (5) is well known to be polynomial-time solvable, e.g., by the Hungarian algorithm [1].

By letting $\mathbb{B} := \bigcup_{i \in \mathbb{I}} (\mathbb{B}_i \cup \mathbb{B}_i^d)$ with $|\mathbb{B}_i^d| := \max\{|\mathbb{A}| - |\mathbb{B}_i|, 0\}$, where \mathbb{B}_i and \mathbb{B}_i^d are respectively the sets of real batteries and dummy batteries, and $\omega_{ab} := \alpha_1 \theta_{ab} + \alpha_2 d_{ab} + \max\{\rho_b - \tau_{ab}, 0\}$, $(a, b) \in \mathbb{E}$, where ρ_b is the time when battery b turns available, an instance of (4) implies an instance of (5) in the sense that an optimal matching of (5) yields an optimal station assignment of (4). For more intuition, see [2].

We then propose our online station assignment algorithm by building on the online version of (5), which is formalized as

Online matching problem

online decision variables: $x_a, a \in \mathbb{A}$

constraints: (5b)-(5d)

aggregate weight: $W(x) := \sum_{(a,b) \in \mathbb{E}} \omega_{ab} x_{ab}$

Let $a_k, k = 1, 2, \dots, A$, be the k^{th} EV in chronological order that requests battery swapping. Denote the offline minimum weight maximum matching after a_k 's emergence as x_k^{off} , and the corresponding online matching by our algorithm as x_k^{on} . Let M_k^{off} and M_k^{on} be the corresponding station assignments with respect to x_k^{off} and x_k^{on} . Let $x^{\text{off}} := x_A^{\text{off}}, x^{\text{on}} := x_A^{\text{on}}, M^{\text{off}} := M_A^{\text{off}}$ and $M^{\text{on}} := M_A^{\text{on}}$. For the online station assignment problem, our algorithm is *competitive* if its competitive ratio $\gamma_M := \sup \frac{C(M^{\text{on}})}{C(M^{\text{off}})}$ is bounded, where the supremum is taken over all input instances. Similarly, let $\gamma_x := \sup \frac{W(x^{\text{on}})}{W(x^{\text{off}})}$ be the competitive ratio of our algorithm for the online matching problem. Our online station assignment algorithm is explicitly demonstrated in Algorithm 1.

Algorithm 1: Online station assignment algorithm

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1 Input:  $(a_k, k = 1, 2, \dots, A)$ , and  $(\omega_{a_k b}, (a_k, b) \in \mathbb{E})$  after  $a_k$  emerges;
2 Output:  $M^{\text{on}}$ ;
3 Initialization:  $k \leftarrow 1, x_0^{\text{off}} \leftarrow \emptyset, x_0^{\text{on}} \leftarrow \emptyset, M_0^{\text{on}} \leftarrow \emptyset$ ;
4 while  $a_k$  emerges do
5   Compute  $x_k^{\text{off}}$ ;
6    $P_k \leftarrow x_k^{\text{off}} \oplus x_{k-1}^{\text{off}}$ ;
7   Remedy  $x_k^{\text{off}}$  such that  $P_k$  consists of only an odd-length path from  $a_k$  to a
   battery vertex  $b_k$ ;
8   Match  $a_k$  to  $b_k$  to form  $x_k^{\text{on}}$  from  $x_{k-1}^{\text{on}}$ ;
9   Assign station  $i_{b_k}$  to EV  $a_k$  to obtain  $M_k^{\text{on}}$  from  $M_{k-1}^{\text{on}}$ ;
10   $k \leftarrow k + 1$ ;
11 end while
12 return  $M^{\text{on}} \leftarrow M_A^{\text{on}}$ 

```

THEOREM 3.1. *Algorithm 1 achieves a competitive ratio of $\gamma_x = 2A - 1$ for the online matching problem, i.e., $\frac{W(x^{\text{on}})}{W(x^{\text{off}})} \leq 2A - 1$, if either of the following conditions holds:*

- (1) EV cost consists of only travel time, i.e., $\alpha_2 = 0$, and moreover, $\alpha_1 \geq 1$;
- (2) There is a positive correlation between travel time and distance, i.e., for $\forall a, a' \in \mathbb{A}, b \in \mathbb{B}$, $\theta_{ab} \leq \theta_{a'b} \iff d_{ab} \leq d_{a'b}$, and moreover, $d_{a'b} - d_{ab} \geq \frac{1-\alpha_1}{\alpha_2} (\theta_{a'b} - \theta_{ab})$.

Although the competitive ratio attained by Algorithm 1 seems unduly pessimistic, it is tight as the following theorem states.

THEOREM 3.2. *No deterministic online algorithm is able to achieve a competitive ratio better than $2A - 1$ for the online matching problem.*

It turns out the competitive analysis of Algorithm 1 in terms of the online matching problem is also extendable to the primal online station assignment problem:

THEOREM 3.3. *Algorithm 1 achieves a tight competitive ratio of $\gamma_M = 2A - 1$ for the online station assignment problem, given the same conditions in Theorem 3.1.*

4 CONCLUSION

An online station assignment algorithm for battery swapping of (commercial) EVs is proposed that aims to minimize the cost of EVs and the congestion at service stations. We prove that the online algorithm that builds on online matching achieves a tight (optimal) competitive ratio of $2A - 1$ under mild conditions, where A is the number of battery swapping requests.

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