Scheduling of EV Battery Swapping—Part II: Distributed Solutions

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Abstract—In Part I of this paper, we formulate an optimal scheduling problem for battery swapping that assigns to each electric vehicle (EV) a best station to swap its depleted battery based on its current location and state of charge. The schedule aims to minimize a weighted sum of EVs’ travel distance, electricity generation cost over both station assignments and power flow variables, subject to EV range constraints, grid operational constraints, and ac power flow equations. We propose there a centralized solution based on second-order cone programming relaxation of optimal power flow and generalized Benders decomposition that is applicable when global information is available. In this paper, we propose two distributed solutions based on the alternating direction method of multipliers and dual decomposition, respectively, that are suitable for systems where the distribution grid, stations, and EVs are managed by separate entities. Our algorithms allow these entities to make individual decisions, but coordinate through privacy-preserving information exchanges to solve a convex relaxation of the global problem. We present simulation results to show that both algorithms converge quickly to a solution that is close to optimum after discretization.

Index Terms—Distributed algorithms, electric vehicle (EV), joint battery swapping, optimal power flow (OPF).

I. INTRODUCTION

A. Motivation

In Part I [1] of this paper, we formulate an optimal scheduling problem for battery swapping that assigns to each electric vehicle (EV) a best station to swap its depleted battery based on its current location and state of charge. The station assignments not only determine EVs’ travel distance, but also impact significantly the power flows on a distribution network because batteries are large loads. The schedule aims to minimize a weighted sum of EVs’ travel distance and electricity generation cost over both station assignments and power flow variables, subject to EV range constraints, grid operational constraints, and ac power flow equations. This joint battery swapping and optimal power flow (OPF) problem is nonconvex and computationally difficult because ac power flow equations are nonlinear and the station assignment variables are binary.

We propose in Part I a centralized solution based on second-order cone programming (SOCP) relaxation of the OPF, which deals with the nonconvexity of power flow equations, and generalized Benders decomposition, which deals with the binary nature of station assignment variables. When the relaxation of the OPF is exact, this approach computes a global optimum. It is, however, suitable only for vertically integrated systems where the distribution grid, stations, and EVs are managed centrally by the same operator, as is the taxi programs of State Grid in China. As EVs proliferate and battery swapping matures, an equally (if not more) likely business model will emerge where the distribution grid is managed by a utility company, stations are managed by a station operator (or multiple station operators), and EVs may be managed by individual drivers (or multipleEV groups, e.g., taxi companies in the electric taxi case). In particular, the set of EVs to be scheduled may include a large number of private cars in addition to commercial fleet vehicles.

The centralized solution of Part I will not be suitable for these future scenarios, for two reasons. First, the operator requires global information such as the grid topology, impedances, operational constraints, background loads, availability of fully charged batteries at each station, locations, and states of charge of EVs, etc. However, in the future the distribution grid, stations and EVs will likely be operated by separate entities that do not
As discussed in Part I, the SOCP relaxation of our problem is usually exact. The proposed distributed algorithms, however, may return station assignments that are not binary due to the relaxation of binary variables, which suggest a probabilistic station assignment for an EV. We prove an upper bound on the number of such EVs with nonbinary station assignments. The bound guarantees that the discretization can be readily implemented and also justifies the final solution is close to optimum.

**B. Literature**

See Part I for discussions on relevant literature on EV charging and battery swapping. The distributed solutions here are motivated by the need to preserve private information of different entities operating the distribution grid, stations, and EVs. Privacy in future grids is a key challenge facing both utilities and end users [2], e.g., see [3]–[6] for privacy concerns on smart meters and [7]–[9] for privacy concerns on EVs. Distributed algorithms preserve privacy as global information is not needed for local computations. Liu et al. [5] schedules thermostatically controlled loads and batteries in a household to hide its actual load profiles such that no sensitive information can be inferred from electricity usage. Yang et al. [6] designs an online control algorithm of batteries that only uses the current load requirement and electricity price to optimize the tradeoff between smart meter data privacy and users’ electricity cost. Liu et al. [10] proposes a consensus-based distributed speed advisory system that optimally determines a common vehicle speed for a given area in a privacy-aware manner to minimize the total emission of fuel vehicles or the total energy consumption of EVs. Other applications can be found in data mining [11], cloud computing [12], etc. To the best of our knowledge, this paper is the first to discuss the distributed scheduling of EV battery swapping in light of binary station assignments and grid operation.

**II. PROBLEM FORMULATION**

We now summarize the joint battery swapping and OPF problem in Part I, using the notations defined there.

Assignments of stations to EVs for battery swapping are represented by the binary variables \( u := (u_{aj}, a \in A, j \in \mathbb{N}_w) \), where

\[
u_{aj} = \begin{cases} 
1, & \text{if station } j \text{ is assigned to } EV \ a \\
0, & \text{otherwise}.
\end{cases}
\]

The assignments \( u \) must satisfy the following conditions.

1) The assigned station must be in every EV’s driving range

\[
u_{aj}d_{aj} \leq \gamma_0 c_a, \quad j \in \mathbb{N}_w, a \in A.
\]  \hspace{1cm} (1a)

2) Exactly one station is assigned to every EV

\[
\sum_{j \in \mathbb{N}_w} u_{aj} = 1, \quad a \in A.
\]  \hspace{1cm} (1b)

3) Every assigned station has enough fully charged batteries

\[
\sum_{a \in A} u_{aj} \leq m_j, \quad j \in \mathbb{N}_w.
\]  \hspace{1cm} (1c)

Fig. 1. Distributed framework.
The assignments $u$ will add charging loads to a distribution network at buses in $\mathbb{N}_w$ that supply electricity to stations. The net power injections $s_j = p_j + jq_j$ depend on the assignments $u$ according to

$$p_j = \begin{cases} p_j^a - p_j^b - r \left( M_j - m_j + \sum_{a \in \mathcal{A}} u_{aj} \right), & j \in \mathbb{N}_w \\ p_j^a - p_j^b, & j \in \mathbb{N}/\mathbb{N}_w \end{cases} \quad (2a)$$

$$q_j = q_j^a - q_j^b, \quad j \in \mathbb{N}. \quad (2b)$$

An active distribution network is modeled by the DistFlow equations from [13]

$$\sum_{k: (i,j) \in \mathcal{E}} S_{ijk} = S_{ij} - z_{ij} l_{ij} + s_j, \quad j \in \mathbb{N} \quad (3a)$$

$$v_j - v_k = 2 \text{Re}(z_{jk}^H S_{jk}) - |z_{jk}|^2 l_{jk}, \quad j \rightarrow k \in \mathcal{E} \quad (3b)$$

$$v_j l_{jk} = |S_{jk}|^2, \quad j \rightarrow k \in \mathcal{E}. \quad (3c)$$

The power flow quantities must satisfy the following constraints on grid operation.

1) Voltage stability

$$v_j \leq v_j' \leq v_j, \quad j \in \mathbb{N}. \quad (4a)$$

2) Generation capacity

$$p_j^a \leq p_j^b \leq p_j^d, \quad j \in \mathbb{N} \quad (4b)$$

$$q_j^a \leq q_j^b \leq q_j^d, \quad j \in \mathbb{N}. \quad (4c)$$

3) Line transmission capacity

$$|S_{jk}| \leq \overline{S}_{jk}, \quad j \rightarrow k \in \mathcal{E}. \quad (4d)$$

The joint battery swapping and OPF problem is to minimize a weighted sum of total generation cost in the distribution network and total travel distance of EVs over both station assignments and power flow variables as

$$\min_{u, s, v, l, S} \sum_{j \in \mathbb{N}} f_j(p_j^a) + \alpha \sum_{a \in \mathcal{A}} \sum_{j \in \mathbb{N}_w} d_{aj} u_{aj} \quad (5)$$

s.t. \hspace{1em} (1)(2)(3)(4), \hspace{1em} $u \in \{0, 1\}^{AN_w}$. \hspace{1em} (5)

### III. DISTRIBUTED SOLUTIONS

#### A. Relaxations

The joint battery swapping and OPF problem (5) is computationally difficult for two reasons: The quadratic equality (3c) is nonconvex and the assignment variables $u$ are binary. To deal with the first difficulty, we replace (3c) by an inequality to relax the feasible set into a second-order cone, i.e., replace (3) in the problem (5) by

$$\sum_{k: (i,j) \in \mathcal{E}} S_{ijk} = S_{ij} - z_{ij} l_{ij} + s_j, \quad j \in \mathbb{N} \hspace{1em} (6a)$$

$$v_j - v_k = 2 \text{Re}(z_{jk}^H S_{jk}) - |z_{jk}|^2 l_{jk}, \quad j \rightarrow k \in \mathcal{E} \hspace{1em} (6b)$$

$$v_j l_{jk} \geq |S_{jk}|^2, \quad j \rightarrow k \in \mathcal{E}. \hspace{1em} (6c)$$

Fixing any assignments $u \in \{0, 1\}^{AN_w}$, the optimization problem is then a convex problem. If an optimal solution to the SOCP relaxation attains equality in (6c), it also satisfies (3), and is therefore, optimal (for the given $u$). In this case, we say that the SOCP relaxation is exact. Sufficient conditions are known that guarantee the exactness of the SOCP relaxation; see [14] and [15] for a comprehensive tutorial and references therein. Even when these conditions are not satisfied, the SOCP relaxation for practical radial networks is still often exact, as confirmed also by our simulations in both Part I and this paper.

To deal with the second difficulty, we use generalized Benders decomposition in Part I. This approach computes an optimal solution when the SOCP relaxation is exact, but the computation is centralized and is suitable only when a single organization, e.g., State Grid in China, operates all of the distribution grid, stations and EVs. In this paper, we develop distributed solutions that are suitable for systems where these three are operated by separate entities that do not share their private information. To this end, we relax the binary assignment variables $u$ to real variables $u \in [0, 1]^{AN_w}$. The constraints (1) are then replaced by

$$u_{aj} = 0 \text{ if } d_{aj} > \gamma_0 c_a, \quad j \in \mathbb{N}_w, \ a \in \mathcal{A} \quad (7a)$$

$$\sum_{j \in \mathbb{N}_w} u_{aj} = 1, \ a \in \mathcal{A} \quad (7b)$$

$$\sum_{a \in \mathcal{A}} u_{aj} \leq m_j, \ j \in \mathbb{N}_w. \quad (7c)$$

In summary, in this paper, we solve the following convex relaxation of (5):

$$\min_{u, s, v, l, S} \sum_{j \in \mathbb{N}} f_j(p_j^a) + \alpha \sum_{a \in \mathcal{A}} \sum_{j \in \mathbb{N}_w} d_{aj} u_{aj} \quad (8)$$

s.t. \hspace{1em} (2)(4)(6)(7), \hspace{1em} $u \in [0, 1]^{AN_w}$. \hspace{1em} (8)

This problem has a convex objective and convex quadratic constraints. After an optimal solution $(x^*, u^*)$ of (8) is obtained, we check if $x^*$ attains equality in (6c). We also discretize $u_{aj}^*$ into $\{0, 1\}$, e.g., by setting for each EV $a$ a single large $u_{aj}^*$ to 1 and the rest to 0 heuristically. An alternative is to randomize the station assignments using $u^*$ as a probability distribution. Whichever method is employed, it should guarantee the discretized station assignments are feasible. As we will show later, the discretization is readily implementable and achieves binary station assignments close to optimum.

#### B. Distributed Solution via the ADMM

The relaxation (8) decomposes naturally into two subproblems, one on station assignments over $u$ and the other on OPF over $(s, s^a, v, l, S)$. The station assignment subproblem will be solved by a station operator that operates the network of stations. The OPF subproblem will be solved by a utility company. Our goal is to design a distributed algorithm for them to jointly solve (8) without sharing their private information.

These two subproblems are coupled only in (2a) where the utility company needs the charging load $s_j^f = r(M_j - m_j + \sum_{a \in \mathcal{A}} u_{aj} x_{aj}^a)$.
\[ \sum_{a \in A} u_{aj} \) of station \( j \) in order to compute the net real power injection \( p_j \). This quantity depends on the total number of EVs that each station \( j \) is assigned to and is computed by the station operator. Their computation can be decoupled by introducing an auxiliary variable \( w_j \) at each bus (station) \( j \) that represents the utility company’s estimate of the quantity \( r \left( M_j - m_j + \sum_{a \in A} u_{aj} \right) \), and requiring that they be equal at optimality.

Specifically, recall the station assignment variables \( u \), and denote the power flow variables by \( x := (w, s, q, v, l, s) \) where \( w := (r \left( M_j - m_j + \sum_{a \in A} u_{aj} \right), j \in \mathbb{N}_w) \). Separate the objective function by defining

\[
\begin{align*}
f(x) &:= \sum_{j \in \mathbb{N}} f_j(p_j) \\
g(u) &:= \alpha \sum_{a \in A} \sum_{j \in \mathbb{N}_v} d_{aj} u_{aj}.
\end{align*}
\]

Replace the coupling constraints (2) by constraints local to bus \( j \) as follows:

\[
\begin{align*}
p_j &= \begin{cases} 
p_j^p - p_j^b - w_j, & j \in \mathbb{N}_w \\
p_j^p - p_j^b, & j \in \mathbb{N}/\mathbb{N}_w
\end{cases} \quad (9a) \\
q_j &= q_j^p - q_j^b, & j \in \mathbb{N}. \quad (9b)
\end{align*}
\]

Denote the local constraint set for \( x \) by

\[
\mathbb{X} := \{ x \in \mathbb{R}^{(|\mathbb{N}_w| + 5)|\mathbb{N}_w| + 3|\mathbb{R}|} : x \text{ satisfies (4)(6)(9)} \}.
\]

Denote the local constraint set for \( u \) by

\[
\mathbb{U} := \{ u \in \mathbb{R}^{AN_e} : u \text{ satisfies (7)} \}.
\]

To simplify notation, define \( u_j := \sum_{a \in A} u_{aj}, \) for \( j \in \mathbb{N}_w \). Then, the relaxation (8) is equivalent to

\[
\begin{align*}
\min_{x,u} & \quad f(x) + g(u) \\
\text{s.t.} & \quad x \in \mathbb{X}, u \in \mathbb{U} \\
& \quad w_j = r \left( M_j - m_j + u_j \right), j \in \mathbb{N}_w. \quad (10c)
\end{align*}
\]

We now apply the ADMM to (10). Let \( \lambda := (\lambda_j, j \in \mathbb{N}_w) \) be the Lagrange multiplier vector corresponding to the current coupling constraint (10c), and define the augmented Lagrangian as

\[
L_\rho(x, u, \lambda) := f(x) + g(u) + h_\rho(w, u, \lambda) \quad (11a)
\]

where \( h_\rho \) depends on \( (x, u) \) only through \((w_j, u_j, j \in \mathbb{N}_w)\) as follows:

\[
h_\rho(w, u, \lambda) := \sum_{j \in \mathbb{N}_w} \lambda_j [w_j - r \left( M_j - m_j + u_j \right)] + \rho \sum_{j \in \mathbb{N}_w} [w_j - r \left( M_j - m_j + u_j \right)]^2 \quad (11b)
\]

and \( \rho \) is the step size for dual variable \( \lambda \) updates. The standard ADMM procedure is to iteratively and sequentially update

\[
\begin{align*}
\text{utility company:} & \quad \text{updates } x(n) \\
\text{station operator:} & \quad \text{updates } (u(n), \lambda(n))
\end{align*}
\]

where

\[
(x(n), u(n)): \text{for } n = 0, 1, \ldots,
\]

\[
x(n + 1) := \arg \min_{x \in \mathbb{X}} f(x) + h_\rho(w(n), u(n), \lambda(n)) \quad (12a)
\]

\[
u(n + 1) := \arg \min_{u \in \mathbb{U}} g(u) + h_\rho(w(n + 1), u, \lambda(n)) \quad (12b)
\]

\[
\lambda_j(n + 1) := \lambda_j(n) + \rho [w_j(n + 1) - r \left( M_j - m_j + u_j(n + 1) \right)], j \in \mathbb{N}_w. \quad (12c)
\]

**Remark 1:**

1) The \( x \)-update (12a) is carried out by the utility company and involves minimizing a convex objective with convex quadratic constraints. The \( (u, \lambda) \)-updates (12b), (12c) are carried out by the station operator and the \( u \)-update minimizes a convex quadratic objective with linear constraints. Both can be efficiently solved.

2) The \( x \)-update by the utility company in iteration \( n + 1 \) needs \((u(n), \lambda(n))\) from the station operator. From (11b), the station operator does not need to communicate the detailed assignments \( u(n) = (u_{aj}(n), a \in A, j \in \mathbb{N}_w) \) to the utility company but only the charging load \( s_j^f \) to each station \( j \).

3) The \( (u, \lambda) \)-updates by the station operator in iteration \( n + 1 \) need the utility company’s estimate \( w(n + 1) \) of \( r \left( M_j - m_j + u_j(n + 1) \right), j \in \mathbb{N}_w \).

4) The reason why the \( x \)-update by the utility company needs \((u_j(n), j \in \mathbb{N}_w)\) and the \( u \)-update by the station operator needs \( w(n + 1) \) lies in the (quadratic) regularization term in \( h_\rho \). This becomes unnecessary for the dual decomposition approach in Section III-C without the regularization term.

The communication structure is illustrated in Fig. 2. In particular, private information of the utility company, such as distribution network parameters \((z_{jk}, (j, k) \in E)\), network states \((s(n), s^D(n), v(n), l(n), S(n))\), cost functions \(f\), and operational constraints, as well as private information of the station operator, such as the total numbers of batteries \((M_j, j \in \mathbb{N}_w)\), the numbers of available fully charged batteries \((m_j, j \in \mathbb{N}_w)\), and how many EVs or where they are or their states of charge, and the detailed assignments \( u(n)\), do not need to be communicated.

When the cost functions \( f_j \) are closed, proper and convex and \( L_\rho(x, u, \lambda) \) has a saddle point, the ADMM iteration (12) converges in that, for any \( j \in \mathbb{N}_w \), the mismatch \(|w_j(n) - r \left( M_j - m_j + u_j(n + 1) \right)| \to 0 \) and the objective function \( f(x(n)) + g(u(n)) \) converges to its minimum value [16]. This does not automatically guarantee that \((x(n), u(n))\)
converges to an optimal solution to (8).\(^2\) If \((x(n), u(n))\) indeed converges to a primal optimal solution \((x^*, u^*)\), \(u^*\) may generally not be binary. We can use a heuristic to derive binary station assignments from \(u^*\), as mentioned previously. Fortunately, the following result shows that the number of EVs with nonbinary assignments is bounded and small in \(u^*\). See Appendix A for its proof.

**Theorem 1:** It is always possible to find an optimal solution \((x^*, u^*)\) to the relaxation (8) in which the number of EVs \(a\) with \(u^*_a < 1\) for any \(a \in \mathbb{N}_w\) is at most \(N_w (N_w - 1)/2\).

In practice, the number \(N_w\) of stations is much smaller than the number \(A\) of EVs that request battery swapping, and hence, the number of nonbinary assignments that need to be discretized will be small. Simulations in Section IV further suggest that the discretized assignments are close to optimum.

**C. Distributed Solution via Dual Decomposition**

The ADMM-based solution assumes the station operator directly controls the station assignments to all EVs. This requires that the station operator know the locations \((d_{a,j})\), states of charge \((c_a)\), and performance \((\gamma_a)\) of EVs. Moreover, the charging load \(s_j^e = r(M_j - m_j + u_j(n))\) of each station \(j\) needs to be provided to the utility company. We now present another solution based on dual decomposition that is more suitable in situations where it is undesirable or inconvenient to share private information between the utility company, the station operator, and EVs.

In the original relaxation (8), the update of the net power injections \(p_j\) in (2) by the utility company involves \(u_j\), which is updated by the station operator. These two computations are decoupled in the ADMM-based solution by introducing an auxiliary variable \(w_j\) for each \(j \in \mathbb{N}_w\) at the utility company and relaxing the constraint \(w_j = r(M_j - m_j + u_j)\). In addition, the station assignments \(u\) must satisfy \(u_j \leq m_j\) in (7c).

This is enforced in the ADMM-based solution by the station operator that computes \(u\) for all EVs. To fully distribute the computation to individual EVs, we dualize \(u_j \leq m_j\) as well. Let \(\lambda := (\lambda_j, j \in \mathbb{N}_w)\) and \(\mu := (\mu_j, j \in \mathbb{N}_w)\) be the Lagrange multiplier vectors for the constraints \(w_j = r(M_j - m_j + u_j)\) and \(u_j \leq m_j, \ j \in \mathbb{N}_w\). Intuitively, \(w\) and \(\lambda\) decouple the computation of the utility company and that of individual EVs through coordination with the station operator. Additionally, \(\mu\) decouples and coordinates all EVs’ decisions so that EVs do not need direct communication among themselves to ensure that their decisions \(u_{a,j}\) collectively satisfy \(u_j \leq m_j\).

Consider the Lagrangian of (10) with these two sets of constraints relaxed

\[
L(x, u, \lambda, \mu) := f(x) + g(u) + \sum_{j \in \mathbb{N}_w} \lambda_j (w_j - r(M_j - m_j + u_j)) + \sum_{j \in \mathbb{N}_w} \mu_j (u_j - m_j) \tag{13}
\]

and the dual problem of (10) as

\[
\max_{\lambda, \mu \geq 0} D(\lambda, \mu) := \min_{x \in \mathbb{X}, u \in \mathbb{U}} L(x, u, \lambda, \mu)
\]

where the constraint set \(\mathbb{U}\) on \(u\) is

\[
\mathbb{U} := \{u \in \mathbb{R}^{N_w} : u \text{ satisfies (7a) and (7b)}\}.
\]

Let \(u_a := (u_{a,j}, j \in \mathbb{N}_w)\) denote the vector of EV \(a\)’s decision on which station to swap its battery. Then, the dual problem is separable in power flows \(x\) and as individual EVs’ decisions \(u_a\) as

\[
D(\lambda, \mu) = V(\lambda) + \sum_{a \in \mathbb{A}} U_a(\lambda, \mu) \tag{14a}
\]

where the problem \(V(\lambda)\) solved by the utility company is

\[
V(\lambda) := \min_{x \in \mathbb{X}} \left( f(x) + \sum_{j \in \mathbb{N}_w} \lambda_j w_j \right) \tag{14b}
\]

and the problem \(U_a(\lambda)\) solved by each individual EV \(a\) is

\[
U_a(\lambda, \mu) := \min_{u_a \in \mathbb{U}_a} \sum_{j \in \mathbb{N}_w} (\alpha_d a_j - r \lambda_j + \mu_j) u_{a,j} \tag{14c}
\]

where the constraint set \(\mathbb{U}_a\) on \(u_a\) is

\[
\mathbb{U}_a := \left\{u_a \in \mathbb{R}^{N_w} : u_{a,j} = 0 \text{ if } d_{a,j} > \gamma_a c_a, j \in \mathbb{N}_w \right\}
\]

Note that (14c) has closed-form solutions. For instance, if there exists a unique optimal solution to \(U_a(\lambda, \mu)\), i.e., for any EV \(a\) there is a unique \(j^*_a(\lambda, \mu)\) defined as

\[
\tilde{j}_a^*(\lambda, \mu) := \arg \min_{j : d_{a,j} \leq \gamma_a c_a} \{\alpha d_{a,j} - r \lambda_j + \mu_j\}
\]

then the optimal solution can be uniquely determined as

\[
u_{a,j}(\lambda, \mu) := \begin{cases} 1, & \text{if } j = \tilde{j}_a^*(\lambda, \mu) \\ 0, & \text{if } j \neq \tilde{j}_a^*(\lambda, \mu) \end{cases}
\]

i.e., it simply chooses the unique station \(j^*_a\) within EV \(a\)’s driving range that has the minimum cost \(\alpha d_{a,j} - r \lambda_j + \mu_j\).

From (13), the standard dual algorithm for solving (10) is, for \(j \in \mathbb{N}_w\),

\[
\lambda_j(n + 1) := \lambda_j(n) + p_1(n) \times [w_j(n) - r(M_j - m_j + u_j(n))]
\]

\[
\mu_j(n + 1) := \max\{\mu_j(n) + p_2(n)(u_j(n) - m_j), 0\}
\]

where \(p_1(n), p_2(n) > 0\) are diminishing step sizes, and from (14), we have

\[
x(n) := \arg \min_{x \in \mathbb{X}} \left( f(x) + \sum_{j \in \mathbb{N}_w} \lambda_j(n) w_j \right) \tag{15c}
\]

and for \(a \in \mathbb{A}\),

\[
u_a(n) := \arg \min_{u_a \in \mathbb{U}_a} \sum_{j \in \mathbb{N}_w} (\alpha d_{a,j} - r \lambda_j(n) + \mu_j(n)) u_{a,j} \tag{15d}
\]

\(^2\)In the theory, ADMM may converge and circulate around the set of optimal solutions, but never reach one. In practice, a solution within a given error tolerance is acceptable.
Remark 2:
1) The \( x \)-update (15c) is carried out by the utility company and involves minimizing a convex objective with convex quadratic constraints. The only information that is non-local to the utility company for its \( x \)-update is one of the dual variables \( \lambda (n) \) computed by the station operator.
2) The \( u_a \)-update (15d) is carried out by each individual EV. Each EV requires both the dual variables \((\lambda(n), \mu(n))\) from the station operator for its update.
3) The dual updates (15a), (15b) are carried out by the station operator that uses a (sub)gradient ascent algorithm to solve the dual problem \( \max_{\lambda, \mu \geq 0} D(\lambda, \mu) \). It requires \( w(n) \) from the utility company and individual decisions \( u_a(n) \) from EVs \( a \).

The communication structure is illustrated in Fig. 3. In particular, EVs are completely decoupled from the utility company and among themselves. Unlike the ADMM-based solution, the station operator knows only the battery swapping decisions of EVs \( a \), but not their private information such as locations \( (d_{aj}) \), states of charge \( (c_a) \) or performance \( (\gamma_a) \).

Since the relaxation (8) is convex, strong duality holds if Slater’s condition is satisfied. Then, when the aforementioned (sub)gradient algorithm converges to a dual optimal solution \((\lambda^*, \mu^*)\), any primal optimal point is also a solution to the corresponding \( x \)-update (15c) and \( u_a \)-update (15d) [17], [18]. Suppose \((x(n), u_a(n), a \in \mathcal{A})\) indeed converges to a primal optimal solution \((x^*, u_a^*, a \in \mathcal{A})\), then typically \((u_a^*, a \in \mathcal{A})\) is not binary. However, the bound in Theorem 1 still holds that guarantees easy discretization and suggests the final discretized stations assignments are close to optimum.

Remark 3: The two solutions have their own advantages and can be adapted to different application scenarios. The ADMM-based solution requires a station operator that is trustworthy and can access EVs’ private information. Since the station operator optimizes station assignments on behalf of all EVs, no computation is required on each EV, and meanwhile communication is only required between the station operator and the utility company. In contrast, the solution based on dual decomposition does not require sharing EVs’ private information with the station operator. It, however, necessitates computation capabilities on all EVs. In addition, communication is needed both between the station operator and the utility company and between the station operator and each EV.
IV. NUMERICAL RESULTS

We test the two distributed solutions on the same 56-bus radial distribution feeder of Southern California Edison (SCE) in Part I. Details about the feeder can be found in [19]. Similar setups from Part I are adopted to demonstrate the algorithm performance. Table I lists the main parameters. The number of EVs that request battery swapping is $A = 400$. We simulate two cases with different $m_j$'s (see Table I(b)). EVs' current locations are randomized uniformly within a 4 km × 4 km square area and their destinations are ignored. We use Euclidean distances $d_{a,j}$ and assume all EVs can reach any of the four stations. The constant charging rate is $r = 0.01$ MW [20], and the weight is $\alpha = 0.02$ $$/km$$ [21]$. Simulations are run on a laptop with Intel Core i7-3632QM CPU at 2.20 GHz, 8-GB RAM, and 64-bit Windows 10 OS.

A. Convergence

The convergence of the ADMM in case (i) is demonstrated in Fig. 4. Fig. 4(a) and (b) shows, respectively, that the Lagrange multiplier vector $\lambda$ and the residual of the relaxed equality constraint (10c) converge rapidly. Case (ii) behaves similarly. Each iteration that computes the three steps of (12) takes on average 0.477 s by Gurobi. For the dual decomposition algorithm, Fig. 5(a) and (b) shows the convergence of its two Lagrange multiplier vectors $\lambda$ and $\mu$, respectively, in case (ii). $\lambda$ maintains

3 The units of the real power, reactive power, cost, distance, and weight in this paper are MW, Mvar, $\$, km, and $$/km$$, respectively.
the consensus between the utility company and EVs at convergence, and \( \mu \) guarantees (7c) is satisfied when it converges. Dual decomposition usually takes more iterations to converge due to the additionally required coordination among all EVs. For case (i), results are similar except that \( \mu \) remains 0 during computation as (7c) is always satisfied. Each iteration of the dual decomposition algorithm involves the centralized update of (15a) and (15b) and the parallelized computation of (15c) and (15d). Each iteration takes on average 0.212 s by Gurobi.

\section*{B. Suboptimality (Comparison With Centralized Solution)}

In case (i), both algorithms obtain a solution in which the station assignments to two EVs, marked black in Fig. 6(a), are nonbinary: \( u_{242} = [0.707 \ 0.293 \ 0.000 \ 0.000] \) and \( u_{367} = [0.230 \ 0.000 \ 0.770 \ 0.000] \). This is consistent with Theorem 1. If we simply round \( u_{243} \) and \( u_{367} \) to binary values, the resulting solution turns out to coincide with a globally optimal solution computed using the centralized solution in Part I.

In case (ii), we reduce available fully charged batteries at each station to activate (7c). Fig. 6(b) shows the solution achieved by both algorithms. The solution turns out to be globally optimal for the original problem (5); in particular, all station assignments are binary. EVs, to which the station assignments are altered due to the bound imposed on battery availability of each station, are marked cyan in Fig. 6(b). The intuition is that an active (7c) sometimes can help eliminate nonbinary assignments to EVs. This is often the case in practice where battery availability is uneven across stations.

\section*{C. Exactness of SOCP Relaxation}

In most cases, that we have simulated, including cases reported here, the SOCP relaxation is exact, i.e., the solutions computed by the two distributed algorithms attain equality in (6c), and therefore, satisfy power flow equations. Partial data for case (ii) are listed in Table II.

\section*{D. Scalability}

We follow the same setup in part I to demonstrate the scalability of the two distributed algorithms, i.e., we first augment the number of EVs, while the number of stations is fixed, and
then, turn the other way round. The computation time that is shown in Figs. 7 and 8 is averaged over ten simulation runs with randomly generated cases. Approximately, the computational effort of both algorithms increases linearly as EVs (or stations) scale up. Compared with the centralized solution in part I, the required computation time of the distributed algorithms is less sensitive to the EV scale, which is intuitive, but turns out more sensitive to the station scale. This results from the fact that the consensus that the distributed algorithms strive toward has to be achieved at each station. Generally, more iterations are needed as more stations are involved.

V. CONCLUDING REMARKS

This paper is an extension of Part I that basically solves the same optimal scheduling problem for battery swapping. Instead of a centralized solution that requires global information, two distributed solutions based on the ADMM and dual decomposition, respectively, are proposed. These solutions are more suitable for systems where the distribution grid, stations, and EVs are operated by separate entities that do not share their private information. They allow these entities to make individual decisions but coordinate through privacy-preserving information exchanges to jointly solve a relaxation of the global problem. Some of the station assignments in a relaxed solution may not be binary and need to be discretized, but we prove that their number is small. Numerical tests on the SCE 56-bus distribution feeder demonstrate the algorithm performance and also suggest that the final discretized station assignments are close to optimum.

APPENDIX A

PROOF OF THEOREM 1

We refer to EV a as a critical EV if its station assignment satisfies $u_{aj} < 1$ for all $j \in \mathbb{N}_w$. We first show the following lemma, and then, prove Theorem 1. Let $(u, y) := (u, s, s^0, v, \ell, S)$.

**Lemma 1:** It is always possible to find an optimal solution $(u^*, y^*)$ to the relaxation (8) where no critical EV shares two stations, i.e., there do not exist $a, b \in A$ and $j, k \in \mathbb{N}_w$ such that $u_{aj}^*, u_{ak}^*, u_{bj}^*, u_{bk}^* > 0$.

**Proof of Lemma 1:** Fix any $(u, y)$ that is feasible for (8). If $u_{aj}^*, u_{ak}^*, u_{bj}^*, u_{bk}^* > 0$, for some $a, b \in A$ and $j, k \in \mathbb{N}_w$, we will construct station assignments $u'$ that satisfy the lemma such that $(u', y)$ is also feasible for (8) but has a lower or equal objective value. This proves the lemma.

Let $B_a := u_{aj} + u_{ak}$, $B_b := u_{bj} + u_{bk}$, $B_j := u_{aj} + u_{bj}$, and $B_k := u_{ak} + u_{bk}$. The interpretation of these quantities is that $rB_a$ and $rB_b$ are the charging loads of EVs a and b, respectively, and $rB_j$ and $rB_k$ are their load distributions at stations j and k, respectively. Clearly, $B_a + B_b = B_j + B_k$. Without loss of generality, we can assume either case 1: $B_a \geq B_j \geq B_k \geq B_b$ or case 2: $B_j \geq B_a \geq B_k \geq B_b$ holds. We now construct $u'$ assuming case 1 holds. The construction is similar if case 2 holds instead.

We consider four disjoint subcases and construct $u'$ for each subcase.

1. **EV a is closer to station j but farther away from station k** than $b(d_{aj} \leq d_{bj}, d_{ak} \leq d_{bk})$: Let $u_{aj}' = B_j$, $u_{ak}' = B_k - B_j$, $u_{bj}' = 0$, $u_{bk}' = B_b$, and the other variables remain the same as in $(u, y)$. This means that the assignments $u'$ send EV b to station k but not station j, and also increase the likelihood of EV a going to station j while decreasing that to station $k$. Since
   
   \[ u_{aj}' + u_{ak}' = B_j + B_k - B_b = u_{aj} + u_{ak} \]
   
   \[ u_{bj}' + u_{bk}' = B_k - B_b = u_{bj} + u_{bk} \]
   
   \[ u_{aj}' + u_{bj}' = B_j = u_{aj} + u_{bj} \]
   
   \[ u_{ak}' + u_{bk}' = B_k - B_b = u_{ak} + u_{bk} \]

   $(u', y)$ is feasible (8). Moreover,
   \[
   \sum_{c=a, b} \sum_{i=j, k} d_{ci}u_{ci}'
   \]
   \[
   = d_{aj}B_j + d_{ak}(B_k - B_b) + d_{bk}B_b
   \]
   \[
   = d_{aj}(u_{aj} + u_{bj}) + d_{ak}(u_{ak} - u_{bj}) + d_{bk}(u_{bj} + u_{bk})
   \]
   \[
   \leq \sum_{c=a, b} \sum_{i=j, k} d_{ci}u_{ci} - u_{bj}(d_{ak} - d_{bk})
   \]
   \[
   \leq \sum_{c=a, b} \sum_{i=j, k} d_{ci}u_{ci}'
   \]

   where the first inequality uses $d_{aj} \leq d_{bj}$ and the second inequality uses $d_{ak} \leq d_{bk}$. Therefore, $(u', y)$ has a lower or equal objective value than $(u, y)$. 1.2 EV b is closer to station j but farther away from station $k$ than $a(d_{bj} \leq d_{aj}, d_{ak} \leq d_{bk})$: This case is symmetric to subcase 1.1.

1.3 EV a is closer than b to both stations $(d_{aj} \leq d_{bj}, d_{ak} \leq d_{bk})$: We either have $d_{bj} - d_{bk} \leq d_{aj} - d_{ak}$ or $d_{bj} - d_{bk} > d_{aj} - d_{ak}$. In the former case, let $u_{aj}' = B_j - B_b$, $u_{ak}' = B_k$, $u_{bj}' = B_b$, and $u_{bk}' = 0$. Then,

   \[
   \sum_{c=a, b} \sum_{i=j, k} d_{ci}u_{ci}'
   \]
   \[
   = \sum_{c=a, b} \sum_{i=j, k} d_{ci}u_{ci} + (d_{ak} - d_{bk} + d_{bj} - d_{aj})u_{bk}
   \]
   \[
   \leq \sum_{c=a, b} \sum_{i=j, k} d_{ci}u_{ci}
   \]

   Similar to subcase 1.1, $(u', y)$ is feasible and has a lower or equal objective value. In the latter case, let $u_{aj}' = B_j$, $u_{ak}' = B_k - B_b$, $u_{bj}' = 0$, and $u_{bk}' = B_b$. Then, $(u', y)$ is feasible and has a lower objective value.

1.4 EV b is closer than a to both stations $(d_{bj} \leq d_{aj}, d_{bk} \leq d_{ak})$: This case is symmetric to subcase 1.3.

This completes the proof of the lemma.

**Proof of Theorem 1:** Fix an optimal solution $(u^*, y^*)$ to the relaxation (8) that satisfies Lemma 1. By definition, a critical EV splits its charging load between at least two different stations. An upper bound on the number of critical EVs is, therefore, the maximum number of critical EVs that we can assign the $N_w$ stations without violating Lemma 1.
Consider the set $C_1$ of critical EVs under the charging loads $u^*$ that split their charging loads between station $i = 1$ and (at least) another station $j = 2, \ldots, N_w$. Lemma 1 implies that there are at most $N_w - 1$ critical EVs in $C_1$ since the assignments $u^*$ are optimal. Consider next the set $C_2$ of critical EVs not in $C_1$ that split their charging loads between station $i = 2$ and (at least) another station $j = 3, \ldots, N_w$. There are at most $N_w - 2$ critical EVs in $C_2$. Similarly there are at most $N_w - i$ critical EVs in the set $C_i$ that are not in $u^*_{i-1} C_k$ that split their charging loads between station $i$ and (at least) another station $j > i$. Hence, the maximum number of such critical EVs is $(N_w - 1) + (N_w - 2) + \cdots + 1 = \frac{1}{2} N_w (N_w - 1)$. This completes the proof of Theorem 1.

REFERENCES


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