# Scheduling of EV Battery Swapping-Part II: Distributed Solutions 

Pengcheng You ${ }^{\oplus}$, Student Member, IEEE, Steven H. Low ${ }^{{ }^{\circ}}$, Fellow, IEEE, Liang Zhang ${ }^{\bullet}$, Student Member, IEEE, Ruilong Deng ${ }^{\bullet}$, Member, IEEE, Georgios B. Giannakis ${ }^{\left({ }^{( }\right)}$, Fellow, IEEE, Youxian Sun, and Zaiyue Yang ${ }^{\bullet}$, Member, IEEE


#### Abstract

In Part I of this paper, we formulate an optimal scheduling problem for battery swapping that assigns to each electric vehicle (EV) a best station to swap its depleted battery based on its current location and state of charge. The schedule aims to minimize a weighted sum of EVs' travel distance and electricity generation cost over both station assignments and power flow variables, subject to EV range constraints, grid operational constraints, and ac power flow equations. We propose there a centralized solution based on second-order cone programming relaxation of optimal power flow and generalized Benders decomposition that is applicable when global information is available. In this paper, we propose two distributed solutions based on the alternating direction method of multipliers and dual decomposition, respectively, that are suitable for systems where the distribution grid, stations, and EVs are managed by separate entities. Our algorithms allow these entities to make individual decisions, but coordinate through privacypreserving information exchanges to solve a convex relaxation of the global problem. We present simulation results to show that both algorithms converge quickly to a solution that is close to optimum after discretization.


Index Terms-Distributed algorithms, electric vehicle (EV), joint battery swapping, optimal power flow (OPF).

[^0]
## I. INTRODUCTION

## A. Motivation

IN PART I [1] of this paper, we formulate an optimal scheduling problem for battery swapping that assigns to each electric vehicle (EV) a best station to swap its depleted battery based on its current location and state of charge. The station assignments not only determine EVs' travel distance, but also impact significantly the power flows on a distribution network because batteries are large loads. The schedule aims to minimize a weighted sum of EVs' travel distance and electricity generation cost over both station assignments and power flow variables, subject to EV range constraints, grid operational constraints, and ac power flow equations. This joint battery swapping and optimal power flow (OPF) problem is nonconvex and computationally difficult because ac power flow equations are nonlinear and the station assignment variables are binary.

We propose in Part I a centralized solution based on secondorder cone programming (SOCP) relaxation of the OPF, which deals with the nonconvexity of power flow equations, and generalized Benders decomposition, which deals with the binary nature of station assignment variables. When the relaxation of the OPF is exact, this approach computes a global optimum. It is, however, suitable only for vertically integrated systems where the distribution grid, stations, ${ }^{1}$ and EVs are managed centrally by the same operator, as is the electric taxi programs of State Grid in China. As EVs proliferate and battery swapping matures, an equally (if not more) likely business model will emerge where the distribution grid is managed by a utility company, stations are managed by a station operator (or multiple station operators), and EVs may be managed by individual drivers (or multiple EV groups, e.g., taxi companies in the electric taxi case). In particular, the set of EVs to be scheduled may include a large number of private cars in addition to commercial fleet vehicles.

The centralized solution of Part I will not be suitable for these future scenarios, for two reasons. First, the operator requires global information such as the grid topology, impedances, operational constraints, background loads, availability of fully charged batteries at each station, locations, and states of charge of EVs, etc. However, in the future the distribution grid, stations and EVs will likely be operated by separate entities that do not

[^1]

Fig. 1. Distributed framework.
share their private information. Second, generalized Benders decomposition solves a mixed-integer convex problem centrally and is computationally expensive. It is hard to scale it to compute in real-time optimal station assignments and an (relaxed) OPF solution in future scenarios where the numbers of EVs and stations are large. In this paper, we develop distributed solutions that preserve private information and are more suitable for general scenarios.

Instead of generalized Benders decomposition, we relax the binary station assignment variables to real variables in $[0,1]$. With both the relaxation of OPF and the relaxation of binary variables, the resulting approximate problem of joint battery swapping and OPF is a convex program. This allows us to develop two distributed solutions where separate entities make their individual decisions but coordinate through information exchanges that do not involve their private information in order to jointly solve the global problem. The first solution based on the alternating direction method of multipliers (ADMM) is for systems where the distributed grid is managed by a utility company and all stations and EVs are managed by a station operator. Here, the utility company maintains a local estimate of some aggregate assignment information that is computed by the station operator, and they exchange the estimate and the aggregate information to attain consensus. The second solution based on dual decomposition is for systems where the distributed grid is managed by a utility company, all stations are managed by a station operator, and all EVs are individually operated. The utility company still sends its local estimate to the station operator while the station operator does not need to send the utility company the aggregate assignment information, but only some Lagrange multipliers. The station operator also broadcasts Lagrange multipliers to all EVs and individual EVs respond by sending the station operator their choices of stations for battery swapping based on the received Lagrange multipliers and their current locations and driving ranges. In both approaches, given the aggregate assignment information and Lagrange multipliers that are exchanged, different entities only need their own local states (e.g., power flow variables) and local data (e.g., impedance values, battery states, EV locations, and driving ranges) to iteratively compute their own decisions. See Fig. 1 for the distributed framework.

As discussed in Part I, the SOCP relaxation of our problem is usually exact. The proposed distributed algorithms, however, may return station assignments that are not binary due to the relaxation of binary variables, which suggest a probabilistic station assignment for an EV. We prove an upper bound on the number of such EVs with nonbinary station assignments. The bound guarantees that the discretization can be readily implemented and also justifies the final solution is close to optimum.

## B. Literature

See Part I for discussions on relevant literature on EV charging and battery swapping. The distributed solutions here are motivated by the need to preserve private information of different entities operating the distribution grid, stations, and EVs. Privacy in future grids is a key challenge facing both utilities and end users [2], e.g., see [3]-[6] for privacy concerns on smart meters and [7]-[9] for privacy concerns on EVs. Distributed algorithms preserve privacy as global information is not needed for local computations. Liu et al. [5] schedules thermostatically controlled loads and batteries in a household to hide its actual load profiles such that no sensitive information can be inferred from electricity usage. Yang et al. [6] designs an online control algorithm of batteries that only uses the current load requirement and electricity price to optimize the tradeoff between smart meter data privacy and users' electricity cost. Liu et al. [10] proposes a consensus-based distributed speed advisory system that optimally determines a common vehicle speed for a given area in a privacy-aware manner to minimize the total emission of fuel vehicles or the total energy consumption of EVs. Other applications can be found in data mining [11], cloud computing [12], etc. To the best of our knowledge, this paper is the first to discuss the distributed scheduling of EV battery swapping in light of binary station assignments and grid operation.

## II. Problem Formulation

We now summarize the joint battery swapping and OPF problem in Part I, using the notations defined there.

Assignments of stations to EVs for battery swapping are represented by the binary variables $u:=\left(u_{a j}, a \in \mathbb{A}, j \in \mathbb{N}_{w}\right)$, where

$$
u_{a j}= \begin{cases}1, & \text { if station } j \text { is assigned to EV a } \\ 0, & \text { otherwise }\end{cases}
$$

The assignments $u$ must satisfy the following conditions.

1) The assigned station must be in every EV's driving range

$$
\begin{equation*}
u_{a j} d_{a j} \leq \gamma_{a} c_{a}, \quad j \in \mathbb{N}_{w}, a \in \mathbb{A} \tag{1a}
\end{equation*}
$$

2) Exactly one station is assigned to every EV

$$
\begin{equation*}
\sum_{j \in \mathbb{N}_{w}} u_{a j}=1, \quad a \in \mathbb{A} \tag{1b}
\end{equation*}
$$

3) Every assigned station has enough fully charged batteries

$$
\begin{equation*}
\sum_{a \in \mathbb{A}} u_{a j} \leq m_{j}, \quad j \in \mathbb{N}_{w} \tag{1c}
\end{equation*}
$$

The assignments $u$ will add charging loads to a distribution network at buses in $\mathbb{N}_{w}$ that supply electricity to stations. The net power injections $s_{j}=p_{j}+\mathbf{i} q_{j}$ depend on the assignments $u$ according to

$$
\begin{align*}
& p_{j}=\left\{\begin{array}{l}
p_{j}^{g}-p_{j}^{b}-r\left(M_{j}-m_{j}+\sum_{a \in \mathbb{A}} u_{a j}\right), \quad j \in \mathbb{N}_{w} \\
p_{j}^{g}-p_{j}^{b}, \quad j \in \mathbb{N} / \mathbb{N}_{w}
\end{array}\right.  \tag{2a}\\
& q_{j}=q_{j}^{g}-q_{j}^{b}, \quad j \in \mathbb{N} \tag{2b}
\end{align*}
$$

An active distribution network is modeled by the DistFlow equations from [13]

$$
\begin{align*}
\sum_{k:(j, k) \in \mathbb{E}} S_{j k} & =S_{i j}-z_{i j} l_{i j}+s_{j}, \quad j \in \mathbb{N}  \tag{3a}\\
v_{j}-v_{k} & =2 \operatorname{Re}\left(z_{j k}^{H} S_{j k}\right)-\left|z_{j k}\right|^{2} l_{j k}, \quad j \rightarrow k \in \mathbb{E}  \tag{3b}\\
v_{j} l_{j k} & =\left|S_{j k}\right|^{2}, \quad j \rightarrow k \in \mathbb{E} \tag{3c}
\end{align*}
$$

The power flow quantities must satisfy the following constraints on grid operation.

1) Voltage stability

$$
\begin{equation*}
\underline{v}_{j} \leq v_{j} \leq \bar{v}_{j}, \quad j \in \mathbb{N} \tag{4a}
\end{equation*}
$$

2) Generation capacity

$$
\begin{array}{ll}
\underline{p}_{j}^{g} \leq p_{j}^{g} \leq \bar{p}_{j}^{g}, & j \in \mathbb{N} \\
\underline{q}_{j}^{g} \leq q_{j}^{g} \leq \bar{q}_{j}^{g}, & j \in \mathbb{N} \tag{4c}
\end{array}
$$

3) Line transmission capacity

$$
\begin{equation*}
\left|S_{j k}\right| \leq \bar{S}_{j k}, \quad j \rightarrow k \in \mathbb{E} \tag{4d}
\end{equation*}
$$

The joint battery swapping and OPF problem is to minimize a weighted sum of total generation cost in the distribution network and total travel distance of EVs over both station assignments and power flow variables as

$$
\begin{align*}
\min _{\substack{u, s, s, \\
\text { vil,S}}} & \sum_{j \in \mathbb{N}} f_{j}\left(p_{j}^{g}\right)+\alpha \sum_{a \in \mathbb{A}} \sum_{j \in \mathbb{N}_{w}} d_{a j} u_{a j} \\
\text { s.t. } & (1)(2)(3)(4), u \in\{0,1\}^{A N_{w}} \tag{5}
\end{align*}
$$

## III. Distributed Solutions

## A. Relaxations

The joint battery swapping and OPF problem (5) is computationally difficult for two reasons: The quadratic equality (3c) is nonconvex and the assignment variables $u$ are binary. To deal with the first difficulty, we replace (3c) by an inequality to relax the feasible set into a second-order cone, i.e., replace (3) in the problem (5) by

$$
\begin{align*}
\sum_{k:(j, k) \in \mathbb{E}} S_{j k} & =S_{i j}-z_{i j} l_{i j}+s_{j}, \quad j \in \mathbb{N}  \tag{6a}\\
v_{j}-v_{k} & =2 \operatorname{Re}\left(z_{j k}^{H} S_{j k}\right)-\left|z_{j k}\right|^{2} l_{j k}, \quad j \rightarrow k \in \mathbb{E}  \tag{6b}\\
v_{j} l_{j k} & \geq\left|S_{j k}\right|^{2}, \quad j \rightarrow k \in \mathbb{E} \tag{6c}
\end{align*}
$$

Fixing any assignments $u \in\{0,1\}^{A N_{w}}$, the optimization problem is then a convex problem. If an optimal solution to the SOCP relaxation attains equality in (6c), it also satisfies (3), and is therefore, optimal (for the given $u$ ). In this case, we say that the SOCP relaxation is exact. Sufficient conditions are known that guarantee the exactness of the SOCP relaxation; see [14] and [15] for a comprehensive tutorial and references therein. Even when these conditions are not satisfied, the SOCP relaxation for practical radial networks is still often exact, as confirmed also by our simulations in both Part I and this paper.

To deal with the second difficulty, we use generalized Benders decomposition in Part I. This approach computes an optimal solution when the SOCP relaxation is exact, but the computation is centralized and is suitable only when a single organization, e.g., State Grid in China, operates all of the distribution grid, stations and EVs. In this paper, we develop distributed solutions that are suitable for systems where these three are operated by separate entities that do not share their private information. To this end, we relax the binary assignment variables $u$ to real variables $u \in[0,1]^{A N_{w}}$. The constraints (1) are then replaced by

$$
\begin{align*}
u_{a j} & =0 \text { if } d_{a j}>\gamma_{a} c_{a}, j \in \mathbb{N}_{w}, a \in \mathbb{A}  \tag{7a}\\
\sum_{j \in \mathbb{N}_{w}} u_{a j} & =1, a \in \mathbb{A}  \tag{7b}\\
\sum_{a \in \mathbb{A}} u_{a j} & \leq m_{j}, j \in \mathbb{N}_{w} \tag{7c}
\end{align*}
$$

In summary, in this paper, we solve the following convex relaxation of (5):

$$
\begin{align*}
\min _{\substack{u, s, s \\
v, i, S \\
v,}} & \sum_{j \in \mathbb{N}} f_{j}\left(p_{j}^{g}\right)+\alpha \sum_{a \in \mathbb{A}} \sum_{j \in \mathbb{N}_{w}} d_{a j} u_{a j} \\
\text { s.t. } & (2)(4)(6)(7), u \in[0,1]^{A N_{w}} . \tag{8}
\end{align*}
$$

This problem has a convex objective and convex quadratic constraints. After an optimal solution $\left(x^{*}, u^{*}\right)$ of (8) is obtained, we check if $x^{*}$ attains equality in (6c). We also discretize $u_{a j}^{*}$ into $\{0,1\}$, e.g., by setting for each EV $a$ a single large $u_{a j}^{*}$ to 1 and the rest to 0 heuristically. An alternative is to randomize the station assignments using $u^{*}$ as a probability distribution. Whichever method is employed, it should guarantee the discretized station assignments are feasible. As we will show later, the discretization is readily implementable and achieves binary station assignments close to optimum.

## B. Distributed Solution via the ADMM

The relaxation (8) decomposes naturally into two subproblems, one on station assignments over $u$ and the other on OPF over $\left(s, s^{g}, v, l, S\right)$. The station assignment subproblem will be solved by a station operator that operates the network of stations. The OPF subproblem will be solved by a utility company. Our goal is to design a distributed algorithm for them to jointly solve (8) without sharing their private information.

These two subproblems are coupled only in (2a) where the utility company needs the charging load $s_{j}^{e}=r\left(M_{j}-m_{j}+\right.$
$\sum_{a \in \mathbb{A}} u_{a j}$ ) of station $j$ in order to compute the net real power injection $p_{j}$. This quantity depends on the total number of EVs that each station $j$ is assigned to and is computed by the station operator. Their computation can be decoupled by introducing an auxiliary variable $w_{j}$ at each bus (station) $j$ that represents the utility company's estimate of the quantity $r\left(M_{j}-m_{j}+\sum_{a \in \mathbb{A}} u_{a j}\right)$, and requiring that they be equal at optimality.

Specifically, recall the station assignment variables $u$, and denote the power flow variables by $x:=\left(w, s, s^{g}, v, l, S\right)$ where $w:=\left(r\left(M_{j}-m_{j}+\sum_{a \in \mathbb{A}} u_{a j}\right), j \in \mathbb{N}_{w}\right)$. Separate the objective function by defining

$$
\begin{aligned}
f(x) & :=\sum_{j \in \mathbb{N}} f_{j}\left(p_{j}^{g}\right) \\
g(u) & :=\alpha \sum_{a \in \mathbb{A}} \sum_{j \in \mathbb{N}_{w}} d_{a j} u_{a j} .
\end{aligned}
$$

Replace the coupling constraints (2) by constraints local to bus $j$ as follows:

$$
\begin{align*}
& p_{j}=\left\{\begin{array}{l}
p_{j}^{g}-p_{j}^{b}-w_{j}, \quad j \in \mathbb{N}_{w} \\
p_{j}^{g}-p_{j}^{b}, \quad j \in \mathbb{N} / \mathbb{N}_{w}
\end{array}\right.  \tag{9a}\\
& q_{j}=q_{j}^{g}-q_{j}^{b}, \quad j \in \mathbb{N} . \tag{9b}
\end{align*}
$$

Denote the local constraint set for $x$ by

$$
\mathbb{X}:=\left\{x \in \mathbb{R}^{\left(\left|\mathbb{N}_{w}\right|+5|\mathbb{N}|+3|\mathbb{E}|\right)}: x \text { satisfies }(4)(6)(9)\right\}
$$

Denote the local constraint set for $u$ by

$$
\mathbb{U}:=\left\{u \in \mathbb{R}^{A N_{w}}: u \text { satisfies (7) }\right\}
$$

To simplify notation, define $u_{j}:=\sum_{a \in \mathbb{A}} u_{a j}$, for $j \in \mathbb{N}_{w}$. Then, the relaxation (8) is equivalent to

$$
\begin{array}{rl}
\min _{x, u} & f(x)+g(u) \\
\text { s.t. } & x \in \mathbb{X}, u \in \mathbb{U} \\
& w_{j}=r\left(M_{j}-m_{j}+u_{j}\right), j \in \mathbb{N}_{w} \tag{10c}
\end{array}
$$

We now apply the ADMM to (10). Let $\lambda:=\left(\lambda_{j}, j \in \mathbb{N}_{w}\right)$ be the Lagrange multiplier vector corresponding to the current coupling constraint (10c), and define the augmented Lagrangian as

$$
\begin{equation*}
L_{\rho}(x, u, \lambda):=f(x)+g(u)+h_{\rho}(w, u, \lambda) \tag{11a}
\end{equation*}
$$

where $h_{\rho}$ depends on $(x, u)$ only through $\left(w_{j}, u_{j}, j \in \mathbb{N}_{w}\right)$ as follows:

$$
\begin{align*}
h_{\rho}(w, u, \lambda):= & \sum_{j \in \mathbb{N}_{w}} \lambda_{j}\left[w_{j}-r\left(M_{j}-m_{j}+u_{j}\right)\right] \\
& +\frac{\rho}{2} \sum_{j \in \mathbb{N}_{w}}\left[w_{j}-r\left(M_{j}-m_{j}+u_{j}\right)\right]^{2} \tag{11b}
\end{align*}
$$

and $\rho$ is the step size for dual variable $\lambda$ updates. The standard ADMM procedure is to iteratively and sequentially update


Fig. 2. Communication between utility company and station operator.

$$
\begin{align*}
&(x, u, \lambda): \text { for } n= 0,1, \ldots, \\
& x(n+1):= \arg \min _{x \in \mathbb{X}} f(x)+h_{\rho}(w, u(n), \lambda(n))  \tag{12a}\\
& u(n+1):=\arg \min _{u \in \mathbb{U}} g(u)+h_{\rho}(w(n+1), u, \lambda(n))  \tag{12b}\\
& \lambda_{j}(n+1):= \lambda_{j}(n)+\rho\left[w_{j}(n+1)\right. \\
&\left.-r\left(M_{j}-m_{j}+u_{j}(n+1)\right)\right], j \in \mathbb{N}_{w} . \tag{12c}
\end{align*}
$$

## Remark 1:

1) The $x$-update (12a) is carried out by the utility company and involves minimizing a convex objective with convex quadratic constraints. The ( $u, \lambda$ )-updates (12b), (12c) are carried out by the station operator and the $u$-update minimizes a convex quadratic objective with linear constraints. Both can be efficiently solved.
2) The $x$-update by the utility company in iteration $n+1$ needs $(u(n), \lambda(n))$ from the station operator. From (11b), the station operator does not need to communicate the detailed assignments $u(n)=\left(u_{a j}(n), a \in \mathbb{A}, j \in \mathbb{N}_{w}\right)$ to the utility company but only the charging load $s_{j}^{e}=$ $r\left(M_{j}-m_{j}+u_{j}(n)\right)$ of each station $j$.
3) The $(u, \lambda)$-updates by the station operator in iteration $n+1$ need the utility company's estimate $w(n+1)$ of $\left(r\left(M_{j}-m_{j}+u_{j}(n+1)\right), j \in \mathbb{N}_{w}\right)$.
4) The reason why the $x$-update by the utility company needs ( $u_{j}(n), j \in \mathbb{N}_{w}$ ) and the $u$-update by the station operator needs $w(n+1)$ lies in the (quadratic) regularization term in $h_{\rho}$. This becomes unnecessary for the dual decomposition approach in Section III-C without the regularization term.
The communication structure is illustrated in Fig. 2. In particular, private information of the utility company, such as distribution network parameters $\left(z_{j k},(j, k) \in \mathbb{E}\right)$, network states $\left(s(n), s^{g}(n), v(n), l(n), S(n)\right)$, cost functions $f$, and operational constraints, as well as private information of the station operator, such as the total numbers of batteries $\left(M_{j}, j \in \mathbb{N}_{w}\right)$, the numbers of available fully charged batteries $\left(m_{j}, j \in \mathbb{N}_{w}\right)$, how many EVs or where they are or their states of charge, and the detailed assignments $u(n)$, do not need to be communicated.

When the cost functions $f_{j}$ are closed, proper and convex and $L_{\rho}(x, u, \lambda)$ has a saddle point, the ADMM iteration (12) converges in that, for any $j \in \mathbb{N}_{w}$, the mismatch $\left|w_{j}(n)-r\left(M_{j}-m_{j}+u_{j}(n)\right)\right| \rightarrow 0$ and the objective function $f(x(n))+g(u(n))$ converges to its minimum value [16]. This does not automatically guarantee that $(x(n), u(n))$
converges to an optimal solution to (8). ${ }^{2}$ If $(x(n), u(n))$ indeed converges to a primal optimal solution $\left(x^{*}, u^{*}\right), u^{*}$ may generally not be binary. We can use a heuristic to derive binary station assignments from $u^{*}$, as mentioned previously. Fortunately, the following result shows that the number of EVs with nonbinary assignments is bounded and small in $u^{*}$. See Appendix A for its proof.

Theorem 1: It is always possible to find an optimal solution $\left(x^{*}, u^{*}\right)$ to the relaxation (8) in which the number of EVs $a$ with $u_{a j}^{*}<1$ for any $j \in \mathbb{N}_{w}$ is at most $N_{w}\left(N_{w}-1\right) / 2$.

In practice, the number $N_{w}$ of stations is much smaller than the number $A$ of EVs that request battery swapping, and hence, the number of nonbinary assignments that need to be discretized will be small. Simulations in Section IV further suggest that the discretized assignments are close to optimum.

## C. Distributed Solution via Dual Decomposition

The ADMM-based solution assumes the station operator directly controls the station assignments to all EVs. This requires that the station operator know the locations $\left(d_{a j}\right)$, states of charge $\left(c_{a}\right)$, and performance $\left(\gamma_{a}\right)$ of EVs. Moreover, the charging load $s_{j}^{e}=r\left(M_{j}-m_{j}+u_{j}(n)\right)$ of each station $j$ needs to be provided to the utility company. We now present another solution based on dual decomposition that is more suitable in situations where it is undesirable or inconvenient to share private information between the utility company, the station operator and EVs.

In the original relaxation (8), the update of the net power injections $p_{j}$ in (2) by the utility company involves $u_{j}$, which is updated by the station operator. These two computations are decoupled in the ADMM-based solution by introducing an auxiliary variable $w_{j}$ for each $j \in \mathbb{N}_{w}$ at the utility company and relaxing the constraint $w_{j}=r\left(M_{j}-m_{j}+u_{j}\right)$. In addition, the station assignments $u$ must satisfy $u_{j} \leq m_{j}$ in (7c). This is enforced in the ADMM-based solution by the station operator that computes $u$ for all EVs. To fully distribute the computation to individual EVs, we dualize $u_{j} \leq m_{j}$ as well. Let $\lambda:=\left(\lambda_{j}, j \in \mathbb{N}_{w}\right)$ and $\mu:=\left(\mu_{j} \geq 0, j \in \mathbb{N}_{w}\right)$ be the Lagrange multiplier vectors for the constraints $w_{j}=r\left(M_{j}-m_{j}+u_{j}\right)$ and $u_{j} \leq m_{j}, j \in \mathbb{N}_{w}$, respectively. Intuitively, $w$ and $\lambda$ decouple the computation of the utility company and that of individual EVs through coordination with the station operator. Additionally, $\mu$ decouples and coordinates all EVs' decisions so that EVs do not need direct communication among themselves to ensure that their decisions $u_{a j}$ collectively satisfy $u_{j} \leq m_{j}$.

Consider the Lagrangian of (10) with these two sets of constraints relaxed

$$
\begin{align*}
L(x, u, \lambda, \mu):= & f(x)+g(u) \\
& +\sum_{j \in \mathbb{N}_{w}} \lambda_{j}\left(w_{j}-r\left(M_{j}-m_{j}+u_{j}\right)\right) \\
& +\sum_{j \in \mathbb{N}_{w}} \mu_{j}\left(u_{j}-m_{j}\right) \tag{13}
\end{align*}
$$

${ }^{2}$ In the theory, ADMM may converge and circulate around the set of optimal solutions, but never reach one. In practice, a solution within a given error tolerance is acceptable.
and the dual problem of (10) as

$$
\max _{\lambda, \mu \geq 0} D(\lambda, \mu):=\min _{x \in \mathbb{X}, u \in \hat{\mathbb{U}}} L(x, u, \lambda, \mu)
$$

where the constraint set $\hat{\mathbb{U}}$ on $u$ is

$$
\hat{\mathbb{U}}:=\left\{u \in \mathbb{R}^{A N_{w}}: u \text { satisfies }(7 \mathrm{a}) \text { and }(7 \mathrm{~b})\right\} .
$$

Let $u_{a}:=\left(u_{a j}, j \in \mathbb{N}_{w}\right)$ denote the vector of EV $a$ 's decision on which station to swap its battery. Then, the dual problem is separable in power flow variables $x$ as well as individual EVs, decisions $u_{a}$ as

$$
\begin{equation*}
D(\lambda, \mu)=V(\lambda)+\sum_{a \in \mathbb{A}} U_{a}(\lambda, \mu) \tag{14a}
\end{equation*}
$$

where the problem $V(\lambda)$ solved by the utility company is

$$
\begin{equation*}
V(\lambda):=\min _{x \in \mathbb{X}}\left(f(x)+\sum_{j \in \mathbb{N}_{w}} \lambda_{j} w_{j}\right) \tag{14b}
\end{equation*}
$$

and the problem $U_{a}(\lambda)$ solved by each individual $\operatorname{EV} a$ is

$$
\begin{equation*}
U_{a}(\lambda, \mu):=\min _{u_{a} \in \hat{\mathbb{U}}_{a}} \sum_{j \in \mathbb{N}_{w}}\left(\alpha d_{a j}-r \lambda_{j}+\mu_{j}\right) u_{a j} \tag{14c}
\end{equation*}
$$

where the constraint set $\hat{\mathbb{U}}_{a}$ on $u_{a}$ is

$$
\hat{\mathbb{U}}_{a}:=\left\{\begin{array}{c}
u_{a j} \in[0,1], j \in \mathbb{N}_{w} \\
u_{a} \in \mathbb{R}^{N_{w}}: u_{a j}=0 \text { if } d_{a j}>\gamma_{a} c_{a}, j \in \mathbb{N}_{w} \\
\sum_{j \in \mathbb{N}_{w}} u_{a j}=1
\end{array}\right\}
$$

Note that (14c) has closed-form solutions. For instance, if there exists a unique optimal solution to $U_{a}(\lambda, \mu)$, i.e., for any EV $a$ there is a unique $j_{a}^{*}(\lambda, \mu)$ defined as

$$
j_{a}^{*}(\lambda, \mu):=\arg \min _{j: d_{a j} \leq \gamma_{a} c_{a}}\left\{\alpha d_{a j}-r \lambda_{j}+\mu_{j}\right\}
$$

then the optimal solution can be uniquely determined as

$$
u_{a j}^{*}(\lambda, \mu):=\left\{\begin{array}{l}
1, \text { if } j=j_{a}^{*}(\lambda, \mu) \\
0, \text { if } j \neq j_{a}^{*}(\lambda, \mu)
\end{array}\right.
$$

i.e., it simply chooses the unique station $j_{a}^{*}$ within EV $a$ 's driving range that has the minimum cost $\alpha d_{a j}-r \lambda_{j}+\mu_{j}$.

From (13), the standard dual algorithm for solving (10) is, for $j \in \mathbb{N}_{w}$,

$$
\begin{align*}
\lambda_{j}(n+1):= & \lambda_{j}(n)+\rho_{1}(n) \\
& \times\left[w_{j}(n)-r\left(M_{j}-m_{j}+u_{j}(n)\right)\right]  \tag{15a}\\
\mu_{j}(n+1):= & \max \left\{\mu_{j}(n)+\rho_{2}(n)\left(u_{j}(n)-m_{j}\right), 0\right\} \tag{15b}
\end{align*}
$$

where $\rho_{1}(n), \rho_{2}(n)>0$ are diminishing step sizes, and from (14), we have

$$
\begin{equation*}
x(n):=\arg \min _{x \in \mathbb{X}}\left(f(x)+\sum_{j \in \mathbb{N}_{w}} \lambda_{j}(n) w_{j}\right) \tag{15c}
\end{equation*}
$$

and for $a \in \mathbb{A}$,

$$
\begin{equation*}
u_{a}(n):=\arg \min _{u_{a} \in \hat{\mathbb{U}}_{a}} \sum_{j \in \mathbb{N}_{w}}\left(\alpha d_{a j}-r \lambda_{j}(n)+\mu_{j}(n)\right) u_{a j .} \tag{15d}
\end{equation*}
$$



Fig. 3. Communication between utility company, station operator, and EVs.

## Remark 2:

1) The $x$-update $(15 \mathrm{c})$ is carried out by the utility company and involves minimizing a convex objective with convex quadratic constraints. The only information that is nonlocal to the utility company for its $x$-update is one of the dual variables $\lambda(n)$ computed by the station operator.
2) The $u_{a}$-update ( 15 d ) is carried out by each individual EV. Each EV requires both the dual variables $(\lambda(n), \mu(n))$ from the station operator for its update.
3) The dual updates (15a), (15b) are carried out by the station operator that uses a (sub)gradient ascent algorithm to solve the dual problem $\max _{\lambda, \mu \geq 0} D(\lambda, \mu)$. It requires $w(n)$ from the utility company and individual decisions $u_{a}(n)$ from EVs $a$.
The communication structure is illustrated in Fig. 3. In particular, EVs are completely decoupled from the utility company and among themselves. Unlike the ADMM-based solution, the station operator knows only the battery swapping decisions of EVs, but not their private information such as locations $\left(d_{a j}\right)$, states of charge $\left(c_{a}\right)$ or performance $\left(\gamma_{a}\right)$.

Since the relaxation (8) is convex, strong duality holds if Slater's condition is satisfied. Then, when the aforementioned (sub)gradient algorithm converges to a dual optimal solution $\left(\lambda^{*}, \mu^{*}\right)$, any primal optimal point is also a solution to the corresponding $x$-update ( 15 c ) and $u_{a}$-update (15d) [17], [18]. Suppose $\left(x(n), u_{a}(n), a \in \mathbb{A}\right)$ indeed converges to a primal optimal solution $\left(x^{*}, u_{a}^{*}, a \in \mathbb{A}\right)$, then typically $\left(u_{a}^{*}, a \in \mathbb{A}\right)$ is not binary. However, the bound in Theorem 1 still holds that guarantees easy discretization and suggests the final discretized stations assignments are close to optimum.

Remark 3: The two solutions have their own advantages and can be adapted to different application scenarios. The ADMMbased solution requires a station operator that is trustworthy and can access EVs' private information. Since the station operator optimizes station assignments on behalf of all EVs, no computation is required on each EV, and meanwhile communication is only required between the station operator and the utility company. In contrast, the solution based on dual decomposition does

TABLE I
Setup
(a) Distributed generator

| Bus | $\bar{p}_{j}^{g}$ | $\underline{p}_{j}^{g}$ | $\bar{q}_{j}^{g}$ | $\underline{q}_{j}^{g}$ | Cost function |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 0 | 2 | -2 | $0.3 p^{g 2}+30 p^{g}$ |
| 4 | 2.5 | 0 | 1.5 | -1.5 | $0.1 p^{g 2}+20 p^{g}$ |
| 26 | 2.5 | 0 | 1.5 | -1.5 | $0.1 p^{g 2}+20 p^{g}$ |
| 34 | 2.5 | 0 | 1.5 | -1.5 | $0.1 p^{g 2}+20 p^{g}$ |

(b) Station

| Bus | Location | $M_{j}$ | $m_{j}$ |
| :--- | :---: | :---: | :---: |
| 5 | $(1,1)$ | $m_{j}$ | (i) $A$; (ii) $A / 2$ |
| 16 | $(3,1)$ | $m_{j}$ | (i) $A$; (ii) $A / 10$ |
| 31 | $(1,3)$ | $m_{j}$ | (i) $A$; (ii) $A / 4$ |
| 43 | $(3,3)$ | $m_{j}$ | (i) $A$; (ii) $A / 4$ |


(a)

(b)

Fig. 4. Convergence of the ADMM. (a) $\lambda$. (b) Residual of relaxed (10c).
not require sharing EVs' private information with the station operator. It, however, necessitates computation capabilities on all EVs. In addition, communication is needed both between the station operator and the utility company and between the station operator and each EV.


Fig. 5. Convergence of dual decomposition. (a) $\lambda$. (b) $\mu$.

## IV. Numerical Results

We test the two distributed solutions on the same 56-bus radial distribution feeder of Southern California Edison (SCE) in Part I. Details about the feeder can be found in [19]. Similar setups from Part I are adopted to demonstrate the algorithm performance. Table I lists the main parameters. ${ }^{3}$ The number of EVs that request battery swapping is $A=400$. We simulate two cases with different $m_{j}$ 's (see Table I(b)). EVs' current locations are randomized uniformly within a $4 \mathrm{~km} \times 4 \mathrm{~km}$ square area and their destinations are ignored. We use Euclidean distances $d_{a j}$ and assume all EVs can reach any of the four stations. The constant charging rate is $r=0.01 \mathrm{MW}$ [20], and the weight is $\alpha=0.02 \$ / \mathrm{km}$ [21]. Simulations are run on a laptop with Intel Core i7-3632QM CPU at $2.20 \mathrm{GHz}, 8-\mathrm{GB}$ RAM, and 64 -bit Windows 10 OS.

## A. Convergence

The convergence of the ADMM in case (i) is demonstrated in Fig. 4. Fig. 4(a) and (b) shows, respectively, that the Lagrange

[^2]

Fig. 6. Suboptimality in different cases: (a) case (i), (b) case (ii).

TABLE II
Exactness of SOCP Relaxation (Partial Results for Case (il))

| Bus <br> From | To | $v_{j} l_{j k}$ | $\left\|S_{j k}\right\|^{2}$ | Residual |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 2 | 2.582 | 2.582 | 0.000 |
| 2 | 3 | 0.006 | 0.006 | 0.000 |
| 2 | 4 | 2.336 | 2.336 | 0.000 |
| 4 | 5 | 3.413 | 3.413 | 0.000 |
| 4 | 6 | 0.005 | 0.005 | 0.000 |
| 4 | 7 | 2.276 | 2.276 | 0.000 |
| 7 | 8 | 1.984 | 1.984 | 0.000 |
| 8 | 9 | 0.009 | 0.009 | 0.000 |
| 8 | 10 | 1.518 | 1.518 | 0.000 |
| 10 | 11 | 1.318 | 1.318 | 0.000 |

multiplier vector $\lambda$ and the residual of the relaxed equality constraint (10c) converge rapidly. Case (ii) behaves similarly. Each iteration that computes the three steps of (12) takes on average 0.477 s by Gurobi. For the dual decomposition algorithm, Fig. 5(a) and (b) shows the convergence of its two Lagrange multiplier vectors $\lambda$ and $\mu$, respectively, in case (ii). $\lambda$ maintains


Fig. 7. Average computation time of the ADMM: (a) as a function of \#EVs, (b) as a function of \#stations.
the consensus between the utility company and EVs at convergence, and $\mu$ guarantees (7c) is satisfied when it converges. Dual decomposition usually takes more iterations to converge due to the additionally required coordination among all EVs. For case (i), results are similar except that $\mu$ remains 0 during computation as (7c) is always satisfied. Each iteration of the dual decomposition algorithm involves the centralized update of (15a) and (15b) and the parallelized computation of $(15 \mathrm{c})$ and (15d). Each iteration takes on average 0.212 s by Gurobi.

## B. Suboptimality (Comparison With Centralized Solution)

In case (i), both algorithms obtain a solution in which the station assignments to two EVs, marked black in Fig. 6(a), are nonbinary: $u_{242}=[0.7070 .2930 .0000 .000]$ and $u_{367}=$ [0.230 0.0000 .7700 .000$]$. This is consistent with Theorem 1. If we simply round $u_{243}$ and $u_{367}$ to binary values, the resulting solution turns out to coincide with a globally optimal solution computed using the centralized solution in Part I.
In case (ii), we reduce available fully charged batteries at each station to activate (7c). Fig. 6(b) shows the solution achieved by


Fig. 8. Average computation time of dual decomposition: (a) as a function of \#EVs, (b) as a function of \#stations.
both algorithms. The solution turns out to be globally optimal for the original problem (5); in particular, all station assignments are binary. EVs, to which the station assignments are altered due to the bound imposed on battery availability of each station, are marked cyan in Fig. 6(b). The intuition is that an active (7c) sometimes can help eliminate nonbinary assignments to EVs. This is often the case in practice where battery availability is uneven across stations.

## C. Exactness of SOCP Relaxation

In most cases, that we have simulated, including cases reported here, the SOCP relaxation is exact, i.e., the solutions computed by the two distributed algorithms attain equality in (6c), and therefore, satisfy power flow equations. Partial data for case (ii) are listed in Table II.

## D. Scalability

We follow the same setup in part I to demonstrate the scalability of the two distributed algorithms, i.e., we first augment the number of EVs, while the number of stations is fixed, and
then, turn the other way round. The computation time that is shown in Figs. 7 and 8 is averaged over ten simulation runs with randomly generated cases. Approximately, the computational effort of both algorithms increases linearly as EVs (or stations) scale up. Compared with the centralized solution in part I, the required computation time of the distributed algorithms is less sensitive to the EV scale, which is intuitive, but turns out more sensitive to the station scale. This results from the fact that the consensus that the distributed algorithms strive toward has to be achieved at each station. Generally, more iterations are needed as more stations are involved.

## V. Concluding Remarks

This paper is an extension of Part I that basically solves the same optimal scheduling problem for battery swapping. Instead of a centralized solution that requires global information, two distributed solutions based on the ADMM and dual decomposition, respectively, are proposed. These solutions are more suitable for systems where the distribution grid, stations, and EVs are operated by separate entities that do not share their private information. They allow these entities to make individual decisions but coordinate through privacy-preserving information exchanges to jointly solve a relaxation of the global problem. Some of the station assignments in a relaxed solution may not be binary and need to be discretized, but we prove that their number is small. Numerical tests on the SCE 56-bus distribution feeder demonstrate the algorithm performance and also suggest that the final discretized station assignments are close to optimum.

## APPENDIX A

## Proof of Theorem 1

We refer to $\mathrm{EV} a$ as a critical EV if its station assignment satisfies $u_{a j}<1$ for all $j \in \mathbb{N}_{w}$. We first show the following lemma, and then, prove Theorem 1. Let $(u, y):=\left(u, s, s^{g}, v, \ell, S\right)$.

Lemma 1: It is always possible to find an optimal solution $\left(u^{*}, y^{*}\right)$ to the relaxation (8) where no critical EVs share two stations, i.e., there do not exist $a, b \in \mathbb{A}$ and $j, k \in \mathbb{N}_{w}$ such that $u_{a j}^{*}, u_{\mathrm{ak}}^{*}, u_{\mathrm{bj}}^{*}, u_{\mathrm{bk}}^{*}>0$.

Proof of Lemma 1: Fix any $(u, y)$ that is feasible for (8). If $u_{a j}, u_{\mathrm{ak}}, u_{\mathrm{bj}}, u_{\mathrm{bk}}>0$, for some $a, b \in \mathbb{A}$ and $j, k \in \mathbb{N}_{w}$, we will construct station assignments $u^{\prime}$ that satisfy the lemma such that $\left(u^{\prime}, y\right)$ is also feasible for (8) but has a lower or equal objective value. This proves the lemma.

Let $B_{a}:=u_{a j}+u_{\mathrm{ak}}, B_{b}:=u_{\mathrm{bj}}+u_{\mathrm{bk}}, B_{j}:=u_{a j}+u_{\mathrm{bj}}$, and $B_{k}:=u_{\mathrm{ak}}+u_{\mathrm{bk}}$. The interpretation of these quantities is that $r B_{a}$ and $r B_{b}$ are the charging loads of EVs $a$ and $b$, respectively, and $r B_{j}$ and $r B_{k}$ are their load distributions at stations $j$ and $k$, respectively. Clearly, $B_{a}+B_{b}=B_{j}+B_{k}$. Without loss of generality, we can assume either case 1: $B_{a} \geq B_{j} \geq B_{k} \geq B_{b}$ or case 2: $B_{j} \geq B_{a} \geq B_{b} \geq B_{k}$ holds. We now construct $u^{\prime}$ assuming case 1 holds. The construction is similar if case 2 holds instead.

We consider four disjoint subcases and construct $u^{\prime}$ for each subcase.
1.1 $\mathrm{EV} a$ is closer to station $j$ but farther away from station $k$ than $b\left(d_{a j} \leq d_{\mathrm{bj}}, d_{\mathrm{bk}} \leq d_{\mathrm{ak}}\right)$ : Let $u_{a j}^{\prime}=B_{j}, u_{\mathrm{ak}}^{\prime}=$
$B_{k}-B_{b}, u_{\mathrm{bj}}^{\prime}=0, u_{\mathrm{bk}}^{\prime}=B_{b}$, and the other variables remain the same as in $(u, y)$. This means that the assignments $u^{\prime}$ send EV $b$ to station $k$ but not station $j$, and also increase the likelihood of EV $a$ going to station $j$ while decreasing that to station $k$. Since

$$
\begin{aligned}
u_{a j}^{\prime}+u_{\mathrm{ak}}^{\prime} & =B_{j}+B_{k}-B_{b}=u_{a j}+u_{\mathrm{ak}} \\
u_{\mathrm{bj}}^{\prime}+u_{\mathrm{bk}}^{\prime} & =B_{b}=u_{\mathrm{bj}}+u_{\mathrm{bk}} \\
u_{a j}^{\prime}+u_{\mathrm{bj}}^{\prime} & =B_{j}=u_{a j}+u_{\mathrm{bj}} \\
u_{\mathrm{ak}}^{\prime}+u_{\mathrm{bk}}^{\prime} & =B_{k}-B_{b}+B_{b}=u_{\mathrm{ak}}+u_{\mathrm{bk}}
\end{aligned}
$$

$\left(u^{\prime}, y\right)$ is feasible (8). Moreover,

$$
\begin{aligned}
& \sum_{c=a, b} \sum_{i=j, k} d_{\mathrm{ci}} u_{\mathrm{ci}}^{\prime} \\
= & d_{a j} B_{j}+d_{\mathrm{ak}}\left(B_{k}-B_{b}\right)+d_{\mathrm{bk}} B_{b} \\
= & d_{a j}\left(u_{a j}+u_{\mathrm{bj}}\right)+d_{\mathrm{ak}}\left(u_{\mathrm{ak}}-u_{\mathrm{bj}}\right)+d_{\mathrm{bk}}\left(u_{\mathrm{bj}}+u_{\mathrm{bk}}\right) \\
\leq & \sum_{c=a, b} \sum_{i=j, k} d_{\mathrm{ci}} u_{\mathrm{ci}}-u_{\mathrm{bj}}\left(d_{\mathrm{ak}}-d_{\mathrm{bk}}\right) \\
\leq & \sum_{c=a, b} \sum_{i=j, k} d_{\mathrm{ci}} u_{\mathrm{ci}}
\end{aligned}
$$

where the first inequality uses $d_{a j} \leq d_{\mathrm{bj}}$ and the second inequality uses $d_{\mathrm{bk}} \leq d_{\mathrm{ak}}$. Therefore, $\left(u^{\prime}, y\right)$ has a lower or equal objective value than $(u, y)$.
1.2 $\mathrm{EV} b$ is closer to station $j$ but farther away from station $k$ than $a\left(d_{\mathrm{bj}} \leq d_{a j}, d_{\mathrm{ak}} \leq d_{\mathrm{bk}}\right)$ : This case is symmetric to subcase 1.1.
1.3 $\mathrm{EV} a$ is closer than $b$ to both stations $\left(d_{a j} \leq d_{\mathrm{bj}}, d_{\mathrm{ak}} \leq\right.$ $\left.d_{\mathrm{bk}}\right)$ : We either have $d_{\mathrm{bj}}-d_{\mathrm{bk}} \leq d_{a j}-d_{\mathrm{ak}}$ or $d_{\mathrm{bj}}-$ $d_{\mathrm{bk}}>d_{a j}-d_{\mathrm{ak}}$. In the former case, let $u_{a j}^{\prime}=B_{j}-$ $B_{b}, u_{\mathrm{ak}}^{\prime}=B_{k}, u_{\mathrm{bj}}^{\prime}=B_{b}$, and $u_{\mathrm{bk}}^{\prime}=0$. Then,

$$
\begin{aligned}
& \sum_{c=a, b} \sum_{i=j, k} d_{\mathrm{ci}} u_{\mathrm{ci}}^{\prime} \\
= & \sum_{c=a, b} \sum_{i=j, k} d_{\mathrm{ci}} u_{\mathrm{ci}}+\left(d_{\mathrm{ak}}-d_{\mathrm{bk}}+d_{\mathrm{bj}}-d_{a j}\right) u_{\mathrm{bk}} \\
\leq & \sum_{c=a, b} \sum_{i=j, k} d_{\mathrm{ci}} u_{\mathrm{ci}} .
\end{aligned}
$$

Similar to subcase $1.1,\left(u^{\prime}, y\right)$ is feasible and has a lower or equal objective value. In the latter case, let $u_{a j}^{\prime}=B_{j}, u_{\mathrm{ak}}^{\prime}=B_{k}-B_{b}, u_{\mathrm{bj}}^{\prime}=0$, and $u_{\mathrm{bk}}^{\prime}=B_{b}$. Then, $\left(u^{\prime}, y\right)$ is feasible and has a lower objective value.
1.4 EV $b$ is closer than $a$ to both stations $\left(d_{\mathrm{bj}} \leq d_{a j}, d_{\mathrm{bk}} \leq\right.$ $\left.d_{\mathrm{ak}}\right)$ : This case is symmetric to subcase 1.3.
This completes the proof of the lemma.
Proof of Theorem 1: Fix an optimal solution $\left(u^{*}, y^{*}\right)$ to the relaxation (8) that satisfies Lemma 1. By definition, a critical EV splits its charging load between at least two different stations. An upper bound on the number of critical EVs is, therefore, the maximum number of critical EVs that we can assign the $N_{w}$ stations to without violating Lemma 1.

Consider the set $C_{1}$ of critical EVs under the assignments $u^{*}$ that split their charging loads between station $i=1$ and (at least) another station $j=2, \ldots, N_{w}$. Lemma 1 implies that there are at most $N_{w}-1$ critical EVs in $C_{1}$ since the assignments $u^{*}$ are optimal. Consider next the set $C_{2}$ of critical EVs not in $C_{1}$ that split their charging loads between station $i=2$ and (at least) another station $j=3, \ldots, N_{w}$. There are at most $N_{w}-2$ critical EVs in $C_{2}$. Similarly there are at most $N_{w}-i$ critical EVs in the set $C_{i}$ that are not in $\cup_{k=1}^{i-1} C_{k}$ that split their charging loads between station $i$ and (at least) another station $j>i$. Hence, the maximum number of such critical EVs is $\left(N_{w}-1\right)+\left(N_{w}-2\right)+\cdots+1=\frac{1}{2} N_{w}\left(N_{w}-1\right)$. This completes the proof of Theorem 1.

## References

[1] P. You et al. "Scheduling of EV battery swapping, I: Centralized solution," IEEE Trans. Cont. Netw. Syst., 2017, p. 1.
[2] P. McDaniel and S. McLaughlin, "Security and privacy challenges in the smart grid, "IEEE Security Privacy, vol. 7, no. 3, pp. 75-77, May/Jun. 2009.
[3] G. Kalogridis, C. Efthymiou, S. Z. Denic, T. A. Lewis, and R. Cepeda, "Privacy for smart meters: Towards undetectable appliance load signatures," in Proc. IEEE Int. Conf. Smart Grid Commun., 2010, pp. 232-237.
[4] C. Efthymiou and G. Kalogridis, "Smart grid privacy via anonymization of smart metering data," in Proc. IEEE Int. Conf. Smart Grid Commun., 2010, pp. 238-243.
[5] E. Liu, P. You, and P. Cheng, "Optimal privacy-preserving load scheduling in smart grid," in Proc. IEEE Power Energy Soc. Gen. Meeting, 2016, pp. 1-5.
[6] L. Yang, X. Chen, J. Zhang, and H. V. Poor, "Optimal privacy-preserving energy management for smart meters," in Proc. IEEE Conf. Comput. Commun., 2014, pp. 513-521.
[7] H.-R. Tseng, "A secure and privacy-preserving communication protocol for V2G networks," in Proc. IEEE Wireless Commun. Netw. Conf., 2012, pp. 2706-2711.
[8] H. Liu, H. Ning, Y. Zhang, and L. T. Yang, "Aggregated-proofs based privacy-preserving authentication for V2G networks in the smart grid," IEEE Trans. Smart Grid, vol. 3, no. 4, pp. 1722-1733, Dec. 2012.
[9] H. Nicanfar, P. TalebiFard, S. Hosseininezhad, V. Leung, and M. Damm, "Security and privacy of electric vehicles in the smart grid context: Problem and solution," in Proc. ACM Int. Symp. Des. Anal. Intell. Veh. Netw. Appl., 2013, pp. 45-54.
[10] M. Liu, R. H. Ordóñez-Hurtado, F. Wirth, Y. Gu, E. Crisostomi, and R. Shorten, "A distributed and privacy-aware speed advisory system for optimizing conventional and electric vehicle networks," IEEE Trans. Intell. Transp. Syst., vol. 17, no. 5, pp. 1308-1318, May 2016.
[11] C. Clifton, M. Kantarcioglu, J. Vaidya, X. Lin, and M. Y. Zhu, "Tools for privacy preserving distributed data mining," ACM SIGKDD Explorations Newslett., vol. 4, no. 2, pp. 28-34, 2002.
[12] J. Zhou, X. Lin, X. Dong, and Z. Cao, "PSMPA: Patient selfcontrollable and multi-level privacy-preserving cooperative authentication in distributedm-healthcare cloud computing system," IEEE Trans. Parallel Distrib. Syst., vol. 26, no. 6, pp. 1693-1703, Jun. 1, 2015.
[13] M. E. Baran and F. F. Wu, "Optimal sizing of capacitors placed on a radial distribution system," IEEE Trans. Power Del., vol. 4, no. 1, pp. 735-743, Jan. 1989.
[14] S. H. Low, "Convex relaxation of optimal power flow, I: Formulations and relaxations," IEEE Trans. Control Netw. Syst., vol. 1, no. 1, pp. 15-27, Mar. 2014.
[15] S. H. Low, "Convex relaxation of optimal power flow, II: Exactness," IEEE Trans. Control Netw. Syst., vol. 1, no. 2, pp. 177-189, Jun. 2014.
[16] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," Found. Trends Mach. Learn., vol. 3, no. 1, pp. 1-122, 2011.
[17] S. Boyd, L. Xiao, and A. Mutapcic, "Subgradient methods," Lecture Notes of EE3920, Stanford University, Stanford, CA, USA, vol. 2004, Autumn Quarter 2003, pp. 2004-2005.
[18] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge, U.K.: Cambridge Univ. Press, 2004.
[19] M. Farivar, R. Neal, C. Clarke, and S. Low, "Optimal inverter VAR control in distribution systems with high PV penetration," in Proc. IEEE Power Energy Soc. Gen. Meeting, 2012, pp. 1-7.
[20] M. Yilmaz and P. T. Krein, "Review of battery charger topologies, charging, power levels, and infrastructure for plug-in electric and hybrid vehicles," IEEE Trans. Power Electron., vol. 28, no. 5, pp. 2151-2169, May 2013.
[21] U.S. Energy Information Administration, "Annual energy review,"Energy Information Administration, U.S. Department of Energy, Washington, DC, USA, 2011. [Online]. Available: www. eia. doe. gov/emeu/aer


Pengcheng You (S'14) received the B.S. degree in electrical engineering and the bachelor's degree in science and engineering (with Hons.) both from Zhejiang University, Hangzhou, China, in 2003, where he is currently working toward the Ph.D. degree with the College of Control Science and Engineering. He is a member of the Networked Sensing and Control Group, State Key Laboratory of Industrial Control Technology, Zhejiang University.

He was a visiting student with the Singapore University of Technology and Design and California Institute of Technology, and also a Ph.D. intern at Pacific Northwest National Laboratory. His research interests include smart grid and machine learning.


Steven H. Low (F'08) received the B.S. degree in electrical engineering from Cornell University, Ithaca, NY, USA, in 1987, and the Ph.D. degree in electrical engineering from University of California, Berkeley, Berkeley, CA, USA, in 1992.

He has been a Professor with the Department of Computing \& Mathematical Sciences and the Department of Electrical Engineering, California Institute of Technology, Pasadena, CA, since 2000. Before that, he was with AT\&T Bell Laboratories, Murray Hill, NJ, USA, from 1992 to 1996, and with the University of Melbourne, Australia, from 1996 to 2000. His research on communication networks was accelerating more than 1 TB of Internet traffic every second in 2014.

Prof. Low was a Member of the Networking and Information Technology Technical Advisory Group for the US President's Council of Advisors on Science and Technology, in 2006. He has been a corecipient of IEEE best paper awards, on the Editorial Boards of major journals in networking and power systems, and is an Honorary/Chair Professor in Australia, China, and Taiwan.


Liang Zhang (S'13) received the B.Sc. and M.Sc. degrees in electrical engineering from the Shanghai Jiao Tong University, Shanghai, China, in 2012 and 2014, respectively. Since 2014, he has been working toward the Ph.D. degree with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN, USA.

His research interests include large-scale optimization and high-dimensional learning.


Ruilong Deng (S'11-M'14) received the B.Sc. and Ph.D. degrees both in control science and engineering from Zhejiang University, Hangzhou, China, in 2009 and 2014, respectively.

He was a Research Fellow with Nanyang Technological University, Singapore, from 2014 to 2015. He is currently, an AITF Postdoctoral Fellow with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada. His research interests include smart grid, cyber security, and wireless sensor networks.

Dr. Deng serves/served as an Editor for the IEEE ACCESS and Journal of Communications and Networks, and a Guest Editor for the IEEE Transactions on Emerging Topics in Computing, IEEE AcCESS, and IET Cyber-Physical Systems: Theory \& Applications. He also serves/served as a Technical Program Committee Member for the IEEE Global Communications Conference, IEEE International Conference on Communications, IEEE International Conference on Smart Grid Communications, and EAI International Conference on Smart Grids for Smart Cities, etc.


Georgios B. Giannakis (F'97) received the Diploma degree in electrical engineering from the National Technical University of Athens, Athens, Greece, in 1981, and the M.Sc. degree in electrical engineering, the M.Sc. degree in mathematics, and the Ph.D. in electrical engineering from the University of Southern California (USC), Los Angeles, CA, USA, in 1983, 1986, and 1986, respectively.

From 1982 to 1986, he was with the USC. He was with the University of Virginia from 1987 to 1998, and since 1999, he has been a Professor with the University of Minnesota, Minneapolis, MN, USA, where he holds an Endowed Chair in Wireless Telecommunications, a University of Minnesota McKnight Presidential Chair in electrical and computer engineering, and serves as the Director of the Digital Technology Center. His general research interests include the areas of communications, networking and statistical signal processing-subjects on which he has published more than 400 journal papers, 700 conference papers, 25 book chapters, 2 edited books, and 2 research monographs (h-index 125). His current research interests include learning from Big Data, wireless cognitive radios, and network science with applications to social, brain, and power networks with renewables. He is the (co-)inventor of 30 patents issued.

Prof. Giannakis is the (co-)recipient of eight Best Paper awards from the IEEE Signal Processing (SP) and Communications Societies, including the G. Marconi Prize Paper Award in Wireless Communications. He also received the Technical Achievement Awards from the SP Society (2000) and the European Association for Signal Processing (EURASIP) (2005), a Young Faculty Teaching Award, the G. W. Taylor Award for Distinguished Research from the University of Minnesota, and the IEEE Fourier Technical Field Award (2015). He is a Fellow of EURASIP, and has served the IEEE in a number of posts, including that of a Distinguished Lecturer for the IEEE-SP Society.


Youxian Sun received the Diploma degree in chemical engineering from the Department of Chemical Engineering, Zhejiang University, Hangzhou, China, in 1964.

He joined the Department of Chemical Engineering, Zhejiang University, in 1964. From 1984 to 1987, he was an Alexander Von Humboldt Research Fellow and a Visiting Associate Professor with the University of Stuttgart, Stuttgart, Germany. He has been a full Professor with Zhejiang University since 1988. In 1995, he was elevated to an Academician of the Chinese Academy of Engineering. His current research interests include modeling, control, and optimization of complex systems, and robust control design and its applications. He has authored/co-authored over 450 journal and conference papers. He is currently the Director of the Institute of Industrial Process Control and the National Engineering Research Center of Industrial Automation, Zhejiang University.

Prof. Sun is a Fellow of the International Federation of Automatic Control (IFAC). He is the President of the Chinese Association of Automation, and also has served as the Vice-Chairman of IFAC Pulp and Paper Committee and the Vice-President of the China Instrument and Control Society.


Zaiyue Yang (M'10) received the B.S. and M.S. degrees from the Department of Automation, University of Science and Technology of China, Hefei, China, in 2001 and 2004, respectively, and the Ph.D. degree from the Department of Mechanical Engineering, University of Hong Kong, Hong Kong, in 2008, all in automatic control.

He was a Postdoctoral Fellow and a Research Associate with the Department of Applied Mathematics, Hong Kong Polytechnic University, Hong Kong, before joining the College of Control Science and Engineering, Zhejiang University, Hangzhou, China, in 2010. Then, he joined the Department of Mechanical and Energy Engineering, Southern University of Science and Technology, Shenzhen, China, in 2017, where he is currently a Professor. His current research interests include smart grid, signal processing, and control theory.

Prof. Yang is an Associate Editor for the IEEE Transactions on InDUSTRIAL INFORMATICS.


[^0]:    Manuscript received June 25, 2017; revised October 3, 2017; accepted October 26, 2017. Date of publication November 15, 2017; date of current version December 14, 2018. This work was supported in part by the Zhejiang Provincial Natural Science Foundation of China under Grant LR16F030002; in part by the NSF through Grant CCF 1637598, Grant ECCS 1619352 and Grant CNS 1545096; in part by the ARPA-E through Grant DE-AR0000699 and the GRID DATA program; in part by the DTRA through Grant HDTRA 1-15-1-0003; and in part by the Alberta Innovates-Technology Futures (AITF) postdoctoral fellowship. Recommended by Associate Editor L. Schenato. (Corresponding author: Zaiyue Yang.)
    P. You and Y. Sun are with the State Key Laboratory of Industrial Control Technology, Zhejiang University, Hangzhou 310027, China (e-mail: pcyou@zju.edu.cn; yxsun@iipc.zju.edu.cn).
    S. H. Low is with the Engineering and Applied Science Division, California Institute of Technology, Pasadena, CA 91125 USA (e-mail: slow@caltech.edu).
    L. Zhang and G. B. Giannakis are with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455 USA (e-mail: zhan3523@umn.edu; georgios@umn.edu).
    R. Deng is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada T6G 1H9 (e-mail: ruilong@ualberta.ca).
    Z. Yang is with the Department of Mechanical and Energy Engineering, Southern University of Science and Technology, Shenzhen 518055, China. He was with the State Key Laboratory of Industrial Control Technology, Zhejiang University, Hangzhou 310027, China (e-mail: yangzy3@sustc.edu.cn).

    Digital Object Identifier 10.1109/TCNS.2017.2774012

[^1]:    ${ }^{1}$ Throughout this paper, stations refers to battery swapping service stations.

[^2]:    ${ }^{3}$ The units of the real power, reactive power, cost, distance, and weight in this paper are MW, Mvar, $\$$, km, and $\$ / \mathrm{km}$, respectively.

