Real-Time Recommendation Algorithm of Battery Swapping Stations for Electric Taxis

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***Abstract*—This paper proposes a real-time algorithm that recommends battery swapping stations (BSSs) to electric taxis (ETs) that need their batteries swapped. The algorithm takes into consideration available batteries at BSSs, driving ranges of ETs and the current traffic conditions, etc, in order to avoid long queues at BSSs. We consider a basic model that assumes past decisions are perfectly executed, and formulate an optimal ET-to-BSS assignment problem. This problem is an integer program and therefore hard to scale. For real-time implementation, we approximate the optimal assignment problem by a repeated potential game and propose an iterative best response algorithm to compute ET-to-BSS assignments. Preliminary numerical results suggest that our heuristic algorithm solves the optimal assignment problem approximately.**

I. Introduction

The widespread adoption of electric vehicles (EVs) will have a large impact on not only our transportation network, but also our power systems [1]. On the one hand, EVs are large loads that will add stress to the grid if uncontrolled. On the other hand, these loads are flexible and can function as storage that helps reduce operational costs and improve grid stability with proper control [2], [3]. They are also mobile that can potentially help relieve congestion both on the road and on the grid with real-time recommendations of EVs' refueling places [4], [5].

Compared with private EVs, the electrification of taxis is developing faster in China because of governmental promotion [6]. Unlike private EVs that are usually driven only a couple hours a day, electric taxis (ETs) need to operate continuously throughout most of the day. Hence battery swapping is more suitable than charging for ETs [4], [7]–[12]. [4] proposes an online routing and battery reservation method for EVs with swappable batteries to minimize the average delay of all EVs. [7] investigates the optimal charging schedule of a BSS for electric buses. Due to the periodicity of bus operations, the schedule is easier to implement. [8] designs a dynamic operation model of a BSS in the power market based on short-term battery managements, and proposes a market strategy for the BSS. A framework for the optimal design of a BSS in distribution systems is presented in [9]. A life cycle cost criterion is proposed for the optimal cost-benefit analysis and safety operation of the BSS. The battery swapping process of a BSS is modeled by a mixed queueing network with an open queue of EVs and a closed queue of batteries in [10]. The unique steady-state distribution of the queueing network is obtained by solving its balance equations.

One of the current difficulties with ETs is the lack of real-time information to help them choose a BSS to swap their batteries. As a result they typically go to a closest BSS when they need a battery to swap, sometimes only to find a long queue of ETs waiting for battery swapping at the BSS. This problem is currently impeding the uptake of ETs and has not been well addressed, to the best of our knowledge. An advantage of ETs is that they are usually under centralized management which allows communications between a central operation center and each ET. Therefore, in this paper we propose a real-time algorithm that recommends BSSs to ETs in need of battery swapping.

Our contributions are:

- A formulation of an optimal ET-to-BSS assignment problem as an integer program.
- An approximation of the optimal assignment problem as a repeated potential game and a simple iterative best response algorithm to compute ET-to-BSS assignments.

The remainder of this paper is organized as follows. Sec. II describes our system model and formulates the optimal ET-to-BSS assignment problem. Sec. III proposes the potential-game based heuristic algorithm to approximately solve the assignment problem. Sec. IV presents numerical results to demonstrate the performance of our heuristic algorithm. Finally, Sec. V concludes.

II. System model

We take into account a basic model that can be extended to accommodate a more practical scenario by incorporating other factors. Consider a group of ETs managed by an operation center which provides recommendations of BSSs for all ETs in need of battery swapping. Meanwhile, consider a slotted finite time horizon $t \in \mathbb{T} := \{1, \ldots, T\}$, and the recommendation algorithm is run at each time t. Suppose there are J BSSs, $j \in \mathbb{J} := \{1, \ldots, J\}$, which only serve ETs. BSS j is located at position y_j, where y_j is a two-dimensional coordinate on a plane. At each time t, the number of (fully-charged) batteries that are available for swapping is $n_j(t)$. Suppose at each time t, there are $I(t)$ ETs, $i \in \mathbb{I}(t) := \{1, \ldots, I(t)\}$, which the recommendation algorithm has to assign to certain BSSs for battery swapping. Note that only those ETs whose states of

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charge (SoCs) have dropped below a threshold need to be assigned. ET i is characterized by a tuple $(x_i(t), d_i(t), b_i(t))$, where $x_i(t)$ is its current position, $d_i(t)$ is its current destination, and $b_i(t)$ is its current SoC. The recommendation algorithm determines at each time t for each ET i the BSS j at which it should swap its battery, in order to achieve a certain social optimality, as we now explain.

A. Constraints

Let $M_{ij}(t)$ represent the assignment:

$$M_{ij}(t) = \begin{cases} 1 & \text{if ET } i \text{ is assigned to BSS } j \\ 0 & \text{otherwise} \end{cases}$$

We first assume that each ET is assigned to a single BSS, i.e.,

$$\begin{cases} \sum_{j \in \mathbb{J}} M_{ij}(t) = 1, \quad \forall i \in \mathbb{I}(t) \\ M_{ij}(t) \in \{0, 1\}, \quad \forall i \in \mathbb{I}(t), \forall j \in \mathbb{J} \end{cases} \tag{1}$$

Hence the location of the BSS that ET i is assigned to at time t, denoted by $y_i(t)$, can be expressed as

$$y_i(t) := \sum_{j \in \mathbb{J}} y_j \, M_{ij}(t)$$

Let $\hat{n}_j(t)$ denote the *forecast* number of available batteries at BSS j at time t, which is assumed to be given according to batteries' charging states. It is increased when a battery at the BSS becomes fully charged and is decreased when a battery is reserved by the assignment $\boldsymbol{M} := [M_{ij}(t)]_{\forall i,j,t}$. More precisely, at each time t let $\tau(t, \sigma, \delta)$ be the (estimated) travel time from origin σ to destination δ at time t, i.e., the ET at origin σ at time t should arrive at destination δ at time $t + \tau(t, \sigma, \delta)$. We assume the mapping defined by the function $\tau(t, \sigma, \delta)$ is given based on an online travel time estimator with real-time traffic information, e.g., Google Maps Navigation. According to the assignment $\boldsymbol{M}(t) := [M_{ij}(t)]_{\forall i,j}$ at time t, ET i will arrive at a future time $\tau_{ij}(t) := t + \tau(t, x_i(t), y_j)$ at BSS j and reduce its forecast $\hat{n}_j(t + \tau(t, x_i(t), y_j))$ by one. Let $\tau_{ij}^{-1}(t)$ denote the inverse function of $\tau_{ij}(t)$, i.e., $\tau_{ij}^{-1}(t)$ is the time at which ET i that arrives at BSS j at time t was assigned. Hence $\hat{n}_j(t)$ evolves according to

$$\hat{n}_j(t+1) = \hat{n}_j(t) + c_j(t) - \sum_{i \in \mathcal{I}(t)} M_{ij}(\tau_{ij}^{-1}(t)) \tag{2}$$

where $c_j(t)$ is the *forecast* number of batteries that become fully charged at BSS j at (the beginning of) time t, and $\mathcal{I}(t) := \bigcap_{\alpha \leq t} \mathbb{I}(\alpha)$, which is the set of all ETs that have been assigned by t. Let $\hat{\boldsymbol{n}} := [\hat{\boldsymbol{n}}(t)]_{\forall t} := [\hat{n}_j(t)]_{\forall j,t}$ be the vector of forecast numbers of available batteries for all BSSs at all times, which is the key impact parameter.

Only those BSSs that are within the driving range of ET i will be taken into consideration as its recommendation candidates. This driving range mainly depends on ET i's SoC $b_i(t)$ and the traffic profile which we represent by a generic function[1]:

$$r_i(t) := r_i(b_i(t), x_i(t))$$

Only BSSs within this range are eligible for recommendation to ET i, so $M_{ij}(t)$ must satisfy

$$|x_i(t) - y_i(t)| \leq r_i(t), \quad \forall i \in \mathbb{I}(t) \tag{3}$$

where $|\cdot|$ is a measure of distance between two locations $x_i(t)$ and $y_i(t)$. This can be the shortest (path) distance on the road network (more realistic) or the straight line distance between these two points (simpler).

B. Objectives

Given an assignment $\boldsymbol{M}(t)$, one of the costs to ET i is the extra distance it has to travel to have its battery swapped at BSS j:

$$|x_i(t) - y_i(t)| + |y_i(t) - d_i(t)| - |x_i(t) - d_i(t)|$$

If a ET does not have a destination $d_i(t)$ at time t, we can ignore the second term $|y_i(t) - d_i(t)|$ in the objective function. Since the last term is independent of the assignment matrix $\boldsymbol{M}(t)$, we can use $|x_i(t) - y_i(t)| + |y_i(t) - d_i(t)|$ as a proxy for this cost.

The waiting time of ETs is also a cost when computing the assignment $\boldsymbol{M}(t)$. It may be better to assign to an ET a BSS a little farther away to avoid waiting in a long queue at a nearby BSS. We use the ET queue length to represent this cost. To this end, redefine $\hat{n}_j(t)$ to be the number of available batteries ready for swapping minus the number of ETs waiting for battery swapping. Hence $\hat{n}_j(t) = 5$ means there are 5 batteries ready for swapping at time t; $\hat{n}_j(t) = -3$ means there are 3 ETs waiting. Then the ET queue length at BSS j at time t is $-\min\{\hat{n}_j(t), 0\}$.

C. Problem formulation

We are interested in the following problem:

$$\begin{aligned} \min_{\boldsymbol{M}(t)} \quad & \sum_{i \in \mathbb{I}(t)} \left(\left| x_i(t) - \sum_{j \in \mathbb{J}} y_j M_{ij}(t) \right| + \left| \sum_{j \in \mathbb{J}} y_j M_{ij}(t) - d_i(t) \right| \right) \\ & - \sum_{j \in \mathbb{J}} \sum_{s=t}^{T} \alpha_j(s) \min\{\hat{n}_j(s), 0\} \\ \text{s.t.} \quad & \hat{n}_j(t+1) = \hat{n}_j(t) + c_j(t) - \sum_{i \in \mathcal{I}(t)} M_{ij}(\tau_{ij}^{-1}(t)), \\ & \forall j \in \mathbb{J}, t \in \mathbb{T} \\ & \left| x_i(t) - \sum_{j \in \mathbb{J}} y_j M_{ij}(t) \right| \leq r_i(t), \quad \forall i \in \mathbb{I}(t) \\ & \sum_{j \in \mathbb{J}} M_{ij}(t) = 1, \quad \forall i \in \mathbb{I}(t) \\ & M_{ij}(t) \in \{0, 1\}, \quad \forall i \in \mathbb{I}(t), \; j \in \mathbb{J} \end{aligned} \tag{4}$$

[1]In general, the driving range depends also on the energy required on the path, but we approximate this by dependence only on its current location $x_i(t)$.

where $\alpha_j(s) \geq 0,\ j \in \mathbb{J},\ s = t, \ldots, T$, are the weights that depend on BSS j and time s, e.g., smaller weights on future ET queue length because of uncertainty. The problem (4) jointly minimizes the overall extra travel distance and queue length, with consideration of ETs' driving ranges and traffic-based travel time.

Note that problem (4) are decoupled across time, i.e., an optimal $\boldsymbol{M}^*(t)$ is determined at each time t separately. In particular, at time t, we do not change the earlier decisions $\boldsymbol{M}^*(s)$, $s = t-1, t-2, \ldots$. These earlier decisions, however, do affect the decision $\boldsymbol{M}(t)$ at time t and they are *parameters* in problem (4). In addition, in practice the measured $n_j(t)$ can be used to adjust the forecast $\hat{n}_j(t)$, so are other parameters. As a result, at each time we can carry out online optimization to enable real-time implementation.

III. Potential-game based heuristic algorithm

Problem (4) is an integer program that is hard to scale to a large number of ETs. For real-time application, we develop a heuristic algorithm that is efficient and seems to perform well, as we now explain.

We first simplify problem (4) to shorten the time involved in the definition of $\mathcal{I}(t) := \bigcap_{\alpha \leq t} \mathbb{I}(\alpha)$. As defined $\mathcal{I}(t)$ is the set of *all* ETs that have been assigned by time t but have not arrived at their BSSs before time t. For simplification, we redefine $\mathcal{I}(t) := \bigcap_{t-t_m \leq \alpha \leq t} \mathbb{I}(\alpha)$, which is the set of all ETs that have been assigned between time $t - t_m$ and t, i.e., we ignore ETs that have been assigned before time t_m that may impact our decision $M(t)$ at time t. Note that t_m needs to be empirically chosen, since a small t_m may omit already-assigned ETs that are expected to arrive at BSS j at time t or later, while a large t_m will complicate the real-time computation. Moreover, with t_m we do not need to take into account the impact of the current decision $\boldsymbol{M}(t)$ on future forecast number of available batteries at each BSS after time $t + t_m$. Therefore, the time range of (4) is narrowed and thus the computational complexity is significantly reduced.

Next we design a heuristic algorithm to obtain a suboptimal solution. The key idea is to compute for each ET a weight corresponding to its every possible BSS candidate. The weight measures the marginal cost added to the objective value of problem (4) if an ET is assigned to a certain BSS based on the current evolution of available batteries. Then we adjust each ET's assignment iteratively according to its real-time weights to finally reach a suboptimal solution.

Note that (1), (3) are completely decentralized, while (2) couples all the current decisions. Hence in terms of (1) and (3), we can first pick out for each ET i all the possible BSS candidates $j \in \mathbb{J}_i$. Given $[\hat{n}_j(t)]_{\forall j}$, $[c_j(s)]_{\forall j, s=t,\ldots,t+t_m}$ and earlier decisions $[\boldsymbol{M}^*(s)]_{s=t-1,\ldots,t-t_m}$, the evolution of available batteries without the current assignment $\boldsymbol{M}(t)$ is predetermined from (2), which is denoted as $[\bar{n}_j^0(s)]_{\forall j, s=t,\ldots,t+t_m}$. Next we assign ETs $i \in \mathbb{I}(t)$ to reserve certain batteries at BSSs $j \in \mathbb{J}$ in a heuristic way that aims to minimize the overall travel distance and queue length.

First, for each ET $i \in \mathbb{I}(t)$ and each of its BSS candidate $j \in \mathbb{J}_i$, we compute a weight $w_{ij}^0(t) = |x_i(t) - y_j| + |y_j - d_i(t)| - \alpha_j(\tau_{ij}(t)) \min\{\bar{n}_j^0(\tau_{ij}(t)), 0\}$. A smaller weight $w_{ij}^0(t)$ implies a smaller increment to the objective value of problem (4) when ET i is assigned to BSS j (at time t). Then all the BSS candidates $j \in \mathbb{J}_i$ are ranked in increasing order of $w_{ij}^0(t)$. Hence each ET i has a sorted list of BSS candidates with a priority given to smaller weights. We first select the BSSs that rank first in each ET's list and use them as the initial assignments $\boldsymbol{M}^0(t)$ for all ETs.

Suppose the initial assignments are implemented, then the evolution of available batteries is altered into $[\bar{n}_j^1(s)]_{\forall j, s=t,\ldots,t+t_m}$. Based on the newly-updated available batteries, the ET queue length of each BSS at each time is known and may influence the values of some weights, thus rendering the initial assignments less efficient. Hence we iteratively adjust the assignment of each ET by re-calculating its weights[2] and re-selecting the first-rank BSS candidate from its updated list according to the current ET queue length of each BSS. For example, we first adjust the assignment of ET i. Its new weights $[w_{ij}^1(t)]_{j \in \mathbb{J}_i}$ are re-calculated to form an updated list for ET i. If its first-choice BSS candidate is changed, then the initial assignments $\boldsymbol{M}^0(t)$ turn into $\boldsymbol{M}^1(t)$ and a new evolution of available batteries $[\bar{n}_j^2(s)]_{\forall j, s=t,\ldots,t+t_m}$ emerges accordingly. Every ET's adjustment yields a new assignment and then a new evolution of available batteries. See Algorithm 1 for details.

Theorem 1: Algorithm 1 converges to an equilibrium $\boldsymbol{M}(t)$.

Proof: See details in Appendix A. ■

IV. Numerical results

We test the proposed heuristic algorithm with a simple case to illustrate its effectiveness. Consider a 6 km × 6 km plane with 3 randomly distributed BSSs. Their coordinates on the plane are (3.27,0.96), (0.47,4.59) and (2.79,2.09), respectively. At a certain time t, suppose 12 ETs need battery swapping. The characters of each ET, i.e., its current position, destination and SoC are randomly generated according to certain probability distributions. The location setup is shown in Fig. 1(a). For simplicity, let $|\cdot|$ denote the Euclidean distance, and the function $\tau(t, \sigma, \delta)$ is set to be directly proportional to $|\sigma - \delta|$. Assume the driving range of each ET is adequate to cover the whole square plane, so all ETs have the 3 BSSs as candidates. In addition, set $t_m = 3$ and other parameters are given in TABLE I.

As shown in Fig. 1(b), the proposed heuristic algorithm computes an assignment for each ET so that it can reach its destination via the recommended BSS to get its battery swapped without a large extra travel or waiting in a long queue.

Fig. 2 validates the performance of the proposed heuristic algorithm. Within 4 iterations, the objective value quickly converges and its equilibrium value is very close to the optimal

[2] Note that the re-calculation of each ET i's weights is based on the current evolution of available batteries, from which the impact of ET i's assignment needs to be removed.

Algorithm 1: Heuristic algorithm

Input: $[\hat{n}_j(t)]_{\forall j}$, $[c_j(s)]_{\forall j, s=t,\dots,t+t_m}$, and $[\boldsymbol{M}^*(s)]_{s=t-1,\dots,t-t_m}$;
Output: $\boldsymbol{M}(t)$;
Initialization: set index $k=1$, pick out $[\mathbb{J}_i]_{\forall i}$, pre-determine $[\bar{n}_j^0(s)]_{\forall j, s=t,\dots,t+t_m}$, and compute $[w_{ij}^0]_{\forall i,j}$;
Generate for each ET i a sorted list of BSS candidates in increasing order of w_{ij}^0;
Select the first-rank BSS candidates from all ETs' lists to form $\boldsymbol{M}^0(t)$;
Update $[\bar{n}_j^1(s)]_{\forall j, s=t,\dots,t+t_m}$ based on $\boldsymbol{M}^0(t)$;
while *any ET makes changes to* $\boldsymbol{M}(t)$ **do**
  for *ET* $i \in \mathbb{I}(t)$ **do**
    Re-calculate $[w_{ij}^k(t)]_{j\in\mathbb{J}_i}$ to update its list;
    if *the first-rank BSS candidate is changed* **then**
      Re-select the new first-rank BSS candidate to form $\boldsymbol{M}^k(t)$;
      Update $[\bar{n}_j^{k+1}(s)]_{\forall j, s=t,\dots,t+t_m}$ based on $\boldsymbol{M}^k(t)$;
      $\boldsymbol{M}(t) \leftarrow \boldsymbol{M}^k(t)$;
      $k \leftarrow k+1$;
    else
      Stick to the previous selection;
    end if
  end for
end while

TABLE I
PARAMETER SETUP

Parameter \ Time	t	$t+1$	$t+2$	$t+3$
α_j	1	1	1	1
$\bar{n}_1^0$	3	3	0	3
$\bar{n}_2^0$	2	3	3	0
$\bar{n}_3^0$	1	1	1	-1

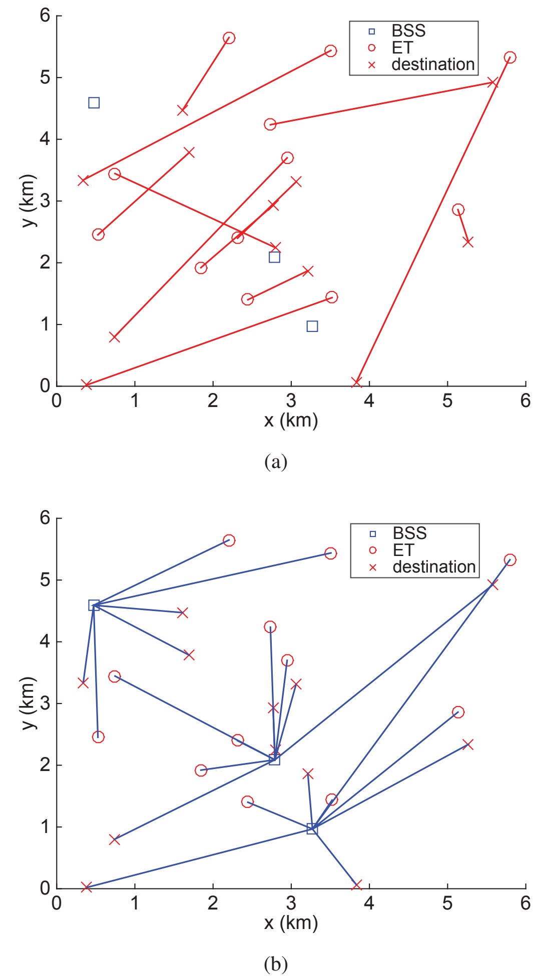

Fig. 1. (a) Location setup. (b) Assignments.

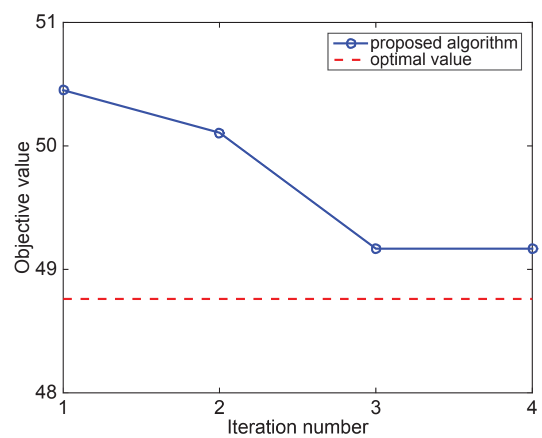

Fig. 2. Convergence.

one, which is obtained by a brute force algorithm. As we can see, the performance gap between the proposed heuristic algorithm and the brute force algorithm is rather small, while the former takes much less computation time than the latter, which has a computational complexity of $O(J^I)$.

V. Conclusion

This paper proposes a real-time recommendation algorithm of BSSs for ETs. Taking into account available batteries at BSSs, driving ranges of ETs and the current traffic conditions, etc, the algorithm attempts to minimize ETs' extra travel distance and avoid long queues at BSSs. A basic model that assumes past decisions are perfectly executed is considered to formulate an optimal ET-to-BSS assignment problem as an integer program. We approximate the problem by a repeated

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potential game and propose an iterative best response algorithm to compute ET-to-BSS assignments. Numerical results validate the performance of our heuristic algorithm.

Appendix A
Proof of Theorem 1

Algorithm 1 can be viewed from the perspective of a repeated potential game [13]. $I(t)$ ETs share a set of J BSSs. Each ET i has its own BSS candidate list $\mathbb{J}_i$, which is equivalent to a strategy set. The ET queue at a BSS corresponds to congestion and is dependent on the number of ETs assigned to the BSS. Moreover, in our model the extra travel distance, which is independent for each ET, is taken into account.

Therefore, (4) is reduced to a potential game as follows:

1) Player set $\mathbb{I}(t)$: it consists of the $I(t)$ ETs in need of battery swapping.
2) Strategy space Z: each ET i selects a strategy j_i, i.e., a BSS candidate, from its strategy set $\mathbb{J}_i$. Then $Z := \times_{i\in\mathbb{I}(t)}\mathbb{J}_i$ represents the strategy space of the game. Denote $z = (j_i, j_{-i})$ as the strategy vector, where j_{-i} is all the other ETs' strategies except ET i.
3) Cost function set $\{C_i(z)\}_{i\in\mathbb{I}(t)}$: each ET i's individual cost is the weighted summation of the travel distance and queue length it has to bear, i.e.,

$$\begin{aligned}C_i(z) = &|x_i(t) - y_{j_i}| + |y_{j_i} - d_i(t)| \\ &- \alpha_{j_i}(\tau_{ij_i}(t)) \min\{\bar{n}^z_{j_i}(\tau_{ij_i}(t)), 0\}\end{aligned} \quad (5)$$

Note that ET i's cost $C_i(z)$ is dependent on strategy vector z.

The procedure of Algorithm 1 is equivalent to applying an iterative best response algorithm in a repeated game framework, where ETs take turn to minimize their costs by defining a strategy and updating it based on the other ETs' strategies, until an equilibrium is reached. It can be proven that such an equilibrium, i.e., a pure Nash equilibrium, exists. First, we define a potential function based on Rosenthal's potential function [13]. For any strategy vector z, let

$$\begin{aligned}\Phi(z) = \sum_{j\in\mathbb{J}} \Bigg[&\sum_{i\in\mathbb{I}^z_j} (|x_i(t) - y_j| + |y_j - d_i(t)|) \\ &- \sum_{s=t}^{t_m} \sum_{k=\bar{n}^z_j(s)<0}^{-1} \alpha_j(s) \min\{k, 0\} \Bigg]\end{aligned} \quad (6)$$

where $\mathbb{I}^z_j$ is the set of ETs that are assigned to BSS j under strategy vector z and $\bar{n}^z_j(s)$ is the evolution of available batteries after strategy vector z is implemented. Note that for the second item in the square brackets of (6), we neglect the cases where $\bar{n}^z_j(s) \geq 0$ because $\min\{k, 0\} = 0$ for all $k \geq 0$.

Lemma 1: Suppose the strategy vector changes from z to z' with an improvement step of ET i reducing its cost by $\Delta > 0$, then $\Phi(z') = \Phi(z) - \Delta$.

Proof: (6) can be calculated by inserting the ETs one after another in any order and summing the costs of the ETs at the time point of their insertion. Let ET i be the last ET that we insert when calculating $\Phi(z)$, then the potential added due to the insertion of ET i corresponds to its cost under strategy vector z. Hence, when z is altered into z' with an reduction of Δ in ET i's cost, Φ is decreased by Δ as well. ■

Lemma 1 shows if a single ET reduces its cost by reselecting a strategy, Φ is decreased by exactly the same amount. Note that

1) $\Delta \geq 0$ and its possible values are discretized and finite.
2) For any strategy vector z, $\Phi(z)$ is bounded as

$$\begin{aligned}\sum_{j\in\mathbb{J}} \Bigg[&\sum_{i\in\mathbb{I}(t)} (|x_i(t) - y_j| + |y_j - d_i(t)|) \\ &- \sum_{s=t}^{t_m} \sum_{k=\bar{n}^0_j(s)-I(t)<0}^{-1} \alpha_j(s) \min\{k, 0\} \Bigg] \geq \Phi(z) \geq 0\end{aligned} \quad (7)$$

Consequently, the number of improvements is upper-bounded and hence finite. Then for the repeated potential game described above, every sequence of improvement steps is finite, which implies the existence of at least one pure Nash equilibrium. Hence Algorithm 1 converges to an equilibrium $\boldsymbol{M}(t)$.

This completes the proof of Theorem 1.

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