Joint Scheduling of Large-Scale Appliances and Batteries Via Distributed Mixed Optimization

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***Abstract*—This paper investigates joint scheduling problem of large-scale smart appliances and batteries (e.g., in a smart building), to minimize electricity payment, user's dissatisfaction and battery loss under kinds of constraints. Due to the binary nature of charge and discharge states of battery, this problem is formulated as a constrained mixed-integer nonlinear program. In order to solve it efficiently, a distributed mixed optimization approach is proposed. First, Lagrangian relaxation is applied to decompose the original problem into two sets of subproblems, each of which corresponds to scheduling on appliance/battery. Then, the battery scheduling subproblem is formulated as a mixed-integer linear program and tackled by Benders decomposition. The main advantages of the proposed approach are the distributed implementation and low computational complexity, as shown by simulations.**

***Index Terms*—Benders decomposition, demand-side management, distributed mixed optimization, Lagrangian relaxation.**

I. Introduction

IN RECENT years, the energy demand keeps increasing and brings great burden to the power grid. Demand-side management (DSM) is an effective solution by smartly scheduling the power usage. It plays a key role in smart grid and has been extensively investigated [1]–[4]. DSM provides various incentive strategies and pricing schemes such as real-time pricing (RTP) [5] and time-of-use pricing (TOU) [6], so that the end users can reduce or shift load from peak periods to off-peak periods. Consequently, it can reduce costs of power plants by smoothing out the peak demand, and produce financial benefits for end users.

In particular, it is well recognized that introducing battery into DSM scheme will render load scheduling more effective, as the battery provides extra flexibility to the entire system [7]. In order to reduce payment of end users, it is a common practice to charge the battery and store more energy during low price periods, while discharge the battery and supply power to other appliances during high price periods. However, it is notable that charging and discharging of battery are two distinct processes, which, therefore, need to be treated separately by introducing binary variables to represent the states of charge and discharge. Some previous studies [7]–[10] have already investigated joint scheduling on small-scale appliances and batteries. In [7], a framework making use of an event driven model predictive control (MPC) approach is proposed to meet the real life dynamics of a household and minimize the consumer's daily energy cost by evaluating the best time to run of appliances and the optimal evolution of battery level of charge. A distributed scheduling system based on artificial neural networks in a house equipped with local solar panels and a storage unit is considered in [8]. The system can schedule the energy consumption of appliances for the next 24 h, but fails to operate in a real-time framework. [9] proposes an appliance scheduling method for residential building energy management controllers by which thermal appliances are scheduled smartly together with thermal mass storage to hedge against high prices and make use of low-price time periods. In addition, optimization models used to manage everyday energy load for both single and multi-user cases are proposed in [10], taking into account distributed energy sources and batteries. However, in many occasions, e.g., smart building, the user may be equipped with a large number of appliances and batteries, such that the optimization variables will be greatly increased and the joint scheduling may become very time consuming or even impractical.

This paper proposes a new approach that can solve the joint scheduling problem in a distributed way and thus is suitable for large-scale appliances and batteries. In addition, we improve the battery model and consider battery losses in charge and discharge processes. The user's dissatisfaction is also taken into account. At last, joint scheduling on large-scale appliances and batteries becomes a constrained mixed-integer nonlinear program (MINP), which is generally very hard to solve.

Our main contributions focus on an efficient solution to the constrained MINP, which can be summarized as follows:

1) To deal with the coupling constraints, the joint scheduling problem is decoupled into two sets of subproblems through Lagrangian relaxation (LR) [11]. Each subproblem corresponds to scheduling on appliance/battery. LR enables distributed implementation of optimization algorithms, which greatly reduces computational complexity.
2) The battery scheduling subproblem is formulated as a mixed-integer linear program, which is efficiently tackled by Benders decomposition. Benders Decomposition is also realized in parallel computation, which reduces time of calculation.

The paper is organized as below. Section II describes system model and problem formulation. Section III divides joint sched-

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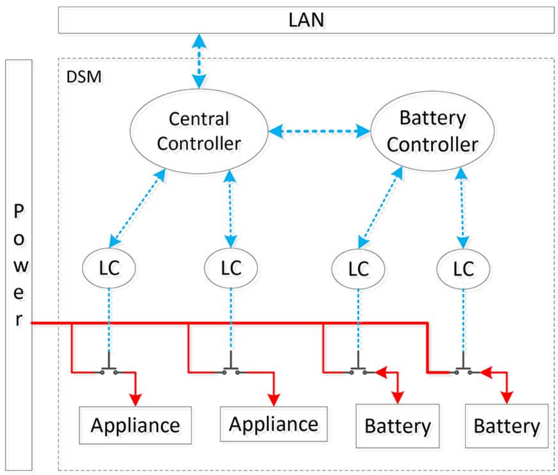

Fig. 1. Energy and information exchange in residential DSM.

uling problem into two sets of subproblems via LR. The solutions to battery scheduling are given in Section IV, followed by performance verification through simulations in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, we consider the scenario depicted in Fig. 1. The user, e.g., a smart building, is equipped with a large number of appliances and batteries. Each appliance/battery is controlled by a local controller (LC), which can communicate independently with the smart meter through a local area network (LAN) [11]. The smart meter acts as a central controller that coordinates all appliances and batteries. It is assumed that the batteries can store energy from the grid during charging process, and supply energy to other appliances during discharging process. We do not consider the situation that the batteries transmit energy back to the grid.

Let the set $\mathcal{H} \triangleq \{1,\dots,H\}$ denote the equally slotted time horizon of a day, and assume that for any time slot h the electricity price c^h, energy consumption rates of appliances, charging/discharge rates of batteries keep constant during that time slot. Meanwhile, assume c^h to be known in advance.

A. Appliance Model

In general, smart appliances can be classified into non-shiftable ones with fixed load x_0^h, and shiftable ones that can be scheduled [7] in this paper. Let $\mathcal{A}$ denote the set of shiftable appliances. For each shiftable appliance $a \in \mathcal{A}$, its energy consumption vector is $\boldsymbol{x}_a \triangleq [x_a^1,\dots,x_a^H]$, where x_a^h is a real number denoting the energy consumption of appliance a at time slot h.

Then, let $\mathcal{H}_a \triangleq [\alpha_a, \beta_a]$ denote the available working interval for appliance a. In order to finish a given task, appliance a has to consume at least D_a units of energy, which can be formulated as a time coupling constraint [12]:

$$\sum_{h=\alpha_a}^{\beta_a} x_a^h \geq D_a \quad \forall a \in \mathcal{A}. \tag{1}$$

More detailed models of electric appliances commonly found in a household have been investigated in [13] and we shall extend the results of this paper to those models in our future work.

Meanwhile, x_a^h should also meet the upper and lower bounds

$$\begin{cases} x_a^{\min} \leq x_a^h \leq x_a^{\max} & \forall h \in \mathcal{H}_a \\ x_a^h = 0 & \text{otherwise} \end{cases} \quad \forall a \in \mathcal{A}. \tag{2}$$

Another important factor to be considered is the user's satisfaction. We assume each appliance has a target consumption amount $\bar{x}_a^h$ at slot h, and the dissatisfaction can be modeled as a convex function $V_a^h(x_a^h)$ [14].

B. Battery Model

Let $\mathcal{B}$ denote the set of batteries. Although the user has multiple batteries with different characteristics, it is apparent that the charge/discharge states of all batteries are identical at one time slot. Thus, we do not consider the case that one battery is charging while another is discharging simultaneously.

A pair of binary variables $\epsilon_c^h, \epsilon_d^h \in \{0,1\}$ is introduced to denote the charge and discharge states of the battery set at time slot h. $\epsilon_c^h = 1$ or $\epsilon_d^h = 1$ implies the charging or discharging phase, with the following constraint [10]:

$$\epsilon_c^h + \epsilon_d^h \leq 1 \quad \epsilon_c^h, \epsilon_d^h \in \{0,1\} \quad \forall h \in \mathcal{H}. \tag{3}$$

The charge and discharge rates for battery b at time slot h are denoted by continuous variables $p_{b,c}^h$ and $p_{b,d}^h$. They cannot exceed the electrical limits as follows:

$$\begin{cases} p_{b,c}^h \leq \epsilon_c^h p_{b,c}^{\max} \quad \forall b \in \mathcal{B} \quad \forall h \in \mathcal{H} & \text{(4a)} \\ p_{b,d}^h \leq \epsilon_c^h p_{b,d}^{\max} \quad \forall b \in \mathcal{B} \quad \forall h \in \mathcal{H} & \text{(4b)} \\ p_{b,c}^h \geq 0, p_{b,d}^h \geq 0 \quad \forall b \in \mathcal{B} \quad \forall h \in \mathcal{H} & \text{(4c)} \end{cases}$$

where $p_{b,c}^{\max}$ and $p_{b,d}^{\max}$ are the maximum charge and discharge rates.

Let e_b^h denote the energy level of battery b, which is bounded as

$$e_b^{\min} \leq e_b^h \leq e_b^{\max} \quad \forall b \in \mathcal{B} \quad \forall h \in \mathcal{H}.$$

Meanwhile, the dynamics of e_b^h is governed by [10]

$$e_b^h = e_b^{h-1} + \eta_b^c p_{b,c}^h - \frac{p_{b,d}^h}{\eta_b^d}$$

where η_b^c and η_b^d are the charge and discharge efficiency. Note that the dynamics can be transformed into

$$e_b^h = e_b^0 + \sum_{t=1}^{h} \left(\eta_b^c p_{b,c}^t - \frac{p_{b,d}^t}{\eta_b^d} \right)$$

where e_b^0 is the initial battery level of battery b. Then for $\forall b \in \mathcal{B}, \forall h \in \mathcal{H}$, we can get

$$\begin{cases} e_b^{\min} \leq e_b^0 + \sum_{t=1}^{h} \left(\eta_b^c p_{b,c}^t - \frac{p_{b,d}^t}{\eta_b^d} \right) & \text{(5a)} \\ e_b^0 + \sum_{t=1}^{h} \left(\eta_b^c p_{b,c}^t - \frac{p_{b,d}^t}{\eta_b^d} \right) \leq e_b^{\max}. & \text{(5b)} \end{cases}$$

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As both charging and discharging actions will result in a certain amount of battery loss, the batteries should not be used immoderately for optimal scheduling purpose. Here, the battery loss is defined as follows for battery b with weighting factors $r_{b,c}$ and $r_{b,d}$ [15]:

$$\sum_{h\in\mathcal{H}}\left(r_{b,c}p_{b,c}^h + r_{b,d}p_{b,d}^h\right).$$

C. Energy Management Problem

At time slot h, the balance between the supply and demand of the system can be written as

$$x_0^h + \sum_{a\in\mathcal{A}} x_a^h + \sum_{b\in\mathcal{B}} p_{b,c}^h = \sum_{b\in\mathcal{B}} p_{b,d}^h + g^h \tag{6}$$

where g^h denotes the power purchased from the grid, with the upper and lower bounds

$$0 \le g^h \le g^{\max} \quad \forall h \in \mathcal{H}. \tag{7}$$

Substituting (6) into (7), we can obtain a constraint coupling all appliances and batteries together

$$0 \le x_0^h + \sum_{a\in\mathcal{A}} x_a^h + \sum_{b\in\mathcal{B}} p_{b,c}^h - \sum_{b\in\mathcal{B}} p_{b,d}^h \le g^{\max} \quad \forall h \in \mathcal{H}. \tag{8}$$

Define an $H \times A$ matrix $\boldsymbol{x} \triangleq [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_A]$, a $2H$ matrix $\boldsymbol{\epsilon} \triangleq [\epsilon_c^1, \ldots, \epsilon_c^H, \epsilon_d^1, \ldots, \epsilon_d^H]$, and a $2H \times B$ matrix $\boldsymbol{p} \triangleq [\boldsymbol{p}_1, \ldots, \boldsymbol{p}_B]$, where $\boldsymbol{p}_b \triangleq [p_{b,c}^1, \ldots, p_{b,c}^H, p_{b,d}^1, \ldots, p_{b,d}^H]$. Thus, the ultimate goal of the energy management problem is to simultaneously minimize electricity payment, user's dissatisfaction and battery loss by adjusting $\boldsymbol{x}$, $\boldsymbol{\epsilon}$, and $\boldsymbol{p}$, as follows:

$$\begin{aligned}
&\textbf{primal problem}:\\
&\min_{\boldsymbol{x},\boldsymbol{\epsilon},\boldsymbol{p}} \quad \mathcal{P}(\boldsymbol{x},\boldsymbol{\epsilon},\boldsymbol{p}) = \sum_{h\in\mathcal{H}} c^h g^h + \sum_{h\in\mathcal{H}_a}\sum_{a\in\mathcal{A}} \pi_a^h V_a^h\left(x_a^h\right)\\
&\qquad + \sum_{h\in\mathcal{H}}\sum_{b\in\mathcal{B}}\left(r_{b,c}p_{b,c}^h + r_{b,d}p_{b,d}^h\right)
\end{aligned} \tag{9}$$
$$\text{s.t.} \quad (1), (2), (3), (4a), (4b), (4c), (5a), (5b), (6), (8)$$

where $\sum_{h\in\mathcal{H}} c^h g^h$ is electricity payment and π_a^h is the weighting factor of the user's dissatisfaction. Clearly, the load scheduling problem is a constrained mixed-integer nonlinear program (MINP), which cannot be directly solved via conventional methods. To this end, this problem will be solved by the proposed approach which uses the techniques of dual decomposition and Benders decomposition. Next, we will decompose this problem into two sets of subproblems.

III. Dual Decomposition

In this section, a detailed description of dual decomposition via LR is presented for solving primal problem $\mathcal{P}$ (9) and the summary of the algorithm is made at the end.

A. Lagrangian Relaxation

Note that the constraint (8) couples the variables across all appliances and batteries, which undoubtedly increases the computational difficulty. Thus, LR is introduced to decouple (9) into two sets of subproblems: one for single appliance scheduling and the other for battery scheduling.

Firstly, define the Lagrange multiplier vectors as $\boldsymbol{\lambda} \triangleq [\lambda^1, \ldots, \lambda^H]$ and $\boldsymbol{\mu} \triangleq [\mu^1, \ldots, \mu^H]$. Note that g^h can be substituted using (6), then the Lagrangian is

$$\begin{aligned}
\mathcal{L}(\boldsymbol{x},\boldsymbol{\epsilon},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu}) &= \mathcal{P}(\boldsymbol{x},\boldsymbol{\epsilon},\boldsymbol{p})\\
&\quad + \sum_{h\in\mathcal{H}} \lambda^h \left(x_0^h + \sum_{a\in\mathcal{A}} x_a^h + \sum_{b\in\mathcal{B}} p_{b,c}^h - \sum_{b\in\mathcal{B}} p_{b,d}^h - g^{\max}\right)\\
&\quad + \sum_{h\in\mathcal{H}} \mu^h \left(-x_0^h - \sum_{a\in\mathcal{A}} x_a^h - \sum_{b\in\mathcal{B}} p_{b,c}^h + \sum_{b\in\mathcal{B}} p_{b,d}^h\right)\\
&= \sum_{h\in\mathcal{H}}\sum_{a\in\mathcal{A}} \left[(c^h + \lambda^h - \mu^h)x_a^h + \pi_a^h V_a^h(x_a^h)\right]\\
&\quad + \sum_{h\in\mathcal{H}}\sum_{b\in\mathcal{B}} \left[s_b^h p_{b,c}^h + t_b^h p_{b,d}^h\right] + \urcorner
\end{aligned}$$

where $s_b^h \triangleq c^h + \lambda^h - \mu^h + r_{b,c}$, $t_b^h \triangleq -c^h - \lambda^h + \mu^h + r_{b,d}$, and

$$\urcorner \triangleq \sum_{h\in\mathcal{H}} (c^h + \lambda^h - \mu^h)x_0^h - \sum_{h\in\mathcal{H}} \lambda^h g^{\max}$$

is independent of $\boldsymbol{x}$, $\boldsymbol{\epsilon}$, and $\boldsymbol{p}$.

Then, due to the separable property, the dual function is

$$\begin{aligned}
\mathcal{D}(\boldsymbol{\lambda},\boldsymbol{\mu}) &= \min_{\boldsymbol{x},\boldsymbol{\epsilon},\boldsymbol{p}} \mathcal{L}(\boldsymbol{x},\boldsymbol{\epsilon},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu})\\
&= \sum_{a\in\mathcal{A}} \min_{\boldsymbol{x}_a} \mathcal{L}_1(\boldsymbol{x}_a,\boldsymbol{\lambda},\boldsymbol{\mu}) + \min_{\boldsymbol{\epsilon},\boldsymbol{p}} \mathcal{L}_2(\boldsymbol{\epsilon},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu}) + \urcorner
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{L}_1(\boldsymbol{x}_a,\boldsymbol{\lambda},\boldsymbol{\mu}) &= \sum_{h\in\mathcal{H}} \left[(c^h + \lambda^h - \mu^h)x_a^h + \pi_a^h V_a^h\left(x_a^h\right)\right]\\
\mathcal{L}_2(\boldsymbol{\epsilon},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu}) &= \sum_{h\in\mathcal{H}}\sum_{b\in\mathcal{B}} \left[s_b^h p_{b,c}^h - t_b^h p_{b,d}^h\right].
\end{aligned}$$

Note that primal problem $\mathcal{P}$ (9) has been divided into two sets of subproblem: one is scheduling a single appliance, and the other is scheduling the battery set [since $\boldsymbol{\epsilon}$ in constraints (3), (4a) and (4b) couples all batteries together]

$$\begin{aligned}
&\textbf{subproblems}:\\
&\min_{\boldsymbol{x}_a} \quad \mathcal{S}_1 = \mathcal{L}_1(\boldsymbol{x}_a,\boldsymbol{\lambda},\boldsymbol{\mu})\\
&\text{s.t.} \quad (1), (2)
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\min_{\boldsymbol{\epsilon},\boldsymbol{p}} \quad \mathcal{S}_2 = \mathcal{L}_2(\boldsymbol{\epsilon},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu})\\
&\text{s.t.} \quad (3), (4a), (4b), (4c), (5a), (5b).
\end{aligned} \tag{11}$$

Finally, the dual problem is to maximize the dual function over $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$

$$\begin{aligned}
&\textbf{dual problem}:\\
&\max_{\boldsymbol{\lambda},\boldsymbol{\mu}} \quad \mathcal{D} = \mathcal{D}(\boldsymbol{\lambda},\boldsymbol{\mu})\\
&\text{s.t.} \quad \lambda^h, \mu^h \ge 0 \quad \forall h \in \mathcal{H}.
\end{aligned} \tag{12}$$

Since the binary variable $\boldsymbol{\epsilon}$ exists in primal problem (9), only weak duality is ensured by LR, and the duality gap exists. That

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Algorithm 1: Dual Decomposition

1 Initialize Lagrangian multipliers λ^h, μ^h;
2 **repeat**
3 Central controller broadcasts $\lambda^h(k)$, $\mu^h(k)$ to the LC of appliances and battery controller;
4 With new Lagrangian multipliers, LC and battery controller solve subproblem $\mathcal{S}_1$ (10) and $\mathcal{S}_2$ (11) to obtain $x_a^h(k)$, $p_{b,c}^h(k)$, $p_{b,d}^h(k)$ and send them back;
5 Central controller updates $\lambda^h(k+1)$, $\mu^h(k+1)$ according to (13);
6 **until** *(i) the number of iterations exceeds the maximum number; or (ii) the difference between primal problem $\mathcal{P}$ and dual problem $\mathcal{D}$ is small enough*;

is, $\mathcal{D} < \mathcal{P}$ holds for each feasible solution and $\mathcal{D}$ indeed becomes the lower bound of the solution to $\mathcal{P}$ [16].

B. Subgradient Method

To solve (12), Lagrangian multipliers are adjusted by subgradient method that is an iterative process as follows [17]:

$$\begin{cases} \lambda^h(k+1) = \left[\lambda^h(k) + \gamma_\lambda f_\lambda^h(k)\right]^+ \\ \mu^h(k+1) = \left[\mu^h(k) + \gamma_\mu f_\mu^h(k)\right]^+ \end{cases} \tag{13}$$

where $k \in \mathbb{N}^+$ is the iteration index, $\gamma_\lambda, \gamma_\mu > 0$ are the step sizes adjusting convergence rate, and $f_\lambda^h(k), f_\mu^h(k)$ are subgradients of dual function with respect to $\boldsymbol{\lambda}, \boldsymbol{\mu}$, respectively,

$$\begin{cases} f_\lambda^h(k) = x_0^h + \sum_{a\in\mathcal{A}} x_a^h(k) + \sum_{b\in\mathcal{B}} p_{b,c}^h(k) - \sum_{b\in\mathcal{B}} p_{b,d}^h(k) - g^{\max} \\ f_\mu^h(k) = -x_0^h - \sum_{a\in\mathcal{A}} x_a^h(k) - \sum_{b\in\mathcal{B}} p_{b,c}^h(k) + \sum_{b\in\mathcal{B}} p_{b,d}^h(k) \end{cases}$$

where $x_a^h(k), p_{b,c}^h(k)$ and $p_{b,d}^h(k)$ can be obtained by solving subproblems $\mathcal{S}_1$ (10) and $\mathcal{S}_2$ (11). Since the concavity of dual problem (12) always holds, the subgradient method is guaranteed to obtain the optimal Lagrangian multipliers. The dual decomposition algorithm is summarized as **Algorithm 1**.

IV. Subproblem Solution

In this section, optimization methods are introduced to solve the battery scheduling subproblem. The single appliance scheduling (10) is a classical convex problem which can be readily solved by standard convex optimization techniques [16]. Consequently, the difficulty of solving load scheduling lies on the battery scheduling because of integer variables.

A. Battery Scheduling

The battery scheduling subproblem $\mathcal{S}_2$ (11) is a mixed-integer linear program (MILP), which is in general hard to tackle. Since Benders decomposition [18] is an effective method to solve MILP with guaranteed global optimality, we design parallel computation based on it. That is, integer variables are solved at battery central controller, while continuous variables are solved at each battery's local controller synchronously, so that the computational burden can be greatly reduced. A detailed description of Benders decomposition is presented below for battery scheduling and the summary of the algorithm is made at the end.

The battery scheduling problem is formulated as follows:

$$\begin{aligned} &\textbf{MILP}: \\ &\min_{\boldsymbol{\epsilon},\boldsymbol{p}} \quad Z \\ &\text{s.t.} \quad (3) \text{ integer constraint} \\ &\qquad (4a), (4b) \text{ integer and continuous constraints} \\ &\qquad (4c), (5a), (5b) \text{ continuous constraints} \end{aligned} \tag{14}$$

where $Z \triangleq \sum_{b\in\mathcal{B}} z_b$ and $z_b \triangleq \sum_{h\in\mathcal{H}} [s_b^h p_{b,c}^h + t_b^h p_{b,d}^h]$.

Denote the optimal solution as $\boldsymbol{\epsilon}^*$ and $\boldsymbol{p}^*$ and the optimal objective value as Z^*. Note that three types of constraints appear in MILP: (3) contains only integer variables; (4c), (5a) and (5b) contain only continuous variables; while (4a) and (4b) contain both integer and continuous variables. Clearly, finding the optimal integer $\boldsymbol{\epsilon}^*$ is the most critical part of MILP. When integer variables are determined, MILP reduces to a common Linear Programming (LP) which can be solved as routine. Therefore, once $\boldsymbol{\epsilon}^*$ is found, $\boldsymbol{p}^*$ is also readily obtained.

Benders decomposition is an iterative method to solve MILP and the intuition is as below [19]. First, MILP is decomposed into a master problem (MP) and a subproblem (SP). MP is an integer programming problem, which aims at finding integer variables by considering only integer constraints but ignoring continuous constraints. When integer variables are found, SP is an LP to find the associated optimal continuous variables. Then, MP is modified in order to find more suitable integer variables, by adding new constraints to shrink the search/feasible region. The global optimal solution of MILP can be guaranteed through this iterative process [18].

In detail, MP is formulated as

$$\begin{aligned} &\textbf{MP}: \\ &Z_{\text{lower}} = \min_{\boldsymbol{\epsilon}} Z \\ &\text{s.t. } (3) \\ &\qquad \text{feasibility constraints} \\ &\qquad \text{infeasibility constraints} \end{aligned} \tag{15}$$

where Z_{lower} is the lower bound of Z, because the objective function of (15) is identical to that of (14), but (15) excludes the constraints with continuous variables. Both feasibility and infeasibility constraints are integer constraints, which help find more suitable integer variables. More details of applying them will be presented by (22) and (23) later on.

As the integer variables $\boldsymbol{\epsilon}$ have already been obtained by solving MP, SP contains only continuous constraints and variables which becomes an LP problem as follows:

$$\begin{aligned} &\textbf{SP}: \\ &Z_{\text{upper}} = \min_{\boldsymbol{p}} Z \\ &\text{s.t. } hbox(4a), (4b) \text{ given } \boldsymbol{\epsilon} \\ &\qquad (4c), (5a), (5b) \end{aligned} \tag{16}$$

where Z_{upper} is the upper bound of MILP, because $\boldsymbol{\epsilon}$ obtained by MP may not be optimal yet.

Clearly, Z^* lies between Z_{upper} and Z_{lower}, i.e., $Z_{\text{lower}} \leq Z^* \leq Z_{\text{upper}}$. By performing iterations between MP and SP,

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both Z_{upper} and Z_{lower} will be updated and Z^* will be found eventually when

$$Z_{\text{lower}} = Z^* = Z_{\text{upper}}. \tag{17}$$

The details of iterative process are stated below.

Step 1: Initialization

Set iteration index $k = 0, z_{\text{upper}} = +\infty$ and $z_{\text{lower}} = -\infty$. Set infeasibility and feasibility constraints to null. Choose one feasible $\boldsymbol{\epsilon}(0)$ which satisfies the integer constraint (3).

Step 2: Solving SP (at Iteration k)

Note that without the variable $\boldsymbol{\epsilon}(k)$ coupling all batteries together, SP (16) can be decoupled into B primal SPs for each battery $b \in \mathcal{B}$

$$\begin{aligned}&\textbf{primal SP}:\\ &\min_{\boldsymbol{p}(k)} \quad z_b\\ &\text{s.t.} \quad (4\text{a}), (4\text{b}) \text{ for given } \boldsymbol{\epsilon}(k)\\ &\qquad\quad (4\text{c}), (5\text{a}), (5\text{b}).\end{aligned} \tag{18}$$

Define $\sigma_c^h(k), \sigma_d^h(k), \vartheta^h(k)$ and $\theta^h(k)$ as dual variables for the corresponding constraints (4a), (4b), (5a) and (5b) at slot h, and let $\boldsymbol{\sigma}_c(k), \boldsymbol{\sigma}_d(k), \boldsymbol{\theta}(k), \boldsymbol{\vartheta}(k)$ be their vector form over $\mathcal{H}$. Then, the Lagrangian for primal SP (18) is

$$\begin{aligned}&\mathcal{L}(\boldsymbol{p}(k), \boldsymbol{\sigma}_c(k), \boldsymbol{\sigma}_d(k), \boldsymbol{\theta}(k), \boldsymbol{\vartheta}(k)) = \sum_{h\in\mathcal{H}} \left\{ s^h p_c^h + t^h p_d^h \right.\\ &+ \sigma_c^h(k)\left[p_c^h - \epsilon_c^h(k) p_c^{\max}\right] + \sigma_d^h(k)\left[p_d^h - \epsilon_d^h(k) p_d^{\max}\right]\\ &+ \theta^h(k)\left[e^0 + \sum_{t=1}^{h}\left(\eta^c p_c^t - \frac{p_d^t}{\eta^d}\right) - e^{\max}\right]\\ &\left. +\vartheta^h(k)\left[e^{\min} - e^0 - \sum_{t=1}^{h}\left(\eta^c p_c^t - \frac{p_d^t}{\eta^d}\right)\right]\right\}.\end{aligned}$$

Note that

$$\begin{aligned}&\sum_{h\in\mathcal{H}}\left[\left(\theta^h(k) - \vartheta^h(k)\right)\sum_{t=1}^{h}\left(\eta^c p_c^t - \frac{p_d^t}{\eta^d}\right)\right]\\ &\qquad = \sum_{h\in\mathcal{H}}\left[\left(\eta^c p_c^h - \frac{p_d^h}{\eta^d}\right)\sum_{t=h}^{H}\left(\theta^t(k) - \vartheta^t(k)\right)\right].\end{aligned}$$

Thus the Lagrangian for primal SP (18) can be rewritten as

$$\begin{aligned}&\mathcal{L}(\boldsymbol{p}(k), \boldsymbol{\sigma}_c(k), \boldsymbol{\sigma}_d(k), \boldsymbol{\theta}(k), \boldsymbol{\vartheta}(k))\\ &= \sum_{h\in\mathcal{H}}\left[s^h + \sigma_c^h(k) + \eta^c\sum_{t=h}^{H}\left(\theta^t(k) - \vartheta^t(k)\right)\right] p_c^h\\ &+ \sum_{h\in\mathcal{H}}\left[t^h + \sigma_d^h(k) - \frac{1}{\eta^d}\sum_{t=h}^{H}\left(\theta^t(k) - \vartheta^t(k)\right)\right] p_d^h\\ &+ \sum_{h\in\mathcal{H}}\left[-\epsilon_c^h(k) p_c^{\max}\sigma_c^h(k) - \epsilon_d^h(k) p_d^{\max}\sigma_d^h(k)\right.\\ &\left. + \left(e^0 - e^{\max}\right)\theta^h(k) + \left(e^{\min} - e^0\right)\vartheta^h(k)\right].\end{aligned} \tag{19}$$

To remove integer variables from feasible region of primal SP (18), we define the corresponding dual SP through (19):

$$\begin{aligned}&\textbf{dual SP}:\\ &\max_{\substack{\boldsymbol{\sigma}_c(k), \boldsymbol{\sigma}_d(k),\\ \boldsymbol{\theta}(k), \boldsymbol{\vartheta}(k)}} \quad Y_b = \sum_{h\in\mathcal{H}}\left[-\epsilon_c^h(k) p_c^{\max}\sigma_c^h(k) - \epsilon_d^h(k) p_d^{\max}\sigma_d^h(k)\right.\\ &\qquad\left. +(e^0 - e^{\max})\theta^h(k) + (e^{\min} - e^0)\vartheta^h(k)\right]\\ &\text{s.t.} \quad s^h + \sigma_c^h(k) + \eta^c\sum_{t=h}^{H}\left(\theta^t(k) - \vartheta^t(k)\right) \geq 0\\ &\qquad t^h + \sigma_d^h(k) - \frac{1}{\eta^d}\sum_{t=h}^{H}\left(\theta^t(k) - \vartheta^t(k)\right) \geq 0\\ &\qquad \sigma_c^h(k) \geq 0, \sigma_d^h(k) \geq 0, \theta^h(k) \geq 0, \vartheta^h(k) \geq 0\\ &\qquad \forall h \in \mathcal{H}.\end{aligned} \tag{20}$$

In dual SP (20), integers $\epsilon_d^h(k)$ and $\epsilon_c^h(k)$ have already been solved previous MP, while $\sigma_c^h(k), \sigma_d^h(k), \theta^h(k), \vartheta^h(k)$ are continuous variables to be solved through LP.

Step 3: Modifying and Solving MP (at Iteration $k+1$)

In many cases the integer variables $\boldsymbol{\epsilon}(k)$ are not optimal, however they can be modified by adding new integer constraints into MP, so that the feasible region shrinks and the new integer $\boldsymbol{\epsilon}(k+1)$ gradually approaches the optimal $\boldsymbol{\epsilon}^*$. The new constraints are constructed based on the solution of all B dual SPs (20), as detailed below. It is notable that in this step, $\sigma_c^h(k), \sigma_d^h(k), \theta_b^h(k), \vartheta_b^h(k)$ are known parameters, while $\epsilon_d^h(k+1)$ and $\epsilon_c^h(k+1)$ are integer variables to be solved, for example, by branch and bound method.

1) Any dual SP (20) is infeasible. MILP has either no feasible solution or an unbounded solution, so the original MINP has no physical solution.
2) All dual SPs (20) are bounded. Thus, all primal SPs (18) are feasible due to duality, but $Z_{\text{lower}} \leq Z_{\text{upper}}$. That is, $\boldsymbol{\epsilon}(k)$ is not optimal. In order to improve the solution in this iteration, $\boldsymbol{\epsilon}(k+1)$ cannot be worse than $\boldsymbol{\epsilon}(k)$, in the sense that the new Z_{lower} is larger than previous Z_{lower}s. Based on this idea, the feasibility constraint to be added into MP (15) can be written as
$$Z \geq Z_{\text{lower}}(i), \quad \forall i \in \mathcal{I} \tag{21}$$
where $\mathcal{I}$ denotes the set of iterations where all dual SPs are bounded. In other words, if all the dual SPs in the kth iteration are bounded, k should be added into set $\mathcal{I}$, i.e., $\mathcal{I} = k \bigcup \mathcal{I}$. Meanwhile, since the dual SP is bounded, strong duality between dual SP and primal SP holds, i.e.,
$$\min_{\boldsymbol{p}(k)} z_b = \max_{\boldsymbol{\sigma}_c(k), \boldsymbol{\sigma}_d(k), \boldsymbol{\theta}(k), \boldsymbol{\vartheta}(k)} Y_b.$$
Then, recall (20) and $Z \triangleq \sum_{b\in\mathcal{B}} z_b$, (21) can be elaborated as
$$\begin{aligned}&\textbf{feasibility constraint}:\\ &Z \geq Z_{\text{lower}}(i) = \sum_{b\in\mathcal{B}}\min_{\boldsymbol{p}} z_b(i) = \sum_{b\in\mathcal{B}}\max_{\boldsymbol{\sigma}_c, \boldsymbol{\sigma}_d, \boldsymbol{\theta}, \boldsymbol{\vartheta}} Y_b(i)\\ &\quad = \sum_{b\in\mathcal{B}}\sum_{h\in\mathcal{H}}\left[-\epsilon_c^h(k+1) p_{b,c}^{\max}\sigma_{b,c}^h(i)\right.\\ &\qquad - \epsilon_d^h(k+1) p_{b,d}^{\max}\sigma_{b,d}^h(i) + \left(e_b^0 - e_b^{\max}\right)\theta_b^h(i)\\ &\qquad \left. + \left(e_b^{\min} - e_b^0\right)\vartheta_b^h(i)\right], \forall i \in \mathcal{I}.\end{aligned} \tag{22}$$

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Algorithm 2: Benders Decomposition

/* Initialization */
1 $k \leftarrow 0$, $\mathcal{I} \leftarrow \emptyset$, $\mathcal{J} \leftarrow \emptyset$, $Z_{\text{upper}} \leftarrow \infty$, $Z_{\text{lower}} \leftarrow -\infty$;
2 **while do**
3 solve MP (15) to obtain $\boldsymbol{\epsilon}(k)$ and z_{lower};
4 **if** *feasible solution* **then**
5 **if** $|Z_{\text{upper}} - Z_{\text{lower}}| \le \varepsilon$ **then**
/* Converged */
6 **return** $\boldsymbol{\epsilon}^* = \boldsymbol{\epsilon}(k)$;
7 **end if**
8 **else if** *unbounded solution* **then**
9 choose arbitrary $\boldsymbol{\epsilon}(k)$ from domain of definition;
10 $Z_{\text{lower}} = -\infty$;
11 **else**
12 **return** no feasible solution;
13 **end if**
14 **for** *each battery* $b \in \mathcal{B}$ **do**
/* Parallel computation */
15 solve dual SP (20) with $\epsilon_c^h(k)$, $\epsilon_d^h(k)$ to obtain $\left\{\sigma_{b,c}^h(k); \sigma_{b,d}^h(k); \theta_b^h(k); \vartheta_b^h(k) | \forall h \in \mathcal{H}\right\}$ and z_b;
16 **end for**
17 **if** *feasible solution* **then**
18 $Z_{\text{upper}} = \min\{Z_{\text{upper}}, \sum_{b\in\mathcal{B}} Z_b\}$;
/* Add feasibility constraint */
19 $\mathcal{I} \leftarrow \{k\} \cup \mathcal{I}$;
20 **else if** *unbounded solution* **then**
/* Add infeasibility constraint */
21 $\mathcal{J} \leftarrow \{k\} \cup \mathcal{J}$;
22 **else**
23 **return** no feasible solution;
24 **end if**
25 $k \leftarrow k + 1$;
26 **end while**

3) Any dual SP (20) is unbounded. Thus, the corresponding primal SP (18) is infeasible for $\boldsymbol{\epsilon}(k)$. Therefore, the algorithm needs to avoid finding these infeasible integers again by introducing a new constraint

$$\begin{aligned}\textbf{infeasibility constraint}:\\ 0 \ge \sum_{h\in\mathcal{H}} \Big[& -\epsilon_c^h(k+1) p_{b,c}^{\max} \sigma_{b,c}^h(j) \\ & - \epsilon_d^h(k+1) p_{b,d}^{\max} \sigma_{b,d}^h(j) + \left(e_b^0 - e_b^{\max}\right)\theta_b^h(j) \\ & + \left(e_b^{\min} - e_b^0\right)\vartheta_b^h(j)\Big], \forall b \in \mathcal{U}(j), \forall j \in \mathcal{J} \end{aligned} \tag{23}$$

where $\mathcal{J}$ denotes the set of iterations where any dual SP is unbounded and $\mathcal{U}(j)$ denotes the set of corresponding batteries in the jth iteration. Similarly, if any dual SP in the kth iteration is unbounded, k should be added into set $\mathcal{J}$, i.e., $\mathcal{J} = k \bigcup \mathcal{J}$. The derivation of (23) is in the Appendix.

These two types of constraints are added to MP (15) dynamically, according to the solution of dual SPs (20) in each iteration. Solving MP produces a new Z_{lower} and if $|Z_{\text{upper}} - Z_{\text{lower}}| \le \varepsilon$, where ε is a small positive number, the solution can be regarded as converged and the iterative process is terminated; otherwise, we shall repeat **step 2**. The Benders decomposition algorithm is summarized as **Algorithm 2**.

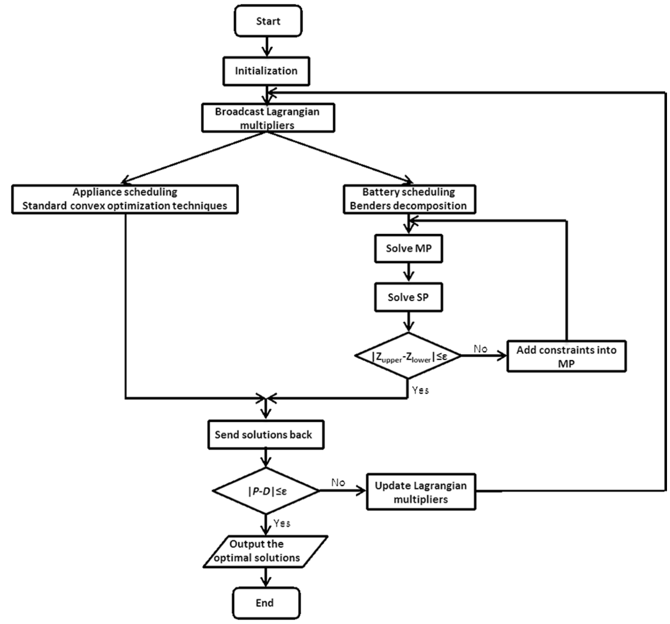

Fig. 2. Flowchart of the proposed approach.

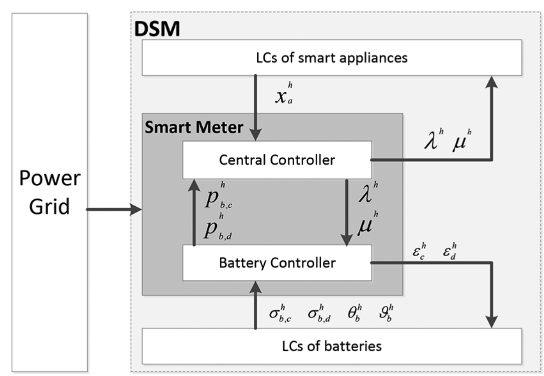

Fig. 3. Framework of distributed implementation.

B. Distributed Implementation

Fig. 2 shows the flowchart of the proposed approach. Clearly, the load scheduling problem can be implemented in a distributed manner. That is, primal problem $\mathcal{P}$ (9) is decomposed into two sets of subproblems. The first type of subproblem $\mathcal{S}_1$ (10) is the scheduling on each appliance, which can be computed by the LC of each appliance. On the other hand, the second type of subproblem $\mathcal{S}_2$ (11) is the scheduling on the battery set, which can be solved by battery controller also in a distributed way. After LCs and battery controller solve subproblems $\mathcal{S}_1$ (10) and $\mathcal{S}_2$ (11), the information will be gathered by smart meter; then, the central controller located at smart meter will update Lagrangian multipliers according to (13), and send the new multipliers back to LCs and battery controller. The multipliers serve as coordination signals that facilitate the global optimality from local optimality. Fig. 3 shows the framework of distributed implementation.

V. Simulations and Results

In this section, extensive simulations have been conducted to demonstrate the efficiency of the proposed approach. Sim-

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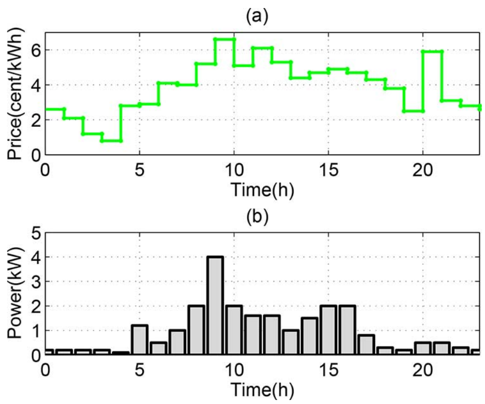

Fig. 4. Electricity price and load of non-shiftable appliances.

TABLE I
PARAMETER SETUP FOR APPLIANCES

Parameter	1	2	3	4	5	Unit
α_a	0	4	17	11	14	h
β_a	23	11	21	15	19	h
D_a	18	16	8	8.75	5	kWh
$x_a^{\max}$	1	2.5	2	2	1.5	kW
$x_a^{\min}$	0.5	1	1	1.5	0	kW
w_a	0.1	0.2	0.1	0.1	0.2	n/a

TABLE II
PARAMETER SETUP FOR BATTERY

Parameter	Value	Unit
$e_b^{\min}$	1.776	kWh
$e_b^{\max}$	7.548	kWh
e_b^{0}	1.776	kWh
$p_{b,c}^{\max}$	1.32	kW
$p_{b,d}^{\max}$	1.77	kW
η_b^c	0.839	n/a
η_b^d	0.764	n/a

ulations are implemented in MATLAB. The dissatisfaction is modeled as a quadratic function [14]:

$$V_a^h\left(x_a^h\right) \triangleq w_a^h\left(x_a^h - \bar{x}_a^h\right)^2$$

where $w_a^h > 0$ is a weight. Particularly, MP (15) and dual SP (20) are solved by Yalmip tool [20] and "linprog" function respectively. The maximum number of iteration is 300. We consider a scenario with 5 shiftable appliances and 1 battery, whose parameters are listed in Tables I and II [15]. The real-time prices are taken from [21] on May 1, 2013, as shown in Fig. 4(a). The load of non-shiftable appliances is shown in Fig. 4(b).

Fig. 5 shows the optimal scheduling on shiftable appliances with consideration of the user's satisfaction. Fig. 6 illustrates the scheduling on battery. Fig. 6(a) shows that the battery is charged in the morning when the electricity price is low. In Fig. 6(b), the battery is discharged at the peak-demand period. Fig. 6(c) shows that the stored energy in the battery changes according to charge/discharge processes. After optimal scheduling on the battery, the user saves 13.62 cents.

In order to validate the efficiency of our propose approach on a larger test system, we additionally consider a scenario with 60

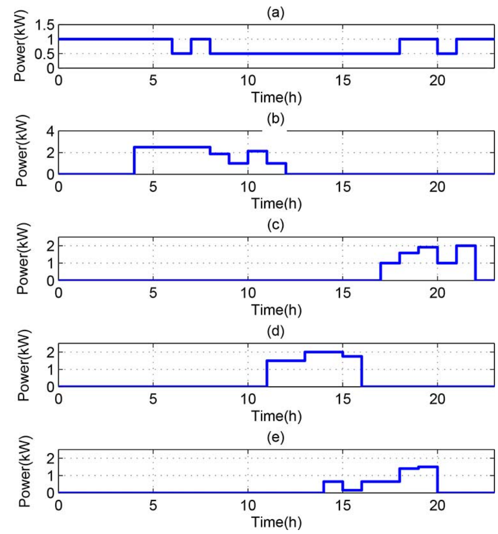

Fig. 5. Optimal scheduling on shiftable appliances.

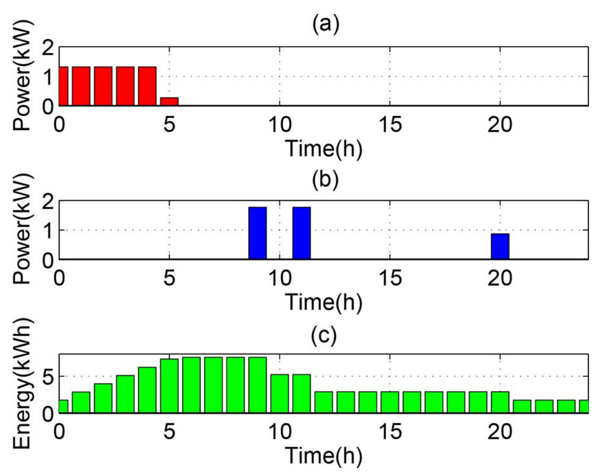

Fig. 6. Optimal battery scheduling.

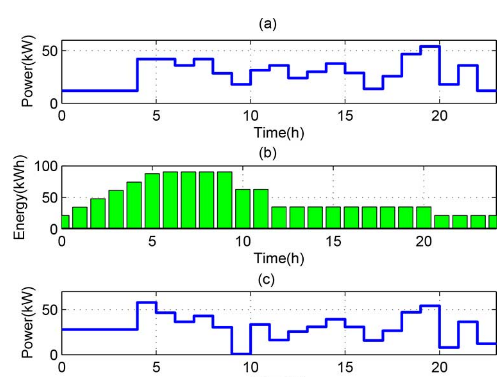

Fig. 7. Optimal scheduling on 60 shiftable appliances and 12 batteries.

shiftable appliances and 12 batteries. The corresponding scheduling result is shown in Fig. 7. Fig. 7(a) illustrates the total power of the 60 scheduled shiftable appliances while the total remaining energy in the 12 batteries is shown in Fig. 7(b). To demonstrate the advantage of joint scheduling on appliances and batteries, the power purchased from the grid is shown in Fig. 7(c). The user reduces expenses by load shifting as well as

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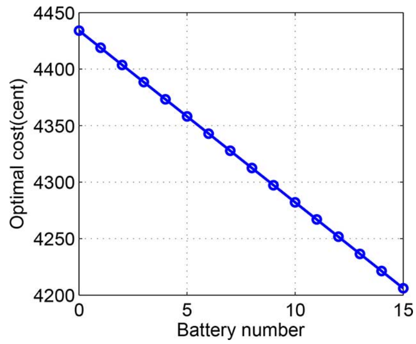

Fig. 8. Battery number impacts on system performance.

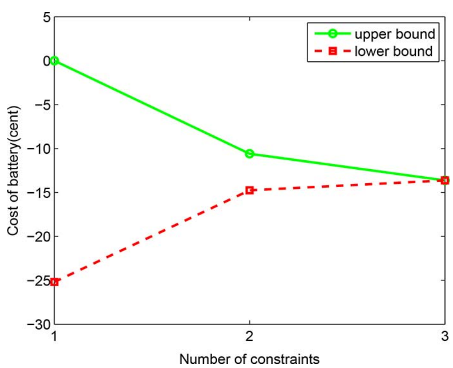

Fig. 9. Convergence of Benders decomposition.

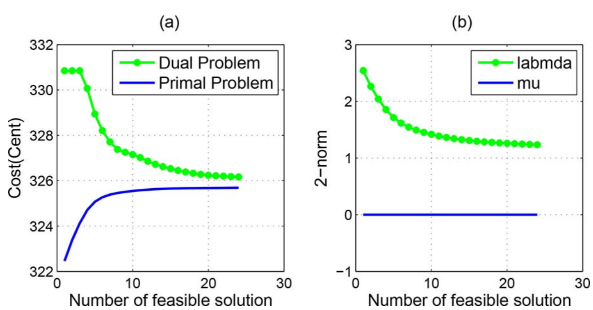

Fig. 10. Convergence of dual decomposition.

charging the batteries during low price periods for power supply during high price periods.

The advantage for adding batteries into load scheduling is obviously shown in Fig. 8. In this simulation, the number of shiftable appliances is fixed at 100, while the number of batteries increases from 0 to 15. The max power the grid can supply is 120 KW. It is clear that the user's cost decreases with the increase of batteries, because more batteries bring more flexibility of scheduling.

Fig. 9 shows the convergence of Benders decomposition algorithm. With feasibility and infeasibility constraints adding into MP (15), the upper bound and lower bound quickly converge to the optimal value. Fig. 10 explains how the proposed approach converges to the optimal solution. In Fig. 10(a), the feasible values of primal problem $\mathcal{P}$ (9) and dual problem $\mathcal{D}$ (12) are plotted. The dual value is always below the primal value due to weak duality; however, the gap is very small. Fig. 10(b) shows the convergence of Lagrangian multipliers.

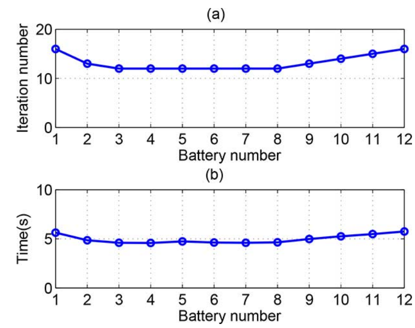

Fig. 11. Scalability of the proposed approach.

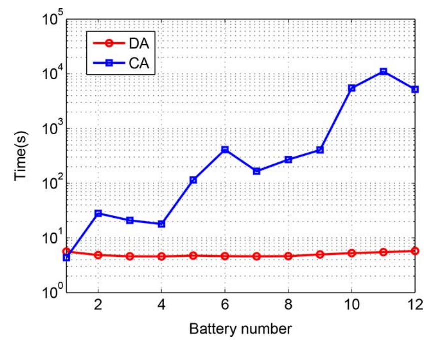

Fig. 12. Comparison of time on DA and CA.

Fig. 11 illustrates the scalability of our proposed approach. In the simulation, the number of batteries increases from 1 to 12, and the number of appliances is kept five times of the battery number. It's obvious that both the iteration number and simulation time keep almost unchanged due to the advantage of parallel computation. For purpose of comparison, Fig. 12 shows the advantage of the proposed distributed approach (DA) over the centralized approach (CA) which is realized by Yalmip tool [20], for large-scale joint scheduling in terms of computing time. In Fig. 12, the computing time of DA and CA with different numbers of batteries is given. It is obviously observed that the computational complexity of CA increases exponentially with the battery number, while DA shows great scalability. In conclusion, the proposed distributed approach is more suitable for large-scale scheduling problem.

VI. Conclusion

In this paper, we propose an approach to improve energy management through efficient joint scheduling on large-scale appliances and batteries. The joint scheduling problem is originally formulated as MINP, which is decoupled via LR into two sets of subproblems: single appliance scheduling and battery scheduling. As single appliance scheduling can be readily solved by common convex optimization methods, the technical challenge of this paper mainly lies on battery scheduling due to the MILP nature, which is then successfully tackled by the proposed Benders decomposition. Meanwhile, due to the distributed nature of the proposed approach, it is particularly suitable for large-scale systems. Extensive simulations are conducted to verify the performance of the proposed approach.

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APPENDIX A
PROOF OF (23)

For simplicity and generality, we drop the iteration index k here. If dual SP (20) is unbounded, the domain of primal SP (18) is empty with the "given" integer $\boldsymbol{\epsilon}$. That is, the original constraints of (18) conflict for $\boldsymbol{\epsilon}$. Thus, we can check the feasibility of (18) by introducing extra variables $\kappa_1^h, \kappa_2^h, \kappa_3^h, \kappa_4^h$, to relax these constraints and formulate the following feasibility check problem (FCP). Note that the feasibility is determined by the constraints, while the objective function is irrelevant in this procedure. Therefore, the objective functions of (18) and FCP are different:

$$\begin{aligned}
&\textbf{FCP}:\\
&\min_{\boldsymbol{p},\boldsymbol{\kappa}} \quad K = \sum_{h\in\mathcal{H}} \left(\kappa_1^h + \kappa_2^h + \kappa_3^h + \kappa_4^h\right)\\
&\text{s.t.} \quad p_{b,c}^h \le \epsilon_c^h p_{b,c}^{\max} + \kappa_1^h\\
&\qquad p_{b,d}^h \le \epsilon_d^h p_{b,d}^{\max} + \kappa_2^h\\
&\qquad e_b^{\min} \le e_b^0 + \sum_{t=1}^{h} \left(\eta_b^c p_{b,c}^t - \frac{p_{b,d}^t}{\eta_b^d}\right) + \kappa_3^h\\
&\qquad e_b^0 + \sum_{t=1}^{h} \left(\eta_b^c p_{b,c}^t - \frac{p_{b,d}^t}{\eta_b^d}\right) \le e_b^{\max} + \kappa_4^h\\
&\qquad \kappa_1^h \ge 0, \kappa_2^h \ge 0, \kappa_3^h \ge 0, \kappa_4^h \ge 0\\
&\qquad p_{b,c}^h \ge 0, p_{b,d}^h \ge 0 \quad \forall b \in \mathcal{B}, \forall h \in \mathcal{H}.
\end{aligned} \tag{24}$$

Clearly, adding a nonnegative variable to the right-hand-side of each constraint indeed relaxes the original constraint. If any of the original constraints is infeasible, we have $\min_{\boldsymbol{p},\boldsymbol{\kappa}} K > 0$ because the corresponding relaxation is effective. Thus, to exclude infeasibility we must have

$$0 \ge \min_{\boldsymbol{p},\boldsymbol{\kappa}} K. \tag{25}$$

The dual form of FCP is

$$\begin{aligned}
&\textbf{dual FCP}:\\
&\max_{\boldsymbol{\sigma}_c,\boldsymbol{\sigma}_d,\boldsymbol{\theta},\boldsymbol{\vartheta}} \quad Y_b = \sum_{h\in\mathcal{H}} \left[-\epsilon_c^h p_{b,c}^{\max}\sigma_{b,c}^h - \epsilon_d^h p_{b,d}^{\max}\sigma_{b\cdot d}^h\right.\\
&\qquad\qquad \left. + \left(e_b^0 - e_b^{\max}\right)\theta_b^h + \left(e_b^{\min} - e_b^0\right)\vartheta_b^h\right]\\
&\text{s.t.} \quad \sigma_{b,c}^h + \eta_b^c \sum_{t=h}^{H} \left(\theta_b^t - \vartheta_b^t\right) \ge 0\\
&\qquad \sigma_{b,d}^h - \frac{1}{\eta_b^d} \sum_{t=h}^{H} \left(\theta_b^t - \vartheta_b^t\right) \ge 0\\
&\qquad 1 - \sigma_{b,c}^h \ge 0, 1 - \sigma_{b,d}^h \ge 0\\
&\qquad 1 - \theta_b^h \ge 0, 1 - \vartheta_b^h \ge 0\\
&\qquad \sigma_{b,c}^h \ge 0, \sigma_{b,d}^h \ge 0,\ \theta_b^h \ge 0,\ \vartheta_b^h \ge 0\\
&\qquad \forall b \in \mathcal{B}, \forall h \in \mathcal{H}.
\end{aligned} \tag{26}$$

Due to strong duality, we have

$$\begin{aligned}
\min_{\boldsymbol{p},\boldsymbol{\kappa}} K = &\sum_{h\in\mathcal{H}} \left[-\epsilon_c^h p_{b,c}^{\max}\sigma_{b,c}^h - \epsilon_d^h p_{b,d}^{\max}\sigma_{b.d}^h\right.\\
&\left. + \left(e_b^0 - e_b^{\max}\right)\theta_b^h + \left(e_b^{\min} - e_b^0\right)\vartheta_b^h\right].
\end{aligned} \tag{27}$$

Finally, taking (27) into (25) gives (23).

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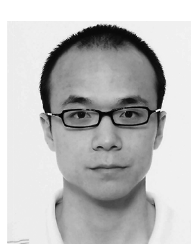

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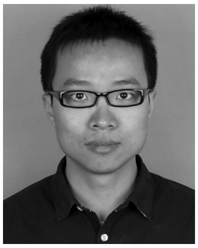

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