Recommendation of Geographic Distributed Charging Stations for Electric Vehicles: A Game Theoretical Approach

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***Abstract*—Electric vehicles (EVs) are vigorously promoted by policy makers in recent years, and expected to penetrate deeply in the near future. It is of great significance to develop efficient charging scheduling algorithms to optimize system operation. In this paper, we investigate the spatial scheduling problem of EVs, which aims to make recommendations of geographic distributed charging stations (CSs) for EVs to minimize the time of travelling and queuing. In order to ensure the fairness of the recommendations, an efficient game-theoretical approach is proposed. Numerical results illustrate the effectiveness of the proposed method in saving time, reducing charging piles' idle rate and EVs' queuing rate.**

I. Introduction

For the sake of economically and environmentally friendly advantages, electric vehicles (EVs) are becoming the promising substitutes of traditional fuel-engined vehicles, vigorously promoted by policy makers, and expected to be widely used in the near future [1, 2]. Hence, the large-scale integration of EVs has attracted great attentions in recent years [3–9]. Because EVs require longer refueling (charging) time and the charging power is considerably large, poor scheduling of the charging behaviour would not only bring new load peak to the grid [3–8], but also increase the queueing time of EVs in the charging stations (CSs) [9]. Consequently, it is particularly meaningful to design a suitable EV charging scheduling mechanism.

The existing charging scheduling mechanisms can be summarized into two categories: the temporal scheduling and the spatial scheduling. The temporal scheduling often aim to make a recommendation of suitable charging time period for EVs, which minimize the total cost of all EVs which perform charging and discharging during the day. [10] proposes a globally optimal scheduling scheme and a locally optimal scheduling scheme for EV charging and discharging, which aim to minimize the total cost of all EVs which perform charging and discharging during the day. The authors in [11] investigate the optimal charging strategies of EVs based on drivers' self-interested charging behavior, traffic congestion, operating expense of CSs and pricing. In [12], [13], the authors study the optimal charging problem to maximize the operating profit of electric taxies by making a temporal scheduling based on uncertain electricity prices and time-varying incomes. [14] proposes a spatial and temporal model of electric vehicle charging demand, based on the fluid dynamic traffic model and the M/M/s queueing theory, for a rapid CS located near a highway exit. Other similar works about scheduling the EVs directly can be reference in [15–20].

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The spatial scheduling is equally important because the power needs to be balanced in both temporal and spatial domains over the grid; however, it receives much less research attentions. The spatial scheduling often aims to make a recommendation of geographic distributed (geo-distributed) CSs for EVs, which saves charging time and expenditure as much as possible. [21] investigates the optimal charging strategies about the spatial dispatch of electric taxies based on the difference of charging prices, queuing time and distance among different CSs. [22] provides a real-time CS recommendation system for electric taxies via large-scale GPS data mining, which guarantees the minimal total time before their recharging starts.

However, it is notable that existing spatial scheduling methods do not consider the future EV arrivals and departures of CSs. Therefore, the queuing time at CSs cannot be accurately obtained, making the spatial scheduling less efficient. What's more, existing methods are mostly based on the assumption of electricity price variation, which in practice may not be frequent and violent enough to affect the charging decisions of EVs. In contrast, EVs are more sensitive to time cost since it will give rise to huge utility loss if they have to wait hours for charging. In this paper, the future EV arrival and departure information are incorporated into the real-time recommendation of geo-distributed CSs for EVs, and the time cost is considered as the main impact factor instead of electricity prices for decision making. Our contributions in this paper can be summarized as follows:

- The CS recommendation problem for EV charging is formulated as a charging cost minimization problem, which comprehensively considers the future arrival and departure information of previous scheduled EVs.
- The CS recommendation problem takes into account time cost including travelling time and queuing time, instead of trivial electricity price.
- The CS recommendation problem is tackled by a game-theoretic approach [23], which converges fast to a Nash-Equilibrium (NE) and ensures the fairness among EVs.

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The remainder of this paper is organized as follows. The system model and the cost function of EVs are presented in the Section II. A game-theoretic approach is proposed to solve the recommendation problem in III. Numerical results are discussed in Section IV, followed by conclusions drawn in Section V.

II. System Model

In this paper we consider a city, in which the CSs are uniformly deployed. We aim to solve the spatial scheduling problem of a set of EVs that plan to charge at the same period. EVs often choose CSs according to the estimated charging cost, which consists of three parts: charging fee, travelling cost, and queuing cost. The charging fee is ignored since it is insignificantly different and even uniform among different stations. Only the travelling cost and queuing cost will be considered in the paper, as shown in Fig. 1.

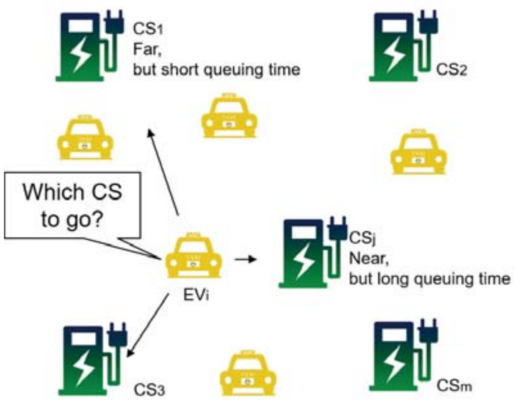

Fig. 1. The factors considered by EVs when selecting CSs

A. Travelling Cost

When an EV selects one CS to refuel its battery energy, the travelling time from the current location to the target CS is defined as its travelling cost, which depends on the distance between EV and CS, and the driving speed along the way. Let $\mathbb{I}(t)$ denote a set of EVs requesting to refuel at time t, and $\mathbb{J}$ denote a set of candidate CSs. Thus, the travelling cost of the ith EV selecting the jth CS is,

$$t_{ij} = d_{ij}/v_{ij}, i \in \mathbb{I}(t), j \in \mathbb{J} \tag{1}$$

where d_{ij} denotes the distance and v_{ij} denotes the driving speed.

B. Queuing Cost

We consider the queuing cost in the jth CS at time slot ι as the queuing time of the new arrival EV has to wait, which can be written into,

$$\omega_j^\iota = \sigma_j \Omega(N_j^\iota), \iota \geq t \tag{2}$$

$$\Omega(N_j^\iota) = \begin{cases} N_j^\iota, & N_j^\iota \geq 0 \\ 0, & N_j^\iota < 0 \end{cases} \tag{3}$$

In above equations,

- σ_j denotes the queuing time factor of the jth CS, i.e., the average time of CS j fully charging a EV, which depends on the number and the charging rate of charging piles in the jth CS.
- $|N_j^\iota|$ denotes, respectively, the number of EVs waiting at the jth CS at time slot ι when $N_j^\iota \geq 0$, and the number of idle charging piles at the jth CS at time slot ι when $N_j^\iota < 0$. The dynamics of N_j^ι can be written as,

$$N_j^\iota = e_j^t + \sum_{\tau=t}^{\iota}(\lambda_j^\tau - \mu_j^\tau) + \sum_{\tau=t}^{\iota} n_j^\tau, \ \iota \geq t \tag{4}$$

  where
  - e_j^t denotes the initial number of EVs waiting for charging in the jth CS at time slot t.
  - λ_j^τ and μ_j^τ denote, respectively, the numbers of EVs which have been assigned to the jth CS in previous decisions and will arrive and leave at time slot τ.
  - n_j^τ denotes the number of EVs which have been assigned to the jth CS in current decision and will arrive at time slot τ.

Note that e_j^t, λ_j^τ and μ_j^τ are associated with previous decisions, thus are seen as known constants at time slot t, while n_j^τ is related to the current decision.

C. The Cost Function

Let s_i denote the strategy of the ith EV, and $s_i = j$ imply the ith EV selecting the jth CS as its target station ($\forall i \in \mathbb{I}(t)$, $\forall j \in \mathbb{J}$). Let set $\vec{s} = \{s_i | i \in \mathbb{I}(t)\} \in \times_{i \in \mathbb{I}(t)} \mathbb{J}$ denote the strategy profile of EV set $\mathbb{I}(t)$. Thus, for any strategy profile $\vec{s}$, the charging cost of the ith EV is the summation of travelling time t_{is_i} and queuing time $\omega_{s_i}^{t+t_{is_i}}$,

$$C_i(\vec{s}) \quad = t_{is_i} + \omega_{s_i}^{t+t_{is_i}} = t_{is_i} + \delta_{s_i} \Omega(N_{s_i}^{t+t_{is_i}}(\vec{s})) \tag{5}$$

where

$$N_{s_i}^{t+t_{is_i}}(\vec{s}) = e_{s_i}^t + \sum_{\tau=t}^{t+t_{is_i}} (\lambda_{s_i}^\tau - \mu_{s_i}^\tau) + \sum_{\tau=t}^{t+t_{is_i}} n_{s_i}^\tau(\vec{s}) \tag{6}$$

where $n_{s_i}^\tau(\vec{s})$ denotes the number of EVs which have been assigned to the s_ith CS and will arrive at time slot τ.

Since each EV is assumed to be selfish and aims to minimize its own utility function, the strategies of all EVs are coupled together. It's not intuitive to untwist the coupling explicitly. Therefore, a game approach is proposed next to ensure the fairness and effectiveness of CS selection.

III. Impartial Solution Based on Nash-Equilibrium

We describe the decision making process of the spatial scheduling as a game, and the impartial solution as a Nash-Equilibrium strategy profile of the game [23–25]. The game is defined as below.

- Player set $\mathbb{I}(t)$: the EV set that needs to be scheduled for charging at time slot t.
- Strategy space $\mathbb{S} := \times_{i \in \mathbb{I}(t)} \mathbb{J}$: the strategy of the ith EV selecting the jth CS is denoted as $s_i = j$. Define $\vec{s} =$

$(s_i, s_{-i}) = \{s_i | i \in \mathbb{I}(t)\} \in \mathbb{S}$ as the strategy profile, where s_{-i} is the strategies of all the other EVs in the set $\mathbb{I}(t)$ except EV i. Also define $(s'_i, s_{-i}) \in \mathbb{S}$ as an alternative strategy profile, where $s'_i \in \mathbb{J}$ denotes any other strategy of EV i except s_i.

- Cost function set $\{C_i(\vec{s})\}_{i\in\mathbb{I}(t)}$: the summation of the travelling time and queuing time for each EV i, as defined in (5).

Then, we have the following theorem.

Theorem 1. *A Nash-Equilibrium exists for above game, from which it is unprofitable to deviate unilaterally for any single player. That is, there exists a strategy profile $\vec{s} \in \mathbb{S}$ such that,*

$$C_i(s_i, s_{-i}) \leq C_i(s'_i, s_{-i}), \forall i \in \mathbb{I}(t) \tag{7}$$

The convergence to a Nash-Equilibrium can be achieved by Algorithm 1.

Proof: We describe the proof in three steps, which are also the essences of Algorithm 1. We first prove an NE exists in any single player set, then show that a new NE can be found when a new player is added. At last it is concluded by an induction method that an NE always exists in this game for any player set. Before approaching these steps, define $\mathbb{SI} \subseteq \mathbb{I}(t)$ as a stable subset of $\mathbb{I}(t)$, which contains an NE strategy profile $\vec{s} \in \times_{i\in\mathbb{SI}}\mathbb{J}$.

Step 1: Any single player subset has an NE strategy, which is also a stable subset. For any single EV subset $\{i\}$, $i \in \mathbb{I}(t)$, a strategy s_i that satisfies $C_i(s_i) \leq C_i(s'_i)$ is its NE strategy. In other words, by enumerating all possible CSs, this EV can easily identify the one with the lowest cost.

Step 2: For any stable subset, when a new player is randomly added, the new subset can always find a new NE strategy and thus remain stable. Randomly pick a EV l ($l \in \mathbb{I}(t) \setminus \mathbb{SI}$), add it into the subset $\mathbb{SI}$. EV l would select CS s_l satisfying $C_l(\vec{s}, s_l) \leq C_l(\vec{s}, s'_l)$ as its strategy, where $(\vec{s}, s_l) = (s_i, s_{-i}, s_l)$, $i \in \mathbb{SI}$ is a strategy profile of the EV set $\mathbb{SI} \cup \{l\}$.

Apparently, EVs which also select s_l but arrive after the newly added EV l will be influenced, because their queue length will increase by one due to the insertion of EV l, i.e.,

$$N_{s_k}^{t+t_{ks_k}}(\vec{s}, s_l) = N_{s_k}^{t+t_{ks_k}}(\vec{s}) + 1 \tag{8}$$

where $k \in \mathbb{B} := \{k | s_k = s_l, t_{ks_k} \geq t_{ls_l}, \forall k \in \mathbb{SI}\}$ denotes the EVs influenced by the newly added EV l. Consequently, the cost of EV k will increase, i.e.,

$$C_k(s_k, s_{-k}, s_l) \geq C_k(s_k, s_{-k}) \tag{9}$$

With the new cost $C_k(s_k, s_{-k}, s_l)$, EV k may or may not change its strategy. All possibilities of its decision fall into the following two categories:

(a) The subset $\mathbb{SI}$ remains stable with the strategy profile $\vec{s}$,

- when $\mathbb{B} = \emptyset$. That is, there is no EV influenced by the newly added EV l;
- when $\mathbb{B} \neq \emptyset$ but no EV in the set $\mathbb{B}$ has an incentive to change its strategy. In other words, although the cost of EV k increases, but the current strategy is still better than other alternative ones.

That is,

$$C_i(s_i, s_{-i}, s_l) \leq C_i(s'_i, s_{-i}, s_l), \forall i \in \mathbb{SI} \cup \{l\} \tag{10}$$

Thus, the new strategy profile $(\vec{s}, s_l) \in \times_{i\in\mathbb{SI}\cup\{l\}}\mathbb{J}$ is an NE strategy profile of the subset $\mathbb{SI} \cup \{l\}$.

(b) The subset $\mathbb{SI}$ becomes unstable, when $\mathbb{B} \neq \emptyset$ and there exists at least one EV in $\mathbb{B}$ which has an incentive to change its strategy.

Sort the EVs in the set $\mathbb{B}$ by arrival in chronological order, and identify the first EV g ($g \in \mathbb{B}$) which has an incentive to change its strategy. When EV g abandons the strategy s_l, all other EVs in the set $\mathbb{B}$ which arrive at CS s_l after EV g would have no incentive to change their strategies now, because the queue length for them would remain unchanged by inserting EV l and then removing EV g. Consequently, their cost also remains unchanged.

Thus, by inserting EV l and then removing EV g from the set $\mathbb{SI}$, the original stable set $\mathbb{SI}$ becomes a new stable set $\mathbb{SI} \cup \{l\} \setminus \{g\}$, and the associated NE strategy profile is (s_{-g}, s_l). In fact, this process means that EV g is replaced by a new EV l which has a higher priority at CS s_l.

(c) Then, EV g needs to find another CS other than s_l. This process is the same as adding EV l into the queue at a certain CS, as described previously. The insertion of EV g at a certain CS may result in the strategy change of another EV at the same CS. Thus, this is a repetitive process. It is apparent that by repeating this process for finite times, eventually all EVs in the subset $\mathbb{SI} \cup \{l\}$ will find suitable CSs and no one will unilaterally change its strategy. That is, $\mathbb{SI} \cup \{l\}$ is a stable subset.

Step 3: According to the induction method, there is an NE for any player set $\mathbb{I}(t)$.

This completes the proof. ■

The procedures described in the proof are summarized in Algorithm 1 to find an NE strategy profile in finite steps.

IV. Numerical Results

In this section, we demonstrate the performances of Algorithm 1 by several cases to illustrate its effectiveness. All the following simulation results are carried out by Matlab R2015b.

In an EV simulation platform, given a $6km \times 6km$ plane, in which 9 CSs each equipped with 6 charging piles with charging rate $R_c = 30kw$ are uniformly deployed, and set $Q = 54kw \cdot h$ as the average refueling energy demand of EVs. Thus, the queuing time factors can be calculated,

$$\sigma_j = Q/(6 * Rc) = 0.3h \tag{11}$$

(1) *The proposed Algorithm 1 has good performance in finding the NE strategy*. Assume there are 50 EVs set with random positions requesting for refueling energy. Set the value of e_j, λ_j, μ_j of the CSs as 0, and the speed of EVs as $0.4km/min$, for simplicity.

As shown in Fig. 2, the proposed algorithm finds the NE strategy quickly in 81 iterations. In the remaining 50 iterations,

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Algorithm 1 Search of Nash-Equilibrium Strategy

Input: $d_{ij}, v_{ij}, \delta_j, e_j, \lambda_j, \mu_j, \forall i \in \mathbb{I}(t), \forall j \in \mathbb{J}$
Output: $\vec{s}$

Initialization: $\mathbb{SI} = null, \vec{s} = null$;
repeat
  Randomly pick a player l, $\forall l \in \mathbb{I}(t) \setminus \mathbb{SI}$;
  $\vec{s}^* \leftarrow$ ITERATOR$(\mathbb{SI}, l, \vec{s})$;
  $\vec{s} \leftarrow \vec{s}^*$;
  $\mathbb{SI}^* \leftarrow \mathbb{SI} \cup \{l\}$;
  $\mathbb{SI} \leftarrow \mathbb{SI}^*$;
until $\mathbb{SI} = \mathbb{I}(t)$

function ITERATOR$(\mathbb{SI}, l, \vec{s})$
  $s_l \leftarrow$ strategy of player l based on $\vec{s}$;
  if $(\vec{s}, s_l)$ is an NE strategy of player set $\mathbb{SI} \cup \{l\}$ **then**
    return $(\vec{s}, s_l)$;
  else
    A player $g \in \mathbb{SI}$ will change strategy unilaterally;
    return ITERATOR$(\mathbb{SI} \cup \{l\} \setminus \{g\}, g, (s_{-g}, s_l))$;
  end if
end function

we traverse the 50 EVs to check their strategy stability, the result shows that all of them do not change strategy any more.

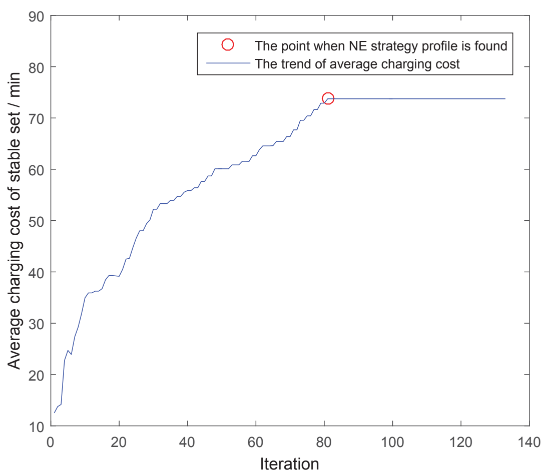

Fig. 2. Trend of average charging cost of stable set

In Fig. 3, the scheduling result is illustrated. The numbers of EVs assigned to different CSs are basically balanced, because Algorithm 1 considers not only the distance but also the queuing length of different CSs. A nearby CS with long queuing length may not be preferred by EVs.

If we set the initial numbers of EVs at CSs 6,5,4 in the middle column as $6, 10, 6$, the scheduling algorithm avoids assigning new EVs to these CSs, as shown in Fig. 4.

(2) *The proposed Algorithm 1 has good performance in assignment, compared with different recommendation methods.* Assume there are 270 EVs (EVs : piles $= 5 : 1$) with random initial positions are set to run in this plane for 10 days with a time interval of $2.5\ min$. As shown in Fig. 5 and Table I, we can observe that:

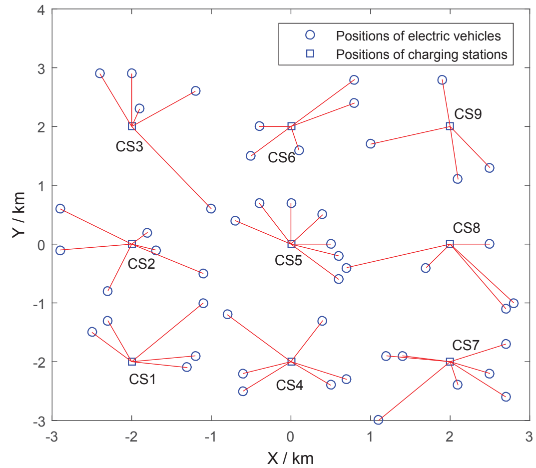

Fig. 3. Recommendation result of test case

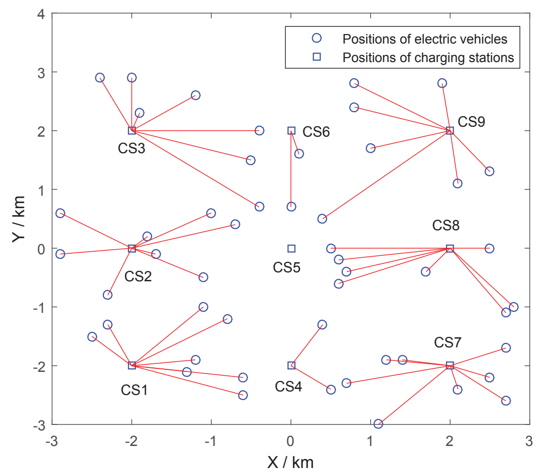

Fig. 4. Recommendation result after setting e_j

- Without scheduling : EVs mainly consider the distance when selecting CSs. The average queuing time is uneven among different CSs. Some CSs are heavily congested, such as CS 1, 2, 4 and 5. Meanwhile, the queuing time is much longer than others. Therefore, the charging infrastructure is utilized inefficiently.
- Algorithm 1 $(\lambda, \mu = 0)$: EVs select CSs recommended by Algorithm 1 without consideration of the future arrival and departure information of EVs scheduled before (i.e., $\lambda, \mu = 0$). The unevenness among CSs is averted, and the queuing time is greatly reduced. However, the recommendation can be further improved.
- Algorithm 1 : EVs select CSs recommended by Algorithm 1 with consideration of the future arrival and departure information of EVs scheduled before (i.e., λ, μ is generated from CSs in the simulation process automatically). The queuing time level is further reduced.

Meanwhile, we also investigate the EV queuing rate and the

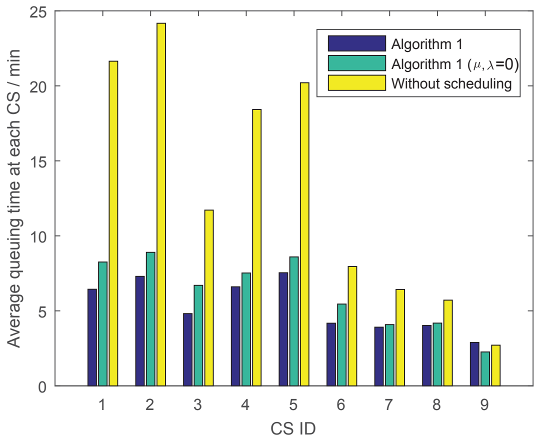

Fig. 5. Average queuing time of CSs using different scheduling methods

charging pile idle rate, which are separately defined as below,

$$EV\ queuing\ rate = \frac{Number\ of\ queuing\ EVs}{Number\ of\ EVs\ in\ CSs} \times 100\% \quad (12)$$

and,

$$Charging\ pile\ idle\ rate = \frac{Number\ of\ idle\ piles}{Number\ of\ total\ piles} \times 100\% \quad (13)$$

As shown in Table I, Algorithm 1 can significantly reduce the queuing of EVs and the idling of charging piles, although the travelling time increases a little. Therefore, the proposed method is capable of improving the utilization rate of charging infrastructure.

TABLE I
SCHEDULING PERFORMANCE OF DIFFERENT METHODS

Item \ Method	Algorithm 1	Algorithm 1 ($\lambda, \mu = 0$)	Without Scheduling
Average travelling time of EVs (min)	4.36	4.18	3.65
Average queuing time of EVs (min)	6.19	7.30	16.43
EV queuing rate (%)	3.83	4.46	8.64
Charging pile idle rate (%)	17.02	17.15	18.27

V. CONCLUSIONS

In this paper, we investigate the spatial charging recommendation problem of EVs, which aim to minimize the cost of charging time consisting of the time spent on travelling and queuing. In order to ensure the fairness of the recommendation, we proposed an efficient recommendation algorithm based on a game-theoretical approach. The simulation results demonstrate that the proposed algorithm has good performances in saving charging time, reducing charging piles idle rate and EVs queuing rate in CSs, and preventing uneven infrastructure usages among CSs.

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