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Coordinated Scheduling Strategy of Charging Station Considering Cost and Efficiency

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Abstract—**From the view point of a charging station (CS), it is important to design a simple, effective and implementable algorithm that reduces the cost, improves the time efficiency and enhances operational stability. Offline algorithms built on global information, in practice, cannot be implemented to achieve the best performance, since current charging rates of existing electric vehicles (EVs) need to be determined in the absence of future information. In the context of a current electricity tariff mechanism, commonly imposed in industry, which additionally charges for peak demand, this paper proposes an online two-stage charging scheduling algorithm (OTCSA) based on observed real time information and historical data to minimize charging cost, reduce charging time, as well as lower the maximum peak power. In the first stage, charging cost is minimized with guarantee to fulfill energy demand of each EV before its departure. The additional cost that penalizes peak demand inherently contributes to flattening the load profile of the CS with the deferrability of EV charging. In the second stage, we squeeze to save more charging time for EVs given the minimal cost. Simulations further validate the three-fold benefits of the proposed approach.**

Index Terms—**Charging station, optimization, electric vehicles, coordinated charging**

I. INTRODUCTION

As a new means of transportation, electric vehicles (EVs) have enormous advantages contrasted with conventional vehicles in emission reduction and energy saving. The fast proliferation of EVs has become a trend in recent years. As EVs surge, their charging demand has also been enormously growing [1]. CSs are among the most common places to charge EVs, where coordination on EV charging has been extensively studied [2]–[6]. Without proper management, the heavy charging load not only poses a serious threat to power grids due to sudden and uncertain spikes [7], but also causes tremendous charging cost and exhausting charging time, which significantly degrades the efficiency of a CS [2]. Considering the deferrability of EV charging, it is imperative to implement effective coordinated scheduling strategies in CSs.

Coordination on EV charging is a well-studied problem [2]–[13]. [8]–[11] all consider the real-time electricity pricing that is defined as a quadratic function of the current load. Among them, [8]–[10] aim to minimize the total charging cost of EVs. Specifically, [8] formulates the problem as a finite-horizon dynamic programming with a continuous state and action space while [9] designs a KKT-based online algorithm that requires zero future information to reduce computational complexity. [10] utilizes a time sliding window mechanism to design a distributed charging scheme that also implements in an online fashion. [11] further takes EV owners' convenience into account and obtains a Pareto-optimal solution by transforming a bi-objective optimization problem into a two-stage problem. In contrast, [12], [13] adopt the predetermined Time-of-Use (TOU) electricity pricing. [12] maximizes the revenue of a CS by purchasing energy from the power grid and selling it to EV owners to fulfill their charging demand. In [13], an adaptive charging network is introduced which adjusts the charging rates of EVs in real time to reduce the total cost to charge EVs. However, most, if not all, of these relevant works only upper bound the total charging load of EVs by physical limits from transformers or lines without considering peak shaving within the bound.

Different from above works, the electricity cost of a CS in our paper consists of two components: expenditure of purchasing energy from the grid and a tariff imposed to charge peak demand over a billing period, e.g., a month. The pricing mechanism encourages CSs to shave their peak loads and strike a tradeoff between these two costs. In order to minimize the cost of a CS and increase time efficiency, we propose an online two-stage charging scheduling algorithm (OTCSA) that is readily implementable. In the first stage, we approximate the global offline problem by a local problem that can be executed online in a receding horizon fashion to address the global temporal coupling within a whole billing period. Besides, energy demands of all EVs is fulfilled before departure with guarantee. In the second stage, we further exploit the flexibility in EV charging to save more charging time while maintaining the minimal charging cost, which can improve the time efficiency of the CS and raise EV owners' satisfaction.

The rest of the paper is organized as follows. We formulate

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the scheduling problem for EV charging and discuss its online approximate solution in Section II. In Section III, we propose a method to save charging time given the first-stage minimal cost. Section IV presents the simulation results. Finally, the paper is concluded in Section V.

II. Optimization of Charging Cost

Charging cost and operational stability, in terms of *stable and steady EV charging load*, are the main concerns of CSs, since the efficient operation of CSs and the low cost of EV charging are key factors that contribute to the popularization of EVs [1].

However, at present, uncoordinated EVs' charging behavior leads to high charging cost and poses a threat to the power grid due to sudden and uncertain load spikes. Therefore, we propose a charging scheduling algorithm to handle the common electricity tariff in industry that charges peak demand in a given billing cycle. Note that the tariff inherently encourages peak shaving. We first present an offline optimization algorithm that assumes full knowledge of global information, which outputs a global optimal solution that is impossible to achieve in real practice but can be seem as a benchmark. Then we proceed to design a simple, yet effective and implementable online algorithm that requires zero future information. As simulations in Section IV suggest, the charging cost and load profile obtained by the proposed online algorithm are close to the posterior global optimum.

A. Problem Formulation

Suppose that I EVs would arrive during a billing period $\mathbb{T} := \{1, 2, \ldots, T\}$, which is a discrete time horizon, and let $\mathbb{I} := \{1, 2, \ldots, I\}$ be the set of EVs. EV i randomly arrives at a CS and submits its energy demand and departure time to the CS. Assume the energy demand of EV i can be and must be fulfilled before its departure by the CS. The charging mission of an EV i can be characterized by a tuple $\pi_i := (a_i, d_i, e_i, x_i^{max})$, where a_i and d_i are the arrival and departure times, respectively, e_i is the amount of energy demand, and x_i^{max} is the charging rate limit. Define x_i^t as the charging rate of EV i at time t. Since EV i can only be charged when it is at the CS, the following constraints must be satisfied:

$$\begin{aligned} &x_i^t \in [0, x_i^{max}], \quad \forall t \in [a_i, d_i], \forall i \in \mathbb{I}, \\ &x_i^t = 0, \quad \forall t \in \mathbb{T} \backslash [a_i, d_i], \forall i \in \mathbb{I}. \end{aligned} \tag{1}$$

To fulfill the charging mission of EV i before its departure, we have

$$e_i \leq \sum_{t \in \mathbb{T}} x_i^t, \quad \forall i \in \mathbb{I}. \tag{2}$$

As shown in Fig. 1, all EV charging loads are supplied by a distribution transformer serving the CS. The transformer has a capacity of P which the total charging load of the CS cannot exceed at any time, i.e.,

$$\sum_{i \in \mathbb{I}} x_i^t \leq P, \quad \forall t \in \mathbb{T}. \tag{3}$$

Note that if there are other loads connected to the distribution transformer, the model can be generalized such that P is time-varying.

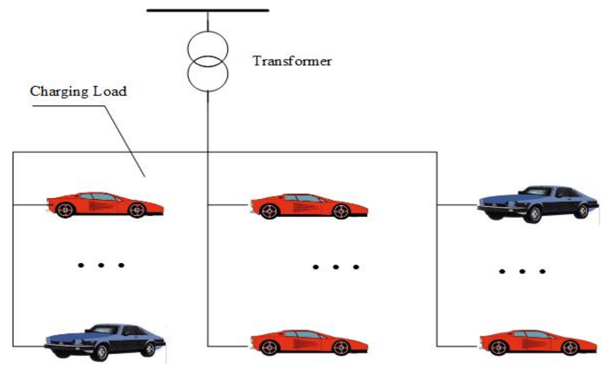


Fig. 1: Illustration of an EV CS

We denote the price of electricity at time t by c^t in a billing period. The prices are external and usually set based on the grid operational conditions, i.e., high prices corresponding to heavy load hours. However, there is still the possibility of inelastic charging demand and EVs may coincide to charge simultaneously that leads to load spikes to the grid. In order to further reduce the peak load, the CS will pay a fee once over a billing period for the peak load at the unit price of $\alpha\ CNY/kW$, which is known *a priori*. We are interested in the following problem that minimizes the total charging cost while ensuring the fulfillment of charging tasks for all EVs:

$$\min_{x} \quad \sum_{i \in \mathbb{I}} \sum_{t \in \mathbb{T}} c^t x_i^t + \alpha \max_{t \in \mathbb{T}} \left\{ \sum_{i \in \mathbb{I}} x_i^t \right\} \tag{4a}$$

$$\text{s.t.} \quad (1)(2)(3) \tag{4b}$$

where $x := (x_i^t, i \in \mathbb{I}, t \in \mathbb{T})$. Suppose all EV information $(\pi_i, i \in \mathbb{I})$ and price information $(c^t, t \in \mathbb{T})$ over a billing cycle are known, solving problem (4) leads to a globally minimal total cost for the CS. Clearly, (4) can be readily solved since it can be transformed into a linear programming easily as shown in the next subsection.

However, (4) is not implementable in practice where future EV information are not available for each current time slot. Therefore, we pursue an online algorithm to solve an approximate of this global problem, which is simple, effective and implementable.

B. Online Solution

To solve the above problem (4) in an online fashion, there are two key issues. *First*, global information including EVs' arrival times and electricity prices in the future are unknown. *Second*, the posterior peak load over a billing cycle is unknown. We introduce a sliding time window mechanism to handle the first issue and the second issue is tackled by predicting locally the global peak load over a billing period.

For convenience, we define $\mathbb{I}_t$ as the set of current EVs in the CS at time t and $\mathbb{T}_t$ as the scheduling time window. Fig. 2 shows an example for better understanding of

$\mathbb{I}_t$ and $\mathbb{T}_t$, where $\mathbb{I}_t = \{EV1, EV2, EV3\}$ and $\mathbb{T}_t = \{t : t_3 \le t \le t_{11}\}$. The sliding time window mechanism

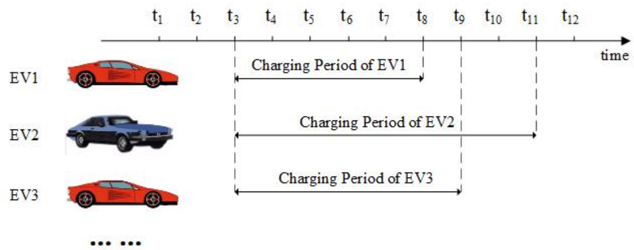


Fig. 2: Illustration of the sliding window mechanism

means that we only consider the current information of EVs in the set $\mathbb{I}_t$ and the time window $\mathbb{T}_t$. As the time window moves on, we re-solve the current local optimization problem (4) to update the charging rates to implement whenever a new EV comes to the CS or a latest electricity price is released.

Note that the cost depends on the peak load over a billing cycle. Let v_{pv} be the previous peak load and v_{pd} be predicted peak load over a billing cycle. To circumvent the difficulty of characterizing the exact peak load, we instead utilize the current peak load and the predicted one from historical data.

As a result, we approximate the global problem (4) by problem (5). Since we only have the current EV information in hand, i.e., $\{\pi_i, i \in \mathbb{I}_t\}$, thus $\max_{t\in\mathbb{T}_t}\left\{\sum_{i\in\mathbb{I}} x_i^t\right\}$ can no longer represent the peak load over the whole billing cycle.

$$\min_{x,v} \quad \sum_{t\in\mathbb{T}_t} c^t \sum_{i\in\mathbb{I}_t} x_i^t + \alpha v \tag{5a}$$

$$\text{s.t.} \quad \sum_{i\in\mathbb{I}_t} x_i^t \le P, \quad \forall t \in \mathbb{T}_t \tag{5b}$$

$$e_i \le \sum_{t\in\mathbb{T}_t} x_i^t, \quad \forall i \in \mathbb{I}_t \tag{5c}$$

$$\sum_{i\in\mathbb{I}_t} x_i^t \le v, \quad \forall t \in \mathbb{T}_t \tag{5d}$$

$$v_0 \le v \tag{5e}$$

$$\begin{aligned} & x_i^t \in [0, x_{max}], \quad \forall t \in [a_i, d_i], \forall i \in \mathbb{I}_t \\ & x_i^t = 0, \quad \forall t \in \mathbb{T}_t \backslash [a_i, d_i], \forall i \in \mathbb{I}_t, \end{aligned} \tag{5f}$$

where v_0 is a constant summarizing the peak load so far and the predicted global peak load as $v_0 := max\{v_{pv}, v_{pd}\}$. We introduce an auxiliary scalar variable v to serve as a proxy for the global peak load. The advantages of this approximation are as follows:

1) (5) is a linear programming problem, which have been well studied and easy to solve.
2) The constraints (5d) and (5e) circumvent the difficulty of characterizing the exact peak load in a billing cycle. For example, if $\sum_{i\in\mathbb{I}_t} x_i^t \le v_0,\ \forall t \in \mathbb{T}_t$, it means the constraint (5d) is redundant and the second term of (5a) is constant as αv_0 and thus negligible.
3) Since the peak load in a billing cycle is relatively easy to predict compared with each EV i's information π_i, we can readily obtain v_{pd} from the historical data. It enables us to make full use of the information that are easy to obtain. Besides, the model is simple to implement in practice.

III. Optimization of Charging Time and Implementation of OTCSA

We define z^t as the total charging load of the CS at time t and $z^{t^*} = \sum_{i\in\mathbb{I}_t} x_i^{t^*}$, where $x_i^{t^*}$ is the optimal charging rate of EV i at time t obtained from solving (5). Note that the value of (5a) is only related to z^t, thus the problem (5) may actually have multiple explicit solutions in terms of x_i^t. For example, as shown in Fig. 3, suppose there are two EVs $\mathbb{I}_t = \{1, 2\}$ and two time slot $\mathbb{T}_t = \{t1, t2\}$ with the same price $c^{t1} = c^{t2}$. Say the solution to (5) is $z^{t1^*} = z^{t2^*} = 20$, then obviously $x_1^{t1} = x_1^{t2} = x_2^{t1} = x_2^{t2} = 10$ denoted as solution 1 and $x_1^{t1} = x_2^{t2} = 20, x_1^{t2} = x_2^{t1} = 0$ denoted as solution 2 are two solutions with the same total charging cost. However, the total charging time they require is different, i.e., EV1 can fulfil the task ahead of schedule.

Fig. 3: Illustration of different solutions with the same cost but the different total charging time

This simple example motivates us to search for the *most time efficient* solution among the optimal solutions from (5). We reschedule the charging rates of EVs to save the total charging time while fixing the charging cost at minimum, which can improve the time efficiency of CSs and EV owners' satisfaction. Finally we summarize the implementation of our two-stage algorithm.

A. Optimization of Charging Time

Our goal is to minimize the total EV charging time while maintaining the optimal cost of (5). This will improve the time efficiency of the CS and EV owners' satisfaction. To this end, we are interested in the following problem:

$$\max_{x} \quad \sum_{t\in\mathbb{T}_t} \sum_{i\in\mathbb{I}_t} \frac{1}{e_i}(d_i - t) x_i^t \tag{6a}$$

$$\text{s.t.} \quad \sum_{i\in\mathbb{I}_t} x_i^t = z^{t*}, \quad \forall t \in \mathbb{T}_t \tag{6b}$$

$$e_i \le \sum_{t\in\mathbb{T}_t} x_i^t, \quad \forall i \in \mathbb{I}_t \tag{6c}$$

$$\begin{aligned} & x_i^t \in [0, x_i^{max}], \quad \forall t \in [a_i, d_i], \forall i \in \mathbb{I}_t \\ & x_i^t = 0, \quad \forall t \in \mathbb{T}_t \backslash [a_i, d_i], \forall i \in \mathbb{I}_t. \end{aligned} \tag{6d}$$

Note that $\frac{1}{e_i}(d_i - t)$ decreases linearly in t, and a small energy demand e_i means a big weight $\frac{1}{e_i}(d_i - t)$. Consequently the objective function (6a) encourages that the CS completes charging tasks of EVs as soon as possible and prioritizes EVs with small energy demand.

Lemma 1. *The problem* (6) *is always feasible, and the optimal charging cost calculated from the first stage remains unchanged.*

Proof. First, it is easy to argue that there is at least one feasible solution to the problem (6). Note that the optimal solution $\{x_i^{t^*}, i \in \mathbb{I}_t, t \in \mathbb{T}_t\}$ to the problem (5) is always a feasible solution to the problem (6), since $\{x_i^{t^*}, i \in \mathbb{I}_t, t \in \mathbb{T}_t\}$ satisfy all the constraints (6b)-(6d). Then the lemma follows as the constraint (6a) fix the optimal charging cost of (5) which is only related to z^t. □

B. Implementation of OTCSA

Here is an overview of the implementation of the two-stage OTCSA. First, we calculate z^{t^*} by solving the problem (5) in the first stage. Then, we solve problem (6) in the second stage to obtain the most time efficient charging rate $\{x_i^t, i \in \mathbb{I}_t, t \in \mathbb{T}_t\}$. The energy demand e_i of EV i is updated as follows:

$$e_i = \begin{cases} 0, & if\ EV\ i\ finishes\ charging \\ e_i, & if\ EV\ i\ arrives \\ e^i - x_i^t, & otherwise \end{cases} \tag{7}$$

OTCSA is summarized in Algorithm 1.

Algorithm 1 OTCSA

Input: c^t, $\{\pi_i \mid i \in \mathbb{I}_t\}$, P, v_0
Output: $x^* := \{x_i^{t*} \mid i \in \mathbb{I}_t, t \in \mathbb{T}_t\}$

```
for t = 1 to T do
    Set a_i = t for EV i currently in the CS, and
    determine I_t, T_t based on time t.
    if  an EV arrives then
            Solve the first-stage optimization problem
        (5) to obtain the optimal {z^{t*} | t ∈ T_t}.
            Solve the second-stage optimization problem
        (6) to obtain the optimal x*.
    end if
    if x is not empty then
        Update {e_i, i ∈ I_t} according to (7).
    end if
end for
```

IV. Simulation Results

We use the real-world charging data from January 2016 to March 2018 provided by the South China Charging Technology Co., Ltd to evaluate the performance of OTCSA.

As shown in Fig. 4, the number of EVs to be charged varies in time. In 12:00-24:00, it is likely that congestion and heavy loads of CSs would occur due to high demand. We define l_i as EV i's laxity:

$$l_i := \frac{(d_i - a_i) * x_i^{max}}{e_i}.$$

If $l_i = 1$ then EV i's energy demand can be satisfied only if it is charged at its peak charging rate from a_i to d_i. If $l_i < 1$ then it is impossible to satisfy EV i's energy demand before its departure. $l_i > 1$ means there is flexibility with EV i's charging task. As is shown in Fig. 5, the average laxity of the real-world charging data is 2.51 and the variance is 1.41. The proportion of charging tasks with $l_i = 1$ is less than 3%, therefore there is enough flexibility to exploit, and OTCSA is expected to reduce the charging cost, improve time efficiency and operational stability.

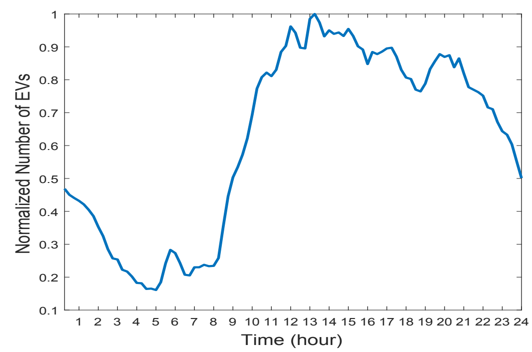


Fig. 4: Normalized Number of EVs

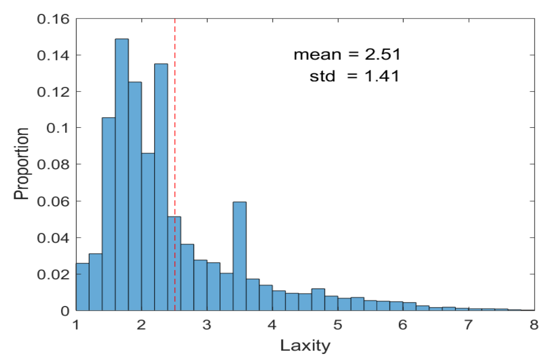


Fig. 5: EVs' Laxity

The electricity price of a large CS in a city like Shenzhen, China [14], consists of two components: TOU electricity prices for the energy purchased from the grid and the peak price to charge the peak load over a billing cycle, i.e., a month. As shown in Fig. 6, TOU electricity prices are divided into three categories: the off-peak price 0.36 CNY/kWh, shoulder price 0.73 CNY/kWh and on-peak price 1.08 CNY/kWh. The peak price is 44 CNY/kW. For convenience, we consider a day of 24 h as a billing period, where each time slot lasts 15 minutes. The peak price is then correspondingly transformed from 44 CNY/kW to 1.446

CNY/kW by assuming that there are 30 days in a month, i.e., α=44/30=1.446 CNY/kW. The peak load over a billing cycle is predicted by calculating the average of the peak load based on historical data. All the convex optimization programs are solved by CVX [15], [16] with Matlab 2016b on a computer with 3.4GHz intel Core i5-7500 CPU and 8 GB memory.

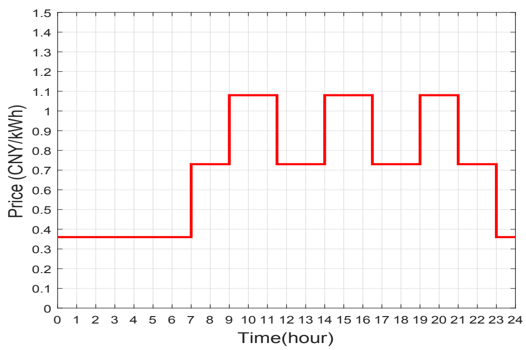


Fig. 6: Illustration of TOU electricity price

we compare the following scheduling strategies:

1) EC: fast charging scheme, which means EV i is charged at the maximum charging rate until the charging task is completed.
2) OFC: The offline optimal charging scheme (4), which achieves the globally minimal total cost by assuming full knowledge of future information.
3) FSC1: The first stage online optimal charging scheme (5) without predicting the peak load over a billing cycle, i.e., $v_{pd} = 0$.
4) FSC2: The first stage online optimal charging scheme (5) with predicting the peak load v_{pd} over a billing cycle.
5) OTCSA: The online two-stage charging scheduling algorithm as shown in Algorithm 1.

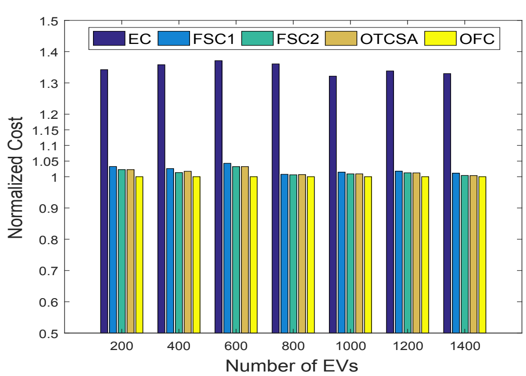


Fig. 7: Illustration of charging cost comparison under different algorithms

We evaluate the charging costs with different numbers of EVs in the CS. The corresponding results for different algorithms are shown in Fig. 7, where the charging costs are normalized with respect to the benchmark of OFC. EC has much higher charging cost due to the lack of scheduling. $0.77\% - 4.29\%$ difference in cost is shown with different numbers of EVs between FSC1 and OFC, while $0.4\% - 3.23\%$ difference in cost is shown with different numbers of EVs between FSC2 and OFC. The difference between FSC2 and OTCSA is much smaller compared with OTCSA or FSC2. The average cost is very close for OTCSA and FSC2, less than 5.05 CNY. *Lemma 1* does not hold in an online setting since the states of charge of EVs are updated in different ways that lead to different overall charging trajectories.

OTCSA has a cost almost the same as FCS2. However, a great amount of charging time is saved, as shown in Fig. 8, due to the second-stage rescheduling. As shown in Fig. 9, when the number of EVs is 1400, OTCSA can reduce the peak load by 37% compared to EC, and the load profile of OFC is similar to that of OTCSA.

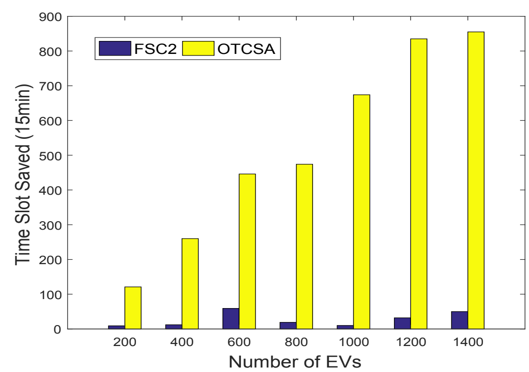


Fig. 8: Illustration of the time slot saved by adding the second optimization

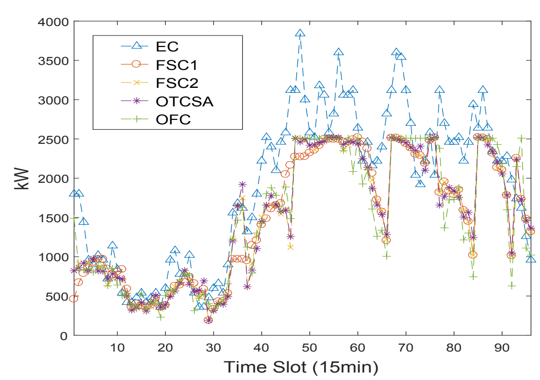
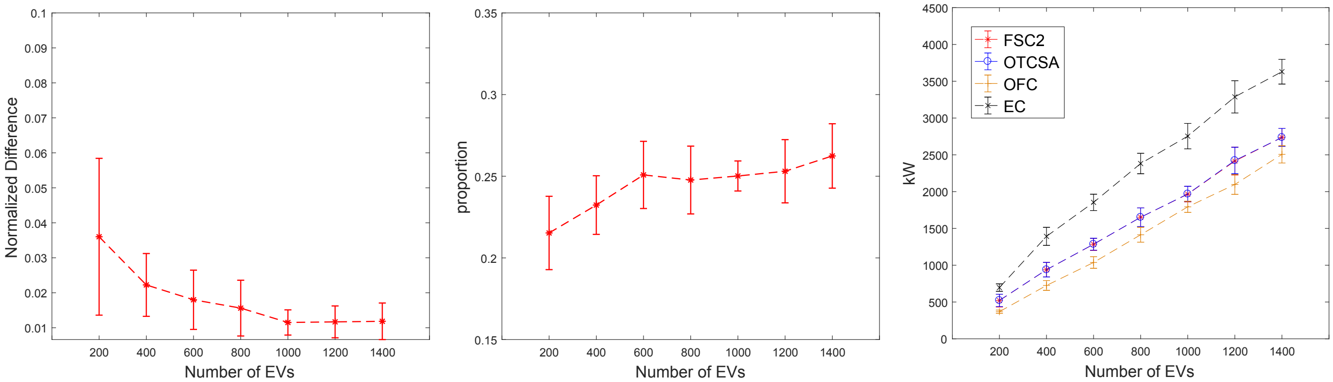


Fig. 9: Illustration of charging power trajectories over a day under different algorithms

We simulate 50 cases for a fixed number of EVs to verify the well performance stability of OTCSA. Fig. 10(a) shows that the normalized charging cost difference between OFC



(a) (b) (c)

Fig. 10: Impact of EV number on (a) normalized cost difference between OFC and OTCSA, (b) proportion of time slots saved of OTCSA more than those of FSC2 to the total charging time, (c) maximum peak power over a billing period

and OTCSA, as the number of EVs increases, tends to smaller and fluctuates very little. The proportion of time slots saved of OTCSA more than those of FSC2 to the total charging time slots is approximately 0.25 as shown in Fig. 10(b). We can see from Fig. 10(c), the maximum peak power over a billing period can be greatly reduced by OFC or OTCSA, and OTCSA has almost no effect on peak power compared to FSC2.

V. Conclusion

Charging cost, time efficiency and lowering the maximum peak power are the main concerns of CSs. To deal with the tariff that additionally charges the peak demand of a CS, we propose OTCSA to minimize the charging cost and time of EVs in two stages. In the first stage, we minimize the charging cost in an online fashion while ensuring that all energy demands of EVs can be fulfilled before departure. In the second stage, we save the charging time of EVs while maintaining the minimal charging cost. Numerical results based on real-world data show that OTCSA achieves a charging cost close to the offline optimum, tremendously reduces the peak load and saves a great amount of charging time.

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