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Counterfactual analysis of default bid market power mitigation strategies in two-stage electricity markets

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ABSTRACT

Market power remains a persistent challenge in many liberalized electricity markets worldwide, driving the adoption of ex-ante and ex-post mitigation measures. Despite locational mitigation tools (e.g., cost-based reference levels or default energy bids), evidence of price manipulation has motivated system-level market power mitigation (MPM) policies. However, the full implications of these rules are not well understood, and limited insight into participant behavior can lead to unintended consequences, including increased market power and welfare losses. We study sequentially cleared electricity markets and analyze a two-stage settlement structure commonly used by system operators (e.g., day-ahead and real-time markets in North America). Our focus is on MPM policies that replace noncompetitive generator offers with operator-estimated default bids, and we model competition between generators and loads with inelastic energy requirements who act strategically in allocating demand across stages under real-time, day-ahead, and simultaneous applications of MPM policies. Motivated by the loss of Nash equilibrium under conventional supply-function bidding, we adopt an alternative mechanism in which generators bid the intercept of an affine supply function. Under real-time MPM, strategic interaction in the day-ahead market drives all demand to real time, producing an undesirable outcome. To test robustness, we incorporate demand uncertainty using a variance-penalized expectation framework. Low risk aversion still leads to substantial real-time clearing, while imbalances in risk preferences further amplify market power. Overall, intercept-function bidding combined with day-ahead and simultaneous MPM policies mitigates generator market power more effectively than real-time substitution alone, although these policies shift some market power toward loads.

1. Introduction

Electricity markets are well known for the persistent exercise of market power, which has continued to appear across regions despite decades of restructuring and liberalization. In North America, empirical studies have shown that even structurally unconcentrated markets experienced capacity withholding and prices above competitive levels (Borenstein et al., 1999; Qu, 2007). Similar behavior has been documented in Europe: Sweeting (2007) found evidence of tacit collusion in the England and Wales pool, while price deviations linked to strategic conduct have been reported in the Spanish (Fabra & Toro, 2005) and German (Müsgens, 2004) wholesale markets. In Australia's National Electricity Market (NEM), generators engaged in rebidding strategies–first withholding capacity to influence early dispatch, then re-entering capacity to exploit settlement prices averaged over multiple dispatch intervals (Dungey et al., 2018). These examples illustrate a wide range of strategic behaviors that can distort prices and reduce social welfare.

To limit such behavior, system operators employ a variety of market power mitigation strategies. European markets, including the UK and Nord Pool, primarily rely on ex-post behavioral monitoring and regulatory investigations (European Union, 2011; Kemp et al., 2018), emphasizing detection and enforcement after the fact. In contrast, North American markets generally employ ex-ante structural measures, such as pivotal-supplier tests and offer caps. For instance, PJM applies a three-pivotal-supplier test and replaces noncompetitive offers with cost-based mitigated offers (LLC, 2024, §2.3.6.1); ISO New England substitutes offers that fail structural or constrained-area tests with unit-specific reference levels (England, 2025, III.A.5.5); and both MISO (Operator, 2013, §64.1.3) and NYISO (Operator, 2025, §23.4.2) utilize reference-level frameworks to benchmark and mitigate submitted offers. Although

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these approaches differ in design, their effectiveness ultimately hinges on how strategic participants adjust their behavior in response. An incomplete understanding of these incentive effects can lead to mitigation designs that unintentionally create new opportunities for exercising market power.

To address these concerns, this paper develops a counterfactual analysis of how strategic participants adapt their behavior under system-level market power mitigation (MPM) policies in a sequential electricity market. We consider a two-stage settlement process that abstracts the forward-spot interaction present in many market designs, and focus on MPM schemes based on default-bid substitution, whereby noncompetitive generator offers are replaced with operator-estimated cost-based bids. While default-bid and reference-level substitution appear in localized, constraint-based form in several U.S. ISOs, CAISO has recently considered extending this principle to the system level (Operator, 2020; Servedio, 2019; Servedio et al., 2020). This initiative is motivated by periods of system-wide market power–identified through residual supply index tests–that cannot be addressed by local, congestion-based mitigation alone, suggesting that similar circumstances could arise for other system operators. Understanding the strategic effects of such system-level policies is complicated by the behavior of the demand side. Even when energy requirements are inelastic, loads behave strategically in sequential markets by allocating demand across stages in response to anticipated prices, and this intertemporal choice can fundamentally alter market outcomes. Previous work (Bansal et al., 2023) showed that under conventional slope-based supply-function bidding, the interaction between strategic loads and strategic generators may lead to the nonexistence of a Nash equilibrium in two-stage markets. This illustrates the type of subtleties that arise when analyzing system-level mitigation rules: strategic responses by loads can destabilize standard bidding models and obscure the incentives created by substitution. These challenges motivate the use of an alternative bidding mechanism that guarantees equilibrium existence while preserving the ability of both generators and loads to participate strategically.

In this paper, we adopt intercept function bidding (Baldick et al., 2004; Chen et al., 2021; Hobbs et al., 2000) as such a mechanism. We model competition between generators and loads with inelastic energy requirements in a two-stage settlement electricity market, where each generator bids the intercept of an affine supply function to maximize profit across both stages, while loads bid demand quantities and seek to minimize their total payment. Within this framework, we study how system-level default-bid substitution affects the resulting equilibria. Since the market operator can estimate generation costs with reasonable accuracy (Servedio et al., 2020), we assume that executing the default-bid MPM policy in either stage substitutes noncompetitive generator bids in that stage with an estimate of their true marginal cost. The resulting strategic behavior depends critically on the stage in which bid substitution is applied.

When substitution occurs only in the real-time market (a real-time MPM policy), generators behave as price-takers in real time, and all strategic interaction shifts to the day-ahead stage, yielding a two-stage Nash game between generators and loads. When substitution occurs in the day-ahead market instead, generators bid truthfully in day-ahead while loads choose their day-ahead allocations strategically, with generators responding in real time; this produces a multi-leader-follower structure in which loads act as leaders and generators as followers, together with within-group Nash competition, consistent with Stackelberg-Nash formulations in related markets (Carvalho et al., 2024; Li et al., 2020). Finally, when substitution is applied in both stages (a simultaneous MPM policy), generators behave truthfully in both markets and loads compete in quantities, resulting in a Nash-Cournot game.

Contributions. The main contributions of this paper are threefold.

1. *A tractable equilibrium framework for sequential electricity markets with strategic demand.* We develop a two-stage equilibrium model in which generators bid intercepts of affine supply functions and loads strategically allocate demand across stages. This formulation guarantees the existence of Nash equilibria under broad conditions and enables closed-form analysis of strategic behavior in settings where conventional slope-based supply-function bidding fails to admit equilibrium. The resulting characterization of the standard (no-MPM) market provides a baseline against which the effects of mitigation policies can be systematically evaluated.
2. *A unified analytical characterization of system-level default-bid MPM policies.* We introduce a general modeling framework for system-level default-bid substitution in sequential markets and derive the resulting equilibrium outcomes for three policy designs: real-time MPM, day-ahead MPM, and simultaneous MPM. Our analysis provides the first closed-form characterization of these equilibria under intercept bidding and reveals how each policy reshapes strategic incentives across market stages. In particular, the framework highlights why real-time MPM induces an undesirable equilibrium in which all demand clears in the real-time market, whereas day-ahead and simultaneous MPM policies mitigate generator market power while preserving substantial day-ahead clearing.
3. *A risk-aware extension via variance-penalized expectations.* To assess the robustness of the undesirable equilibrium identified under real-time MPM in the deterministic model, we extend our framework to incorporate demand uncertainty and heterogeneous risk preferences using a variance-penalized expectation formulation. This stochastic extension isolates the policy for which robustness is most in question–real-time MPM–and allows us to analyze how risk aversion influences stage allocation, price formation, and market power. The analysis shows how low risk aversion preserves the deterministic outcome of predominantly real-time clearing, while higher risk aversion or asymmetric risk preferences can substantially alter incentives and amplify market power.

Related Work:

Our work advances the literature along three dimensions and provides a policy-relevant understanding of electricity market dynamics.

Market power, forward markets, and counterfactual policy analysis. A large body of work has examined the root causes of market power, its susceptibility to strategic behavior, and the role of forward markets in mitigating it. Classical studies such as Allaz and Vila (1993) and Bushnell (2007) show how forward contracting can reduce market power in single-stage settings, while Newbery (2002) demonstrates how under-contracting and insufficient capacity can lead to high real-time prices, even in unconcentrated markets. These works identify structural drivers of market power but do not study market power *mitigation rules* themselves. A smaller set of papers evaluates the effects of specific mitigation mechanisms using counterfactual analysis–for instance, virtual transactions in PJM (Long & Giacomoni, 2020) or vertical integration in the Australian NEM (Gans & Wolak, 2012). Such analyses illustrate the feasibility of policy evaluation but remain rare and focus on targeted interventions rather than system-level mitigation. Our work extends this literature by providing the first analytical counterfactual evaluation of a *system-level* default-bid MPM policy, carried out within a tractable intercept-bidding framework that guarantees equilibrium existence and enables closed-form analysis. This approach clarifies how system-level substitution reshapes strategic incentives in sequential markets–linking classical insights on forward markets and price caps to practical mitigation mechanisms–and shows how participants may adapt their strategies under default-bid substitution (Wu et al., 2023). The framework further highlights that default-bid policies do not inherently limit demand-side market power, and enables system operators to anticipate strategic responses and assess impacts on prices, welfare, and efficiency.

Strategic demand as a critical aspect of market power analysis. A further important aspect of the literature concerns the treatment of demand in electricity-market games. Classical models typically treat demand as passive–either exogenous or merely price-responsive–while strategic behavior is modeled predominantly on the generation side. Early

Bertrand (Hobbs, 1986) and Cournot (Allaz & Vila, 1993) models, as well as the supply function equilibrium (SFE) framework (Klemperer & Meyer, 1989) and its extensions (Anderson & Hu, 2008; Bushnell, 2007), provide insights into strategic supply behavior but largely omit active demand-side participation. More recent work has begun to incorporate strategic loads, either through explicit price-quantity bidding or through intertemporal allocation decisions in sequential markets. For example, You et al. (2019a) analyze strategic inelastic demand, while Emami et al. (2022) study demand-function equilibria–analogous to SFE–showing that strategic demand can amplify price spreads and reduce efficiency. Our work builds on and extends this aspect of the literature by treating demand as an active strategic participant in a sequential market with market-power mitigation. By allowing loads to choose *when* to buy (day-ahead versus real-time), even with inelastic energy requirements, we show that mitigation policies targeting only generators can unintentionally shift market power to the demand side. This highlights the importance of modeling strategic demand explicitly when evaluating the effectiveness of system-level MPM policies.

Risk aversion and its influence on market incentives. A third aspect of the literature examines how risk aversion shapes strategic behavior in electricity markets subject to demand fluctuations, renewable uncertainty, and price volatility. Stochastic optimization models–often using Conditional Value-at-Risk (CVaR)–show that risk preferences influence bidding, forward contracting, and equilibrium prices. For example, Kazempour and Pinson (2016) demonstrate that CVaR-based risk aversion widens price spreads in a two-stage market with renewable uncertainty, while Murphy and Smeers (2010) extend the classical framework of Allaz and Vila (1993) to include capacity constraints and uncertain demand, showing that risk can weaken the power-mitigating effects of forward contracting. Evidence on how risk preferences interact with market design or policy interventions is comparatively limited. One of the few policy-focused studies, Downward et al. (2016), shows that risk-averse participation can interact with asset-transfer policies in nuanced ways, sometimes increasing and sometimes reducing wholesale prices. Our work contributes to this literature by introducing heterogeneous risk preferences into a sequential market with system-level market power mitigation. Using a variance-penalized expectation framework, we incorporate risk sensitivity for both generators and strategic loads and analyze how risk aversion modifies incentives under default-bid substitution. This extension reveals how risk preferences can amplify or dampen market power and materially affect the performance of mitigation policies–an aspect that has received little attention in prior counterfactual policy analysis.

Finally, we position this paper within our broader research agenda on market-power mitigation. Our earlier work (Bansal et al., 2022, 2023) established some of the first analytical foundations for studying default-bid MPM policies. Bansal et al. (2022) analyzed a day-ahead MPM policy, offering initial insights into how default-bid substitution affects strategic behavior in a forward market. Bansal et al. (2023) extended this analysis to real-time mitigation using a deterministic slope-based supply-function framework and highlighted the essential role of strategic demand. These studies examined individual policies in isolation and did not provide a unified treatment of day-ahead, real-time, and simultaneous MPM designs, or incorporate uncertainty. Building on this foundation, the present work develops a tractable two-stage equilibrium framework that integrates all three policies and introduces both intercept bidding and a risk-aware extension to systematically assess their strategic implications.

Paper Organization: The rest of the paper is structured as follows. In Section 2, we formulate the social planner problem, describe the two-stage market, and define participants' behavior. In Section 3, we characterize the market equilibrium in a standard market based on intercept bidding. We model MPM policies and characterize the market equilibrium for different participation behaviors in Section 4. We provide insights on the market outcome in a market with MPM policy and compare it with the standard market in Section 5. To streamline the presentation, we relegate the comparison of the intercept bid-based standard market with the slope bid-based standard market to Appendix A. In Section 6, we discuss the variance penalized expectation framework and further investigate the real-time MPM policy. Finally, conclusions are in Section 7.

Notation: The standard notation $f(x, y)$ denotes a function of independent variables x and y. We use $f(x; y)$ to represent a function of an independent variable x and a parameter y. Also, $|\mathcal{I}|$ represents the cardinality of the set $\mathcal{I}$.

2. Electricity market clearing

In this section, we formulate the underlying social planner problem and then describe the standard two-stage settlement electricity market design, and define participants' behavior. Finally, we define a general market equilibrium in such a market setting.

2.1. Social planner problem

Consider a single-interval two-stage settlement electricity market where a set $\mathcal{G}$ of generators compete with a set $\mathcal{L}$ of inelastic loads. The power dispatch of generator j over the two stages is denoted by $g_j \in \mathbb{R}$ such that

$$g_j := g_j^d + g_j^r \tag{1}$$

where $g_j^d \in \mathbb{R},\ g_j^r \in \mathbb{R}$ denote the dispatch in the two stages, i.e., day-ahead and real-time markets, respectively. In this paper, we use the superscripts d and r to denote the decision variables and market parameters associated with the day-ahead and real-time markets, respectively. The total inelastic demand of load l, denoted by $d_l \in \mathbb{R}^+$, is allocated across two market stages:

$$d_l := d_l^d + d_l^r \tag{2}$$

where $d_l^d \in \mathbb{R},\ d_l^r \in \mathbb{R}$ represent the amounts of load allocated in the day-ahead and real-time markets, respectively. Further, the total inelastic demand across all loads, denoted as $d \in \mathbb{R}^+$, is obtained by summing the individual demands over all loads $l \in \mathcal{L}$, i.e.,

$$d := \sum\nolimits_{l\in\mathcal{L}} d_l \tag{3}$$

The market operator seeks to achieve supply-demand balance, i.e.,

$$\sum\nolimits_{j\in\mathcal{G}} g_j = d \tag{4}$$

The social planner problem that seeks to minimize the cost of dispatching generators to meet aggregate demand is given by:

$$\min_{g_j, j\in\mathcal{G}} \sum\nolimits_{j\in\mathcal{G}} \frac{c_j}{2} g_j^2 \quad \text{s.t. (4)} \tag{5}$$

where we assume a quadratic cost of dispatching generators, parameterized by quadratic coefficients $c_j \in \mathbb{R}^+$. The underlying social planner problem (5) is considered a benchmark, and we will analyze the deviation between market equilibrium and the social planner solution as one of the metrics to study market power.

2.2. Two-stage market mechanism

We now describe a standard two-stage market clearing, as shown in panel (a) of Fig. 1.

Day-ahead Market

Each generator j submits an intercept function, with constant slope $b^d \in \mathbb{R}^+$ and parameterized by $\beta_j^d \in \mathbb{R}$, that indicates the willingness of the generator to participate in the market, given by:

$$g_j^d = b^d \lambda^d - \beta_j^d, \tag{6}$$

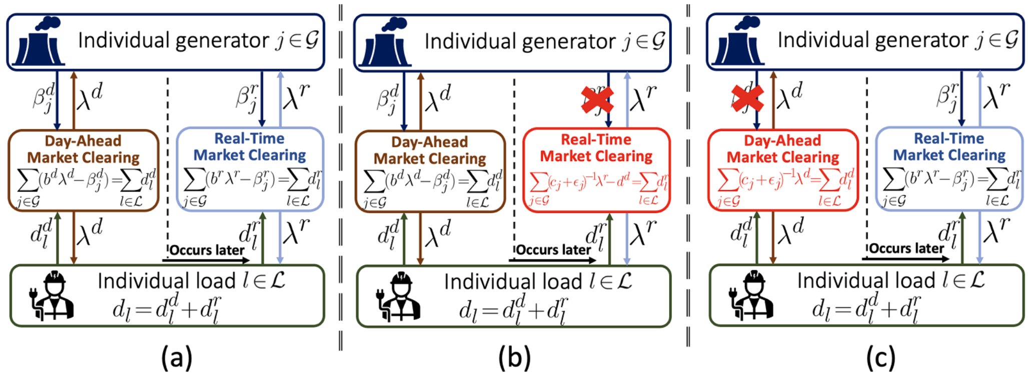

Fig. 1. Two-stage market mechanism in (a) a standard market, (b) a market with a real-time MPM policy, and (c) a market with a day-ahead MPM policy.

where λ^d denotes the day-ahead price. Each load $l \in \mathcal{L}$ in the day-ahead market bids quantity d_l^d. Once all the bids (β_j^d, d_l^d) are received, the market clears with supply-demand balance:

$$\sum_{j\in\mathcal{G}} \left(b^d \lambda^d - \beta_j^d\right) = \sum_{l\in\mathcal{L}} d_l^d. \tag{7}$$

The solution to (7) gives the dispatch and clearing price such that generator j earns $\lambda^d g_j^d$ while load l pays $\lambda^d d_l^d$ in the market settlement process.

Real-time Market

Similar to the day-ahead market, each generator j submits an intercept function, with constant slope $b^r \in \mathbb{R}^+$ and parameterized by $\beta_j^r \in \mathbb{R}$, as:

$$g_j^r = b^r \lambda^r - \beta_j^r, \tag{8}$$

where λ^r denotes the real-time prices. Each load $l \in \mathcal{L}$ in real-time market bids quantity d_l^r. The load allocation in the real-time market is given once the load allocation in the day-ahead market is determined due to the demand inelasticity and (2). Once all the bids (β_j^r, d_l^r) are received, the market clears with supply-demand balance, given by

$$\sum_{j\in\mathcal{G}} \left(b^r \lambda^r - \beta_j^r\right) = \sum_{l\in\mathcal{L}} d_l^r. \tag{9}$$

The solution to (9) determines the dispatch and clearing price such that generator j earns $\lambda^r g_j^r$ while load l pays $\lambda^r d_l^r$ in the market settlement process.

2.3. Participant behavior

We focus on two different forms of participation behavior, i.e., price-taking and price-anticipating, where each generator j (load l) seeks to maximize (minimize) its profit (payment) in the two-stage market. The profit of generator j, denoted by π_j, is given by:

$$\pi_j(g_j^d, g_j^r, \lambda^d, \lambda^r) := \lambda^r g_j^r + \lambda^d g_j^d - \frac{c_j}{2}(g_j^d + g_j^r)^2 \tag{10}$$

Similarly, the payment of load l, denoted by ρ_l, is given by:

$$\rho_l(d_l^d, d_l^r, \lambda^d, \lambda^r) := \lambda^d d_l^d + \lambda^r d_l^r = \lambda^d d_l^d + \lambda^r (d_l - d_l^d) \tag{11}$$

where we substitute the load inelasticity constraint (2).

Price-taking Participation

We first discuss the price-taking participant behavior and then formulate the individual problems of participants. A participant is price-taking in the market if it does not anticipate the impact of its bid on the market prices and accepts the existing prices as given. Given the day-ahead and real-time prices (λ^d, λ^r) in the market, the individual problem of price-taking generator j is:

$$\max_{g_j^d, g_j^r} \ \pi_j(g_j^d, g_j^r; \lambda^d, \lambda^r) \tag{12}$$

and the individual problem of price-taking load l is given by:

$$\min_{d_l^d} \ \rho_l(d_l^d; \lambda^d, \lambda^r) \tag{13}$$

Price-anticipating (Strategic) Participation

We now discuss the price-anticipating participant behavior. A participant is price-anticipating (strategic) in the two-stage market if it can manipulate the prices by anticipating the impact of its bid and other participants' bids in two stages. Given load bids $d_l^d, d_l^r, l \in \mathcal{L}$, and other generators' bids $\beta_k^d, \beta_k^r, k \in \mathcal{G}, k \neq j$, the individual problem of a price-anticipating generator j is given by:

$$\max_{g_j^d, g_j^r} \ \pi_j\left(g_j^d, g_j^r, \lambda^d\left(g_j^d; \overline{g}_{-j}^d, d^d\right), \lambda^r\left(g_j^r; \overline{g}_{-j}^r, d^r\right)\right) \ \text{ s.t. } (7), (9) \tag{14}$$

where $\overline{g}_{-j}^d := \sum_{k\in\mathcal{G}, k\neq j} g_k^d$, and $\overline{g}_{-j}^r := \sum_{k\in\mathcal{G}, k\neq j} g_k^r$. Similarly, the individual problem for price-anticipating load l is given by:

$$\min_{d_l^d} \ \rho_l\left(d_l^d, \lambda^d\left(d_l^d; g_j^d, \overline{d}_{-l}^d\right), \lambda^r\left(d_l^d; g_j^r, \overline{d}_{-l}^r\right)\right) \ \text{ s.t. } (7), (9) \tag{15}$$

where $\overline{d}_{-l}^d := \sum_{l\in\mathcal{L}, k\neq l} d_l^d$, $\overline{d}_{-l}^r := \sum_{l\in\mathcal{L}, k\neq l} d_l^r$.

2.4. Market equilibrium

In this section, we describe the notion of market equilibrium in a two-stage settlement electricity market. In the market, firms make decisions in their best interest without accounting for others' incentives. However, at the equilibrium, the resulting prices are such that the market achieves the supply-demand balance, and no participating firm has any incentive to deviate from its bid. More formally,

Definition 1. A two-stage market is at equilibrium if the participant bids and market clearing prices $(\beta_j^d, \beta_j^r, j \in \mathcal{G}, d_l^d, d_l^r, l \in \mathcal{L}, \lambda^d, \lambda^r)$ in the day-ahead and real-time markets satisfy:

1. The bid β_j^d, β_j^r of generator j maximizes its profit.
2. The allocation d_l^d, d_l^r of load l minimizes its payment.
3. The market clears with prices λ^d given by (7) and λ^r given by (9).

An equilibrium analysis of the market is often used to understand the presence of market power and stability of a market mechanism. Though equilibrium is hard to attain in reality due to the dynamic nature of the market, descriptive and predictive equilibrium outcomes (if possible) provide intuition about the behavior of individual participants (Starr, 2011) and their interplay. We use equilibrium analysis in this paper to analyze the impact of system-level MPM policies on market outcomes.

3. Equilibrium in standard market

In this section, we model the competition between generators and loads in a standard two-stage market without any mitigation policy. The participants bid in both day-ahead and real-time markets. Such a game essentially forms a bi-level game, with the real-time market at the lower level and day-ahead market at the upper level. We analyze such a game backward, starting from the real-time market, for the equilibrium path. The resulting equilibrium is regarded as a benchmark to determine the impact of the system-level MPM policies later.

Competitive Equilibrium

We first consider the case of price-taking participants in the market. We substitute (6) and (8) into (12) to get the individual problem of generator j, given the prices (λ^d, λ^r), as:

$$\max_{\beta_j^d, \beta_j^r} -\beta_j^d \lambda^d - \beta_j^r \lambda^r - \frac{c_j}{2}(\beta_j^d + \beta_j^r)^2 + c_j(b^d \lambda^d + b^r \lambda^r)(\beta_j^d + \beta_j^r) \tag{16}$$

The individual problem of load l is given in the optimization problem (13). We can now characterize the competitive equilibrium in this market setting:

Theorem 1. *A competitive equilibrium in a standard two-stage settlement market without any mitigation policy exists and is given by*

$$\beta_j^d + \beta_j^r = \frac{b^d + b^r - c_j^{-1}}{\sum_{k \in \mathcal{G}} c_k^{-1}} d, \ \forall j \in \mathcal{G} \tag{17a}$$

$$\sum_{j \in \mathcal{G}} (b^d \lambda^d - \beta_j^d) = \sum_{l \in \mathcal{L}} d_l^d, \ \sum_{j \in \mathcal{G}} (b^r \lambda^r - \beta_j^r) = \sum_{l \in \mathcal{L}} d_l^r, \ d_l^d + d_l^r = d_l, \ \forall l \in \mathcal{L} \tag{17b}$$

$$\lambda^d = \lambda^r = \frac{1}{\sum_{j \in \mathcal{G}} c_j^{-1}} d \tag{17c}$$

We provide the proof of the theorem in Appendix B. The competitive equilibrium in Theorem 1 exists non-uniquely, i.e., each load l is indifferent to demand allocation due to equal prices in the two stages.

Nash Equilibrium

We next characterize the Nash equilibrium as a result of competition between price-anticipating participants. We first characterize the interaction between generators and loads in a real-time market for some given allocation in the day-ahead market. This results in a real-time subgame equilibrium that will help compute the Nash equilibrium in the two-stage market.

Theorem 2. *We assume that there is more than one strategic generator in the market, i.e., $|\mathcal{G}| > 1$. The subgame equilibrium (g_j^r, d^r, λ^r) due to the interplay between generators and loads in the real-time market, given the day-ahead market outcome (g_j^d, d_l^d), is an optimal primal-dual solution to an augmented convex social planner problem, as:*

$$\min_{g_j^r} \sum_{j \in \mathcal{G}} \left(\frac{1}{2b^r(|\mathcal{G}| - 1)} g_j^{r\,2} + \frac{c_j}{2} \left(g_j^d + g_j^r \right)^2 \right) \tag{18a}$$

$$\text{s.t.} \ \sum_{j \in \mathcal{G}} g_j^r = \sum_{l \in \mathcal{L}} d_l^r \tag{18b}$$

We provide the proof of the theorem in Appendix C. The strategic participation of generators in real-time shifts the dispatch of generators, captured by the first term in the objective function of the augmented social planner problem in Theorem 2. Since the augmented problem is strictly convex, the subgame equilibrium is unique. Moreover, the subgame equilibrium does not exist if there is only one generator in the market and prices become indefinite.

The following theorem characterizes the Nash equilibrium, where load minimizes its payment as a leader, anticipating the prices in two stages with the knowledge of others' bids. Since analyzing supply function equilibria in closed form is inherently challenging, prior literature has often relied on simplifying assumptions to gain analytical insights (Banal-Estanol & Micola, 2011; Matsui, 2016; Mousavian et al., 2020; Rudkevich et al., 1998; You et al., 2019a). In this case, we first introduce the notion of a symmetric market equilibrium, as defined below:

Definition 2. A market equilibrium that satisfies Definition 1 is said to be symmetric on the generator side if all the generators are homogeneous and make identical decisions in both stages, i.e., $\beta_j^d := \beta^d$, $\beta_j^r := \beta^r$, $\forall j \in \mathcal{G}$.

For tractability and closed-form analysis, we consider the participation of homogeneous generators and analyze the resulting symmetric market equilibrium. The following theorem characterizes the resulting symmetric Nash equilibrium in the market, where each individual generator solves (14) while each individual load solves (15).

Theorem 3. *Let's assume that generators are homogeneous, i.e., $c_j := c$, $\forall j \in \mathcal{G}$. If there is more than one generator participating in the market, i.e., $|\mathcal{G}| > 1$, then a symmetric Nash equilibrium uniquely exists and it is given by:*

$$\beta_j^d = \frac{b^d c}{|\mathcal{G}|} d + \frac{b^r c - \frac{|\mathcal{G}|-2}{|\mathcal{G}|-1}}{b^r c + \frac{|\mathcal{L}|+1}{|\mathcal{G}|-1}} \frac{|\mathcal{L}| + 1}{|\mathcal{G}|(|\mathcal{G}| - 1)} d^d, \ \beta_j^r = \frac{b^r c}{|\mathcal{G}|} d - \frac{|\mathcal{G}| - 2}{|\mathcal{G}|(|\mathcal{G}| - 1)} d^r, \quad \forall j \in \mathcal{G} \tag{19a}$$

$$g_j^d = \frac{1}{|\mathcal{G}|} d^d, \ g_j^r = \frac{1}{|\mathcal{G}|} d^r, \ \forall j \in \mathcal{G} \tag{19b}$$

$$d_l^d = \frac{b^d d_l}{b^d + b^r(|\mathcal{G}| - 1)} + \frac{\frac{b^d}{1 + b^r c(|\mathcal{G}|-1)}}{b^d + b^r(|\mathcal{G}| - 1)} d^r - \frac{b^r}{b^d + b^r(|\mathcal{G}| - 1)} d^d, \quad d_l^r = d_l - d_l^d, \forall l \in \mathcal{L} \tag{19c}$$

$$\lambda^d = \frac{b^r c(|\mathcal{G}| - 1) + 2}{b^r c(|\mathcal{G}| - 1) + 1} \frac{c}{|\mathcal{G}|} d + \frac{\left(\frac{b^r}{b^d} - 1 \right) c + \frac{1}{b^d(|\mathcal{G}|-1)}}{b^r c(|\mathcal{G}| - 1) + 1} \frac{d^d}{|\mathcal{G}|}, \tag{19d}$$

$$\lambda^r = \lambda^d + \frac{\frac{1}{|\mathcal{G}|(|\mathcal{G}|-1)} \left(\frac{|\mathcal{G}|-2}{|\mathcal{G}|-1} - b^r c \right) d}{b^d \left(b^r c + \frac{|\mathcal{L}|+1}{|\mathcal{G}|-1} \right) + b^r \left(b^r c + \frac{1}{|\mathcal{G}|-1} \right) (|\mathcal{G}| + |\mathcal{L}| - 1)} \tag{19e}$$

We provide the proof of the theorem in Appendix D. At the equilibrium, the load allocation across stages depends on the slope of the bidding function, and operators can tune these for a higher allocation in the day-ahead market. Such behavior is desirable, as observed in current market practice, with the majority of demand in the day-ahead market. More specifically, we provide such a condition on the slope of the intercept functions in Corollary 1. Moreover, for $|\mathcal{G}| = 1$, the generator makes arbitrary large bid decisions to drive prices high in the market, and the Nash equilibrium does not exist.

Corollary 1. *The load allocation across the two stages at the Nash equilibrium in a standard market (19) is given by:*

$$d^d = \frac{b^d \left(b^r c + \frac{|\mathcal{L}|+1}{|\mathcal{G}|-1} \right)}{b^d \left(b^r c + \frac{|\mathcal{L}|+1}{|\mathcal{G}|-1} \right) + b^r \left(b^r c + \frac{1}{|\mathcal{G}|-1} \right)(|\mathcal{G}|+|\mathcal{L}|-1)} d, \quad d^r = \frac{b^r \left(b^r c + \frac{1}{|\mathcal{G}|-1} \right)(|\mathcal{G}|+|\mathcal{L}|-1)}{b^d \left(b^r c + \frac{|\mathcal{L}|+1}{|\mathcal{G}|-1} \right) + b^r \left(b^r c + \frac{1}{|\mathcal{G}|-1} \right)(|\mathcal{G}|+|\mathcal{L}|-1)} d \tag{20}$$

Furthermore, for

$$b^d \geq b^r \frac{\left(b^r c + \frac{1}{|\mathcal{G}|-1} \right)(|\mathcal{G}| + |\mathcal{L}| - 1)}{\left(b^r c + \frac{|\mathcal{L}|+1}{|\mathcal{G}|-1} \right)} \implies b^d - b^r \geq b^r \frac{b^r c(|\mathcal{G}| + |\mathcal{L}| - 2) + \frac{|\mathcal{G}|-2}{|\mathcal{G}|-1}}{\left(b^r c + \frac{|\mathcal{L}|+1}{|\mathcal{G}|-1} \right)} \tag{21}$$

the load allocation in the day-ahead market is higher than in the real-time market, i.e., $d^d \geq d^r$.

Since $\mathcal{L} + \mathcal{G} \geq 2$ holds at equilibrium, the above corollary provides a lower bound on the slope of the day-ahead supply function, such that demand allocates more in the day-ahead stage at equilibrium. Specifically, if the price sensitivity of generator dispatch (slope of the intercept bid function) in the day-ahead market is sufficiently higher than in the real-time market, or if the price sensitivity in the real-time market is sufficiently low, then the load prefers to allocate more demand to the real-time market instead.

4. Equilibrium in market with an MPM policy

In this section, we model the impact of system-level MPM policies on market equilibrium. Each generator operates truthfully in the stage with an MPM policy in response to operator intervention in the form of a mitigation policy. With considerable market knowledge of participants' technology, fuel prices, operational constraints, historical prices, etc., ISOs can estimate, if not accurately, a reasonable bound on the operation cost of generators, which is used in substituting their bids with default bids in the presence of an MPM policy. However, each generator is allowed to bid an intercept function in the other stage.

These policies are planned firstly for the real-time followed by the day-ahead market to keep a check on the high risk of market power exercise in the real-time market compared to the day-ahead market. For this paper, we assume that the operator makes an error in estimating the operation cost of a generator in the stage with an MPM policy. We first develop an understanding of the system-level MPM policies and then compare them with the standard market.

4.1. Real-time MPM policy

In this subsection, we model the real-time default-bid MPM policy, as shown in panel (b) of Fig. 1. We then formulate the individual problem for different participation behaviors and characterize the market equilibrium.

Modelling Real-time Default-bid MPM Policy

For the real-time MPM policy, the operator roughly estimates the operation cost of the generator j in the real-time market, given the dispatch in the day-ahead market, i.e.,

$$g_j^r = (c_j + \epsilon_j)^{-1} \lambda^r - g_j^d, \ \forall j \in \mathcal{G} \tag{22}$$

where $\epsilon_j \geq 0$ denotes the estimation error. Summing the Eq. (22) over $j \in \mathcal{G}$ and substituting the two-stage supply-demand balance (4), we get

$$\lambda^r = \frac{d}{\sum_{j \in \mathcal{G}} (c_j + \epsilon_j)^{-1}} \tag{23}$$

Both generators and loads compete in the day-ahead market, and we characterize the resulting equilibrium under different participation modes in the following subsections.

Competitive Equilibrium

We first consider the case of price-taking participants in the market. We substitute (6), (22), and (23) in (12) to get the individual problem of price-taking generator j, given the clearing price λ^d, as:

$$\max_{\beta_j^d} \ \tilde{\pi}_j(\beta_j^d; \lambda^d) := \max_{\beta_j^d} \left(\frac{d}{\sum_{j \in \mathcal{G}} (c_j + \epsilon_j)^{-1}} - \lambda^d \right) \beta_j^d \tag{24}$$

Similarly, substituting (23) in (13) gives the individual problem of load l as:

$$\min_{d_l^d} \ \tilde{\rho}_l(d_l^d; \lambda^d) := \min_{d_l^d} \left(\lambda^d - \frac{d}{\sum_{j \in \mathcal{G}} (c_j + \epsilon_j)^{-1}} \right) d_l^d \tag{25}$$

where the price λ^d is given in the market. Without loss of generality, we assume $\epsilon_j = \epsilon c_j, \ \forall j \in \mathcal{G}$, for a constant parameter $\epsilon \geq 0$. The resulting competitive equilibrium is characterized below:

Theorem 4. *Let's assume $\epsilon_j = \epsilon c_j, \ \forall j \in \mathcal{G}$, for a constant parameter $\epsilon \geq 0$. The competitive equilibrium in a two-stage market with a real-time MPM policy exists and it is given by:*

$$g_j^d + g_j^r = \frac{c_j^{-1}}{\sum_{k \in \mathcal{G}} c_k^{-1}} d, \ \beta_j^d \in \mathbb{R}, \ \forall j \in \mathcal{G} \tag{26a}$$

$$d_l^d + d_l^r = d_l, \ \forall l \in \mathcal{L} \tag{26b}$$

$$\lambda^d = \lambda^r = \frac{1+\epsilon}{\sum_{j \in \mathcal{G}} c_j^{-1}} d \tag{26c}$$

We provide proof of the theorem in Appendix E. (The proof in Appendix E considers arbitrary ϵ_j). In the market, generators prefer higher prices, while loads prefer lower prices, resulting in opposing interests. A set of equilibria exist in the market with equal prices in two stages. However, at such equilibria, loads do not have any incentive to allocate demand in the day-ahead market. Interestingly, the resulting competitive equilibrium still aligns with the social planner problem (5).

Nash Equilibrium

In this section, we characterize the market equilibrium for the competition between price-anticipating participants. Substituting (22) and (23) in (14), we get the individual problem of the price-anticipating generator j that seeks to maximize the profit as:

$$\max_{\beta_j^d, \lambda^d} \ \pi_j\left(\beta_j^d, \lambda^d\left(\beta_j^d; \overline{\beta}_{-j}^d, d^d\right)\right) \ \text{s.t. (7)} \tag{27}$$

Similarly, we substitute (22),(23) in (15) to get the individual problem of the price-anticipating load as:

$$\min_{d_l^d, \lambda^d} \ \rho_l\left(d_l^d, \lambda^d\left(d_l^d; \beta_j^d, \overline{d}_{-l}^d\right)\right) \ \text{s.t. (7).} \tag{28}$$

We analyze the sequential game backward, starting with the real-time market where generators operate truthfully, resulting in fixed clearing prices. Although loads could bid in the real-time market, the bids are fixed by their decisions in the day-ahead market and load inelasticity. Therefore, each participant competes in the day-ahead market for individual interests. The following theorem characterizes the Nash equilibrium.

Theorem 5. *Let's assume $\epsilon_j = \epsilon c_j, \ \forall j \in \mathcal{G}$, for a constant parameter $\epsilon \geq 0$. If there is more than one generator participating in the market, i.e., $|\mathcal{G}| > 1$, the two-stage Nash equilibrium in a market with a real-time MPM policy uniquely exists, as:*

$$g_j^d = 0, \ g_j^r = \frac{c_j^{-1}}{\sum_{k \in \mathcal{G}} c_k^{-1}} d, \ \forall j \in \mathcal{G} \tag{29a}$$

$$\beta_j^d = \frac{(1+\epsilon) b^d}{\sum_{k \in \mathcal{G}} c_k^{-1}} d, \ \forall j \in \mathcal{G} \tag{29b}$$

$$d_l^d = 0, \ d_l^r = d_l, \ \forall l \in \mathcal{L} \tag{29c}$$

$$\lambda^d = \lambda^r = \frac{1+\epsilon}{\sum_{j \in \mathcal{G}} c_j^{-1}} d \tag{29d}$$

We provide proof of the theorem in Appendix F (the proof considers arbitrary ϵ_j). For a non-zero demand allocation in the day-ahead market, generators have the incentive to change their bids while attempting to manipulate prices and extract higher profits. Loads attempt to decrease prices to seek minimum payment simultaneously. The mutual competition to outbid each other results in the same price across stages, and all the demand shifts to the real-time market. Although there is no price difference across stages, i.e., no arbitrage opportunity, and the market dispatch aligns with the social planner optimum, i.e., efficient market equilibrium, such an equilibrium may not be desirable from the operator's perspective. In practice, the day-ahead market accounts for a majority of energy trades.

4.2. Day-ahead MPM policy

In this section, we consider the impact of a day-ahead MPM policy, as shown in panel (c) of Fig. 1.

Modeling Day-ahead Default-bid MPM policy

In this case, the operator estimates the cost of generator dispatch cost in the day-ahead, as:

$$g_j^d = (c_j + \epsilon_j)^{-1} \lambda^d \tag{30}$$

where $\epsilon_j \geq 0$ represents the error in the estimation. Summing the Eq. (30) over $j \in \mathcal{G}$ and using the power-balance in day-ahead market (7) implies that:

$$\lambda^d = \frac{d^d}{\sum_{j\in\mathcal{G}}(c_j + \epsilon_j)^{-1}} \tag{31}$$

Each generator has the flexibility to bid in the real-time market and we characterize the resulting market equilibrium in the following subsection. Such a game essentially constitutes a bi-level game, with multiple leaders (loads) acting in the day-ahead market and followers (generators) in the real-time market. We solve this by formulating individual optimization problems for each participant's payoff (profit or payment) and then simultaneously solving the necessary and sufficient KKT conditions to obtain the equilibrium.

Competitive Equilibrium

We first define the individual problem of participants and then characterize the resulting competitive equilibrium. The individual problem of price-taking generator j is given by:

$$\max_{\beta_j^r} \tilde{\pi}_j(\beta_j^r; \lambda^r) := \max_{\beta_j^r} -\beta_j^r \lambda^r - \frac{c_j}{2}\left(\frac{(c_j+\epsilon_j)^{-1} d^d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}} + b^r\lambda^r - \beta_j^r\right)^2 \tag{32}$$

where we substitute (30),(31) in (12). Similarly, the individual problem of load l is given by (13). The resulting competitive equilibrium is characterized in the theorem below.

Theorem 6. *Let's assume $\epsilon_j = \epsilon c_j,\ \forall j \in \mathcal{G}$, for a constant parameter $\epsilon \geq 0$. The competitive equilibrium in the two-stage market with a day-ahead MPM policy exists:*

$$g_j^d = \frac{1}{1+\epsilon}\frac{c_j^{-1}}{\sum_{k\in\mathcal{G}} c_k^{-1}} d,\ g_j^r = \frac{\epsilon}{1+\epsilon}\frac{1}{c_j}\frac{d}{\sum_{k\in\mathcal{G}} c_k^{-1}},\ \forall j \in \mathcal{G} \tag{33a}$$

$$\beta_j^r = \left(b^r - \frac{1}{c_j}\frac{\epsilon}{1+\epsilon}\right)\frac{d}{\sum_{k\in\mathcal{G}} c_k^{-1}},\ \forall j \in \mathcal{G} \tag{33b}$$

$$d_l^d + d_l^r = d_l;\ d^d = \frac{1}{1+\epsilon} d,\ d^r = \frac{\epsilon}{1+\epsilon} d \tag{33c}$$

$$\lambda^d = \lambda^r = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{33d}$$

The proof of the theorem was first presented in our previous paper (Bansal et al., 2022), we include it here in Appendix G for completeness. Unlike the case of the real-time MPM policy in Theorem 4 with equal prices across stages, the equilibrium in Theorem 6 is unique and incentivizes load to allocate the majority of demand in the day ahead market.

Nash Equilibrium

We next consider the competition between price-anticipating participants in a market with a day-ahead MPM policy. The sequential game where generators operate truthfully in the day-ahead market results in a multi-leader-follower game with loads making decisions in the day-ahead as leaders and generators participating as followers in the real-time market. The following theorem characterizes the Nash equilibrium, where load minimizes its payment as a leader, anticipating the prices in two stages with the knowledge of others' bids.

Theorem 7. *Let's assume $\epsilon_j = \epsilon c_j,\ \forall j \in \mathcal{G}$, for a constant parameter $\epsilon \geq 0$ and that more than one generator is participating in the market under a day-ahead MPM policy, i.e., $|\mathcal{G}| > 1$. Then the Nash equilibrium exists uniquely as:*

$$g_j^d = \left(1+\epsilon\frac{\sum\limits_{k\in\mathcal{G}} C_k^{-1}}{\sum\limits_{k\in\mathcal{G}} c_k^{-1}}\right)^{-1}\left(1 - \frac{1}{|\mathcal{L}|+1}\frac{\sum\limits_{k\in\mathcal{G}} C_k^{-1}}{\sum\limits_{k\in\mathcal{G}} c_k^{-1}}\right)\frac{c_j^{-1}}{\sum\limits_{k\in\mathcal{G}} c_k^{-1}} d,$$

$$g_j^r = \frac{(1+\epsilon(|\mathcal{L}|+1)}{|\mathcal{L}|+1}\left(1+\epsilon\frac{\sum\limits_{k\in\mathcal{G}} C_k^{-1}}{\sum\limits_{k\in\mathcal{G}} c_k^{-1}}\right)^{-1}\frac{C_j^{-1}}{\sum\limits_{k\in\mathcal{G}} c_k^{-1}} d \tag{34a}$$

$$d_l^d = \left(1+\epsilon\frac{\sum\limits_{k\in\mathcal{G}} C_k^{-1}}{\sum\limits_{k\in\mathcal{G}} c_k^{-1}}\right)^{-1}\left(d_l + \left(\frac{1}{|\mathcal{L}|+1} d - d_l\right)\frac{\sum\limits_{k\in\mathcal{G}} C_k^{-1}}{\sum\limits_{k\in\mathcal{G}} c_k^{-1}}\right),\ d_l^r = d_l - d_l^d \tag{34b}$$

$$\lambda^d = \left(1+\epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)^{-1}\left(1 - \frac{1}{|\mathcal{L}|+1}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)\frac{(1+\epsilon)d}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{34c}$$

$$\lambda^r = \frac{1}{1+\epsilon}\lambda^d + \left(1+\epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)^{-1}\left(\epsilon + \frac{1}{|\mathcal{L}|+1}\right)\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{34d}$$

where $C_j = \left(\frac{1}{b^r(|\mathcal{G}|-1)} + c_j\right)$.

The proof of the theorem was first presented in our previous paper (Bansal et al., 2022), and we include it here in Appendix H for completeness. Unlike the standard Nash equilibrium in Theorem 3, in the presence of a day-ahead MPM policy, the resulting Nash equilibrium always leads to higher prices in the real-time market; see (34d). As generators operate truthfully in the day-ahead market, loads exploit this opportunity to allocate higher demand in the day-ahead market to seek lower payment. Generators, with the flexibility to bid in the real-time market, attempt to manipulate and drive prices in the real-time market. The design of the day-ahead MPM policy puts generators in a disadvantageous position as followers in the market.

Corollary 2. *At the Nash equilibrium (34) in a market with a day-ahead MPM policy, the load allocation in the day-ahead and the real-time market is given by:*

$$d^d = \left(1+\epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)^{-1}\left(1 - \frac{1}{|\mathcal{L}|+1}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right) d,$$

$$d^r = \left(1+\epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)^{-1}\left(\epsilon + \frac{1}{|\mathcal{L}|+1}\right)\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}} d \tag{35}$$

Assuming $\epsilon = 0$, the following relation holds,

$d^d \in (0.5d, d),\ d^r \in (0, 0.5d)$

The proof uses the relation $b^r > 0$ and sums up the individual load allocation at the Nash equilibrium (34).

4.3. Simultaneous (real-time and day-ahead) MPM policy

In this subsection, we model the impact of a simultaneous MPM policy, i.e., the impact of applying the MPM policy to both real-time and day-ahead markets. In this case, the operator estimates the cost of the generator dispatch cost in both stages as:

$$g_j^d = (c_j + \epsilon_j)^{-1}\lambda^d \tag{36a}$$

$$g_j^r = (c_j + \epsilon_j)^{-1}\lambda^r - g_j^d \tag{36b}$$

where $\epsilon_j \geq 0$ denotes the estimation error. Summing the Eqs. (36a) and (36b) over $j \in \mathcal{G}$ and using the supply-demand balance (4) and (7), we get

$$\lambda^d = \frac{d^d}{\sum_{j\in\mathcal{G}}(c_j + \epsilon_j)^{-1}} \tag{37a}$$

$$\lambda^r = \frac{d}{\sum_{j\in\mathcal{G}}(c_j + \epsilon_j)^{-1}} \tag{37b}$$

In this case, each load has the flexibility to allocate its demand in either of the stages, and we characterize the resulting market equilibrium in the following subsection.

Table 1
Competitive (CE) and Nash (NE) Equilibrium in standard market and MPM policy markets.

Instance	Standard	Real-time MPM	Day-ahead MPM	Simultaneous MPM
CE	Non-unique equilibrium Solves social planner Arbitrary load allocation Price same as marginal cost	Non-unique equilibrium Partially solves social planner Arbitrary load allocation Price higher than marginal cost	Unique equilibrium Solves social planner Majority of load in day-ahead Price same as marginal cost	Unique equilibrium Solves social planner Total load in day-ahead Price same as marginal cost
NE	Unique & non-efficient equilibrium Load allocation depends on slope –	Unique & efficient equilibrium All load in real-time Undesirable to operator	Unique & non-efficient equilibrium Load allocation depends on error Desired market power mitigation	Unique & non-efficient equilibrium Majority of load in day-ahead Desired market power mitigation

Competitive Equilibrium

For the case of price-taking participation, the individual problem of each load l is given by:

$$\min_{d_l^d} \tilde{\rho}_l(d_l^d; \lambda^d, \lambda^r) := \min_{d_l^d} \left(\lambda^d - \frac{d}{\sum_{j\in\mathcal{G}} (c_j+\epsilon_j)^{-1}} \right) d_l^d \tag{38}$$

where λ^d is assumed to be given in the market. The resulting competitive equilibrium, assuming $\epsilon_j = \epsilon c_j,\ \forall j \in \mathcal{G}$, is characterized as follows:

Theorem 8. *Let's assume $\epsilon_j = \epsilon c_j,\ \forall j \in \mathcal{G}$, for a constant parameter $\epsilon \geq 0$. The competitive equilibrium in a two-stage market with both real-time and day-ahead MPM policy exists, as:*

$$g_j^d = \frac{c_j^{-1}}{\sum_{k\in\mathcal{G}} c_k^{-1}} d,\ g_j^r = 0,\ \forall j \in \mathcal{G} \tag{39a}$$

$$d_l^d = d_l,\ d_l^r = 0\ \forall l \in \mathcal{L} \tag{39b}$$

$$\lambda^d = \lambda^r = \frac{1+\epsilon}{\sum_{j\in\mathcal{G}} c_j^{-1}} d \tag{39c}$$

We provide the proof of the theorem in Appendix I (the proof considers arbitrary ϵ_j). Loads allocate all the demand in the day-ahead market, leading to equal prices across two stages. However, any variations in demand allocation within the real-time market may cause price discrepancies between the two stages. This situation would incentivize load participants to shift towards the stage with the lower prices. As a result, at equilibrium, the prices in both stages remain equal.

Nash Equilibrium

In this subsection, we characterize the market equilibrium for the competition between price-anticipating loads. The resulting competition can be visualized as a Nash-Cournot game among participants. Substituting (37a) and (37b) in (15), we get the individual problem of load l as:

$$\min_{d_l^d} \left(\frac{d^d}{\sum_{j\in\mathcal{G}} (c_j+\epsilon_j)^{-1}} - \frac{d}{\sum_{j\in\mathcal{G}} (c_j+\epsilon_j)^{-1}} \right) d_l^d + \frac{d}{\sum_{j\in\mathcal{G}} (c_j+\epsilon_j)^{-1}} d_l \tag{40}$$

The following theorem, assuming $\epsilon_j = \epsilon c_j,\ \ \forall j \in \mathcal{G}$, characterizes the Nash equilibrium.

Theorem 9. *Let's assume $\epsilon_j = \epsilon c_j,\ \forall j \in \mathcal{G}$, for a constant parameter $\epsilon \geq 0$. The Nash equilibrium in a market with both real-time and day-ahead MPM policy uniquely exists as:*

$$g_j^d = \frac{L}{L+1} \frac{c_j^{-1}}{\sum_{k\in\mathcal{G}} c_k^{-1}} d,\ g_j^r = \frac{1}{L+1} \frac{c_j^{-1}}{\sum_{k\in\mathcal{G}} c_k^{-1}} d,\ \forall j \in \mathcal{G} \tag{41a}$$

$$d_l^d = \frac{1}{L+1} d,\ d_l^r = d_l - \frac{1}{L+1} d,\ \forall l \in \mathcal{L} \tag{41b}$$

$$\lambda^d = \frac{L}{L+1} \frac{(1+\epsilon)}{\sum_{k\in\mathcal{G}} c_k^{-1}} d,\ \lambda^r = \frac{1+\epsilon}{\sum_{k\in\mathcal{G}} c_k^{-1}} d \tag{41c}$$

We provide the proof of the theorem in Appendix J (the proof considers arbitrary ϵ_j). Although the day-ahead prices are lower relative to the real-time prices, load participants do not have any incentive to deviate from the equilibrium. A unilateral deviation of load l in terms of an additional allocation of demand ϕ_l in the day-ahead market results in its increased payment. Interestingly, real-time prices depend only on the total demand and remain unaffected by any such unilateral deviations. Furthermore, the net load payment at Nash equilibrium is lower than in the competitive equilibrium and depends on the number of load participants. As the number of load participants increases, the relative difference tends to zero, with a complexity of $O\left(\frac{1}{L}\right)$.

5. Market analysis

In this section, we analyze the impact of system-level mitigation policies by comparing the resulting market equilibria with standard market equilibrium.

5.1. Equilibrium insights on MPM policies

We first discuss the case of the real-time MPM policy followed by the day-ahead MPM policy, as summarized in Table 1. The mitigation policies in real time result in equal prices across stages. Despite estimation errors, the individual generator dispatch aligns with the social planner dispatch (5) at both competitive (26) and Nash equilibrium (29). However, the resulting clearing price (26c) and (29d) at the equilibrium is higher than the system marginal cost. Moreover, the competitive equilibrium outcome fails to incentivize loads to allocate demand in the day-ahead market (26b) and allows for an arbitrary allocation between stages. On the other hand, Nash equilibrium incentivizes loads to allocate demand to the real-time market entirely (29b), making it undesirable from the operators' perspectives.

The day-ahead MPM policy also results in a unique competitive equilibrium (33) that aligns with the social planner optimum (5) while incentivizing loads to allocate the majority of demand (for a small error in the estimation of cost) to the day-ahead market (33c). At the Nash equilibrium, the mitigation policy leads to generators participating as followers and limiting their market power. Generators participate strategically in real-time, inflating the prices above the system marginal cost (34d). However, loads acting as leaders anticipate the real-time sub-game equilibrium and allocate more demand in the day-ahead market (35). Although a higher demand allocation in the day-ahead market increases the day-ahead clearing prices (34c), it is still below the clearing prices in the real-time market (34d). The loads are favored in the competition with a total payment at Nash equilibrium below the competitive equilibrium levels (assuming estimation error $\epsilon = 0$), as shown in row 1 of Table 2.

Finally, a simultaneous MPM policy, applied in both the day-ahead and real-time markets, results in a Nash-Cournot competition among loads, while generators act truthfully in both stages. The resulting equilibrium is unique, and the policy effectively mitigates generator market power.

Corollary 3. *Assuming estimation error $\epsilon = 0$, in a market with a day-ahead MPM policy, the total generator profit at the Nash equilibrium (34) is always below the competitive equilibrium levels (33).*

Table 2
Comparison of normalized Nash equilibrium (normalized with competitive equilibrium) between a standard market and a day-ahead market policy market (DA-MPM).

Case	Social Cost	Generators Aggregate Profit	Loads Aggregate Payment
DA-MPM	$1+\frac{\frac{\Delta}{(\lvert\mathcal{L}\rvert+1)^2}}{\sum_{j\in\mathcal{G}} c_j^{-1}}$	$1-\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\frac{2\lvert\mathcal{L}\rvert}{(\lvert\mathcal{L}\rvert+1)^2}-\frac{\frac{\Delta}{(\lvert\mathcal{L}\rvert+1)^2}}{\sum_{j\in\mathcal{G}} c_j^{-1}}$	$1-\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\frac{\lvert\mathcal{L}\rvert}{(\lvert\mathcal{L}\rvert+1)^2}$
Standard	1	$1+\frac{2\frac{d^d d^r}{d^2}}{b^r c(\lvert\mathcal{G}\rvert-1)+1}+\frac{2\frac{(d^d)^2}{d^2}}{b^d c(\lvert\mathcal{G}\rvert-1)}+\frac{2\frac{(d^r)^2}{d^2}}{b^r c(\lvert\mathcal{G}\rvert-1)}$	$1+\frac{\frac{d^d d^r}{d^2}}{b^r c(\lvert\mathcal{G}\rvert-1)+1}+\frac{\frac{(d^d)^2}{d^2}}{b^d c(\lvert\mathcal{G}\rvert-1)}+\frac{\frac{(d^r)^2}{d^2}}{b^r c(\lvert\mathcal{G}\rvert-1)}$

where $\Delta := \sum_{j\in\mathcal{G}} \frac{c_j}{C_j^2} - \frac{\left(\sum_{j\in\mathcal{G}} C_j^{-1}\right)^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}$

From the market perspective, the social cost is higher at the Nash equilibrium (34) than the competitive equilibrium (33), as shown in column 1 of Table 2.

Corollary 4. *Assuming generators are homogeneous, i.e.,* $c_j = c,\ \forall j \in \mathcal{G}$, *and estimation error* $\epsilon = 0$, *the social cost at the Nash equilibrium* (34) *is the same as the competitive equilibrium* (33).

The corollary uses the fact that for homogeneous generators $\Delta = 0$, as shown in Table 2. The term Δ is a non-linear function of the cost coefficients of generators and provides a quantitative measure of the heterogeneity in the system.

5.2. Comparison of day-ahead MPM policy with a standard market

We next compare only the equilibrium for a day-ahead MPM policy with equilibria in a standard market, as the real-time MPM policy market equilibrium results in undesirable market outcomes. Unlike a set of competitive equilibria in a standard market (17), the competitive equilibrium in the market with a day-ahead MPM policy is unique. It incentivizes loads to allocate the majority of demand in the day-ahead market (33).

Interestingly, at the Nash equilibrium in a market with a day-ahead MPM policy, clearing prices in real-time is always higher than in the day-ahead market (34d) due to the leader-follower structure and strategic participation of generators in real-time only. However, in the standard market, generators exploit the inelasticity of demand to manipulate the prices at Nash equilibrium in two stages, resulting in higher day-ahead clearing prices (19e) under certain conditions, i.e., the number of generators participating in the market and slope of the intercept function. We study the role of price-anticipating participants in a standard market and market with a day-ahead mitigation policy from the market and individual perspectives, i.e., social cost, generators' profit, and loads' payment in Table 2.

For the sake of comparison between two market settings, we evaluate the Nash equilibrium with the assumption that generates are homogeneous and participate symmetrically in the market. Furthermore, we assume the estimation error to be $\epsilon = 0$. Since generators are homogeneous, the market clears with the minimum cost of dispatch that equals the social planner cost, as shown in column 1 of Table 2. We next look at the individual perspective to evaluate the properties of the Nash equilibrium. In the standard market, generators win the competition at the Nash equilibrium since they always earn a higher profit than the one achieved in the competitive equilibrium level, as shown in row 2 of Table 2. However, in the case of the day-ahead MPM policy, loads win the competition with lower payment at the Nash equilibrium than the competitive equilibrium, as shown in row 1 of Table 2. Although the day-ahead MPM policy does have the intended mitigation effect on the market power of generators, it results in loads exercising market power at the expense of generators.

Fig. 2 compares the (normalized) aggregate profit and (normalized) aggregate payment at the Nash equilibrium in the standard market with a day-ahead MPM policy (DA-MPM) market, respectively. For simplicity, we assume that $b^d = b^r = \frac{1}{c}$ and that the estimation error $\epsilon = 0$. The aggregate generator profit (load payment) at the Nash equilibrium is normalized with the corresponding competitive equilibrium levels, which are the same in both market settings, and analyzed as we increase the number of participants in the market. If the ratio is greater than 1, then it means that generators make more profit and loads have to pay more at the Nash equilibrium when compared to the competitive equilibrium. This means that generators benefit more than loads. On the other hand, if the ratio is less than 1, then loads win the competition and benefit more than the generators. The aggregate profit ratio in the DA-MPM policy market, as given by

$$1-\frac{b^r c(\lvert\mathcal{G}\rvert-1)}{1+b^r c(\lvert\mathcal{G}\rvert-1)}\frac{2\lvert\mathcal{L}\rvert}{(\lvert\mathcal{L}\rvert+1)^2},$$

increases monotonically in the number of loads due to increased competition between loads, signaling a reduction in market power. In contrast, the ratio decreases monotonically in the number of generators due to increased competition between generators. This increased competition with an increase in the number of generators exacerbates their exploitation in the market, as shown by darker colors in the columns of panels (b) and (d) in Fig. 2.

The aggregate profit or payment ratio in the standard market increases with the number of loads and decreases with the number of generators, as shown in panels (a) and (c) in Fig. 2. The generators always win the competition in the standard market with higher profit levels at the Nash equilibrium compared with the competitive equilibrium. However, the day-ahead MPM policy results in the complete mitigation of generator market power, as shown in the comparison of generator normalized aggregate profit in the two markets in panels (a) and (b) in Fig. 2, respectively.

6. Variance penalized model for demand uncertainty

In this section, we model the impact of demand uncertainty on a two-stage market equilibrium under real-time MPM policies. We employ *variance-penalized expectation* (VPE) optimization to balance the trade-off between maximizing expected rewards and managing variance as a measure of risk in uncertain decision-making caused by demand fluctuations. Several studies have applied similar techniques in financial markets for portfolio optimization and, more recently, in the shortest path problem, where the objective is to maximize the expected weight before reaching a target state (Piribauer et al., 2022).

We assume that demand of each individual load l is random and denoted by $\tilde{d}_l$ such that $d_l^d + d_l^r = \tilde{d}_l, \forall l \in \mathcal{L}$. The total two stage demand is denoted by $\tilde{d} := \sum_{l\in\mathcal{L}} \tilde{d}_l$ The market operator seeks to minimize the cost of meeting the supply demand balance. Also, each generator j seeks to maximize its profit as:

$$\max_{g_j^d, g_j^r} \mathbb{E}[\pi_j(g_j^d, g_j^r)] - \delta_j Var(\pi_j(g_j^d, g_j^r)) \tag{42}$$

and load l seeks to minimize its payment as:

$$\min_{d_l^d, d_l^r} \mathbb{E}[\rho_l(d_l^d, d_l^r)] + \eta_l Var(\rho_l(d_l^d, d_l^r)) \tag{43}$$

Here, $\delta_j \in \mathbb{R}^+,\ j \in \mathcal{G}$, and $\eta_l \in \mathbb{R}^+,\ l \in \mathcal{L}$, are model parameters which indicate the risk preference of a participant, described in terms of a

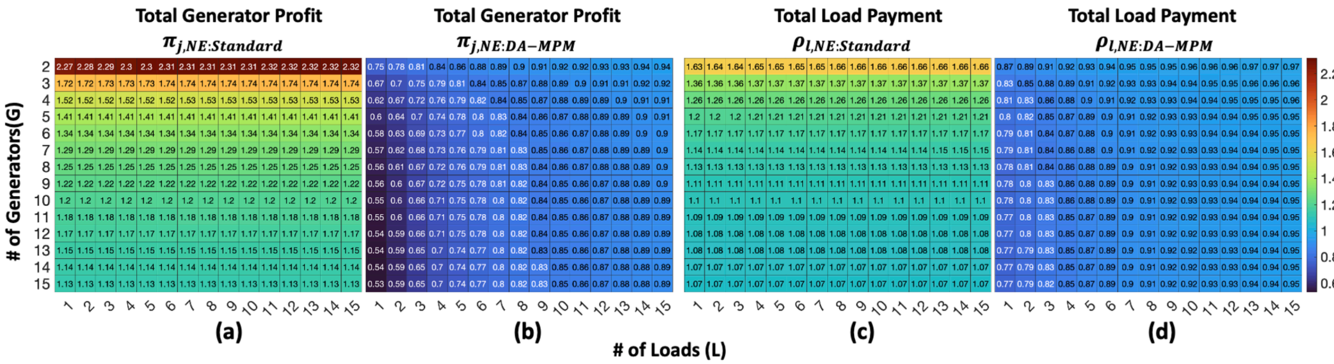

Fig. 2. Total profit and total payment at Nash Equilibrium (NE) normalized with competitive equilibrium (CE): total profit in (a) standard markets and (b) day-ahead MPM (DA-MPM), and total payment in (c) standard markets and (d) day-ahead MPM (DA-MPM).

trade-off between expected rewards and variance. For ease of analysis, we ignore the error in estimating the generator dispatch cost in the stage with the MPM policy in the following analysis.

Competitive Equilibrium

We substitute (6), (22), and (23) in (10) to get the individual profit function of generator j, given the clearing price λ^d, as:

$$\pi_j(\beta_j^d;\lambda^d) = \left(\lambda^d - \frac{\tilde{d}}{\sum_{k\in\mathcal{G}} c_k^{-1}}\right)(b^d\lambda^d - \beta_j^d) + \frac{1}{2c_j}\left(\frac{\tilde{d}}{\sum_{k\in\mathcal{G}} c_k^{-1}}\right)^2 \quad (44)$$

Then the individual problem of each generator j is given by:

$$\max_{\beta_j^d} \mathbb{E}[\pi_j(\beta_j^d;\lambda^d)] - \delta_j Var(\pi_j(\beta_j^d;\lambda^d)) \quad (45)$$

Similarly, we substitute (23) in (11) to get the individual payment function of load l, given the clearing price λ^d, as:

$$\rho_l(d_l^d;\lambda^d) = \left(\lambda^d - \frac{\tilde{d}}{\sum_{k\in\mathcal{G}} c_k^{-1}}\right)d_l^d + \frac{\tilde{d}}{\sum_{k\in\mathcal{G}} c_k^{-1}}\tilde{d}_l \quad (46)$$

and the individual problem of each load l is given by:

$$\min_{d_l^d} \mathbb{E}[\rho_l(d_l^d;\lambda^d)] + \eta_l Var(\rho_l(d_l^d;\lambda^d)) \quad (47)$$

The resulting competitive equilibrium is characterized below:

Theorem 10. *Let's assume $\mu\in\mathbb{R}, \sigma^2\in\mathbb{R}^+$, and $\tilde{\mu}_3\in\mathbb{R}$ denote the mean, variance, and standardized skewness of uncertain demand $\tilde{d}$. Also, let $\delta_j\in\mathbb{R}^+, j\in\mathcal{G}$ and $\eta_l\in\mathbb{R}^+, l\in\mathcal{L}$ denote the variance penalty parameters associated with each generator j and load l, respectively. Then, a competitive equilibrium in a two-stage market with a real-time MPM policy exists and given by:*

$$d_l^d = \frac{\mathbb{E}[\tilde{d}^2\tilde{d}_l] - \mathbb{E}[\tilde{d}]\mathbb{E}[\tilde{d}\tilde{d}_l]}{Var(\tilde{d})} - \frac{1}{2}\frac{\eta_l^{-1}}{\sum_{j\in\mathcal{G}}\delta_j^{-1} + \sum_{k\in\mathcal{L}}\eta_k^{-1}}(\tilde{\mu}_3\sigma + 2\mu) \quad (48a)$$

$$g_j^d = \frac{1}{2}\left(\frac{\delta_j^{-1}}{\sum_{k\in\mathcal{G}}\delta_k^{-1} + \sum_{l\in\mathcal{L}}\eta_l^{-1}} + \frac{c_j^{-1}}{\sum_{k\in\mathcal{G}} c_k^{-1}}\right)(\tilde{\mu}_3\sigma + 2\mu) \quad (48b)$$

$$\lambda^d = \mathbb{E}[\lambda^r] + \frac{1}{\sum_{j\in\mathcal{G}}\delta_j^{-1} + \sum_{l\in\mathcal{L}}\eta_l^{-1}}\frac{(\tilde{\mu}_3\sigma + 2\mu)\sigma^2}{(\sum_{j\in\mathcal{G}} c_j^{-1})^2},\ \lambda^r = \frac{\tilde{d}}{\sum_{k\in\mathcal{G}} c_k^{-1}} \quad (48c)$$

We provide the proof of the theorem in Appendix K. Unlike in the deterministic case of real-time MPM policy in Theorem 4, the equilibrium in Theorem 10 exists uniquely, with the majority of demand in the day-ahead market, given by:

$$d^d = \sum_{j\in\mathcal{G}} g_j^d = \mu + \frac{1}{2}\tilde{\mu}_3\sigma + \frac{1}{2}\frac{\sum_{j\in\mathcal{G}}\delta_j^{-1}}{\sum_{j\in\mathcal{G}}\delta_j^{-1} + \sum_{l\in\mathcal{L}}\eta_l^{-1}}(\tilde{\mu}_3\sigma + 2\mu) \quad (49)$$

Nash Equilibrium

We next characterize the equilibrium for the competition between price-anticipating participants. Similarly, substituting (22), and (23) in (42), we get the individual problem of generator j as:

$$\max_{\beta_j^d} \mathbb{E}[\pi_j(\beta_j^d, \lambda^d(\beta_j^d;\overline{\beta}_{-j}^d, d^d))] - \delta_j Var(\pi_j(\beta_j^d, \lambda^d(\beta_j^d;\overline{\beta}_{-j}^d, d^d))) \quad \text{s.t. } (7) \quad (50)$$

Similarly, substituting (22), and (23) in (43), we get the individual problem of load l as:

$$\max_{d_l^d} \mathbb{E}[\rho_l(d_l^d, \lambda^d(d_l^d;\beta_j^d, \overline{d}_{-l}^d))] - \eta_l Var(\rho_l(d_l^d, \lambda^d(d_l^d;\beta_j^d, \overline{d}_{-l}^d))) \quad \text{s.t. } (7) \quad (51)$$

The resulting Nash equilibrium is characterized as below:

Theorem 11. *Let's assume $\mu\in\mathbb{R}, \sigma^2\in\mathbb{R}^+$, and $\tilde{\mu}_3\in\mathbb{R}$ denote the mean, variance, and standardized skewness of uncertain demand $\tilde{d}$. Let $\delta_j\in\mathbb{R}^+, j\in\mathcal{G}$ and $\eta_l\in\mathbb{R}^+, l\in\mathcal{L}$ denote the variance penalty parameters associated with each generator j and load l, respectively. Also, assume that there are at least two generators, i.e., $|\mathcal{G}|\geq 2$. Then, a Nash equilibrium in a two-stage market with a real-time MPM policy exists and given by:*

$$d^d = \frac{\left(\sum_{l\in\mathcal{L}}\kappa_l^{-1}\right)\left(\sum_{j\in\mathcal{G}}\frac{\omega_j^{-1}\delta_j}{c_j}\right)}{\left(\sum_{j\in\mathcal{G}}\omega_j^{-1} + \frac{\sum_{l\in\mathcal{L}}\kappa_l^{-1}}{\left(1-\frac{1}{|\mathcal{G}|}\right)}\right)}\frac{(\tilde{\mu}_3\sigma+2\mu)\sigma^2}{\left(\sum_{j\in\mathcal{G}} c_j^{-1}\right)^3} + \frac{2\sum_{j\in\mathcal{G}}\omega_j^{-1}}{\left(\sum_{j\in\mathcal{G}}\omega_j^{-1} + \frac{\sum_{l\in\mathcal{L}}\kappa_l^{-1}}{\left(1-\frac{1}{|\mathcal{G}|}\right)}\right)}\frac{\mathbb{E}\left[\tilde{d}^2\left(\sum_{l\in\mathcal{L}}\frac{\eta_l}{\kappa_l}\tilde{d}_l\right)\right] - \mathbb{E}[\tilde{d}]\mathbb{E}\left[\tilde{d}\left(\sum_{l\in\mathcal{L}}\frac{\eta_l}{\kappa_l}\tilde{d}_l\right)\right]}{\left(\sum_{j\in\mathcal{G}} c_j^{-1}\right)^2} \quad (52a)$$

$$g_j^d = \frac{\omega_j^{-1}}{\sum_{k\in\mathcal{G}}\omega_k^{-1}}d^d - \left(1 - \frac{1}{|\mathcal{G}|}\right)\omega_j^{-1}\left(\frac{\sum_{k\in\mathcal{G}}\frac{\omega_k^{-1}\delta_k}{c_k}}{\sum_{k\in\mathcal{G}}\omega_k^{-1}} - \frac{\delta_j}{c_j}\right)\frac{(\tilde{\mu}_3\sigma + 2\mu)\sigma^2}{\left(\sum_{k\in\mathcal{G}} c_k^{-1}\right)^3} \quad (52b)$$

$$\lambda^d = \mathbb{E}[\lambda^r] + \frac{1}{\sum_{j\in\mathcal{G}}\omega_j^{-1}\left(1-\frac{1}{|\mathcal{G}|}\right)}d^d - \frac{\sum_{j\in\mathcal{G}}\frac{\omega_j^{-1}\delta_j}{c_j}}{\sum_{j\in\mathcal{G}}\omega_j^{-1}}\frac{(\tilde{\mu}_3\sigma + 2\mu)\sigma^2}{\left(\sum_{j\in\mathcal{G}} c_j^{-1}\right)^3} \quad (52c)$$

where $\kappa_l := (b^d|\mathcal{G}|)^{-1} + 2\eta_l\frac{Var(\tilde{d})}{\left(\sum_{j\in\mathcal{G}} c_j^{-1}\right)^2}$ and $\omega_j := (b^d|\mathcal{G}|)^{-1} + 2\delta_j\frac{Var(\tilde{d})}{\left(\sum_{j\in\mathcal{G}} c_j^{-1}\right)^2}$, respectively.

We provide the proof of the theorem in Appendix L. In the stochastic setting with uncertain demand, the total load allocation in the day-ahead stage (as given in Eq. (52a)) is determined as a function of the

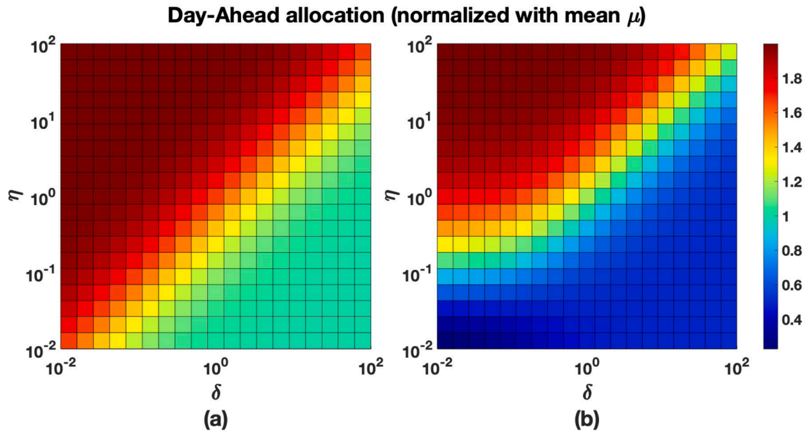

Fig. 3. Day-ahead allocation at (a) competitive equilibrium and (b) Nash equilibrium with respect to homogeneous penalty parameters δ (x-axis) and η (y-axis) associated with generators and loads.

expected demand, scaled by constant coefficients that depend on the penalty parameters $\delta_j, j \in \mathcal{G}$ and $\eta_l, l \in \mathcal{L}$. On one hand, in a largely risk-neutral market setting, where the penalty parameters are close to zero, the day-ahead allocation tends to vanish, thereby shifting most of the load adjustment to the real-time stage. On the other hand, in a strongly risk-averse market with large penalty parameters, assuming homogeneous penalty parameters for simplicity (i.e.,$\delta_j = \delta,\ \forall j \in \mathcal{G},\ \eta_l = \eta,\ \forall l \in \mathcal{L}$), the day-ahead allocation is given by::

$$\lim_{\substack{\delta\to\infty,\\ \eta\to\infty}} d^d \approx \frac{(|\mathcal{L}| + 2|\mathcal{G}|)(|\mathcal{G}| - 1)}{|\mathcal{G}|(|\mathcal{L}| + |\mathcal{G}| - 1)}\left(\frac{1}{2}\tilde{\mu}_3\sigma + \mu\right)$$

However, the forced load allocation in the day-ahead stage results in an increase in day-ahead prices on the order of $O(\delta)$, i.e.,

$$\lim_{\substack{\delta\to\infty,\\ \eta\to\infty}} \lambda^d \to \infty$$

Therefore, the real-time MPM policy may not be desirable from the operator perspective.

6.1. Impact of penalty parameters

We now analyze the impact of the penalty parameters $\delta_j,\ j \in \mathcal{G}$ and $\eta_l,\ l \in \mathcal{L}$ on the competitive and Nash equilibria in Theorems 10 and 11, respectively, using a numerical case study. For ease of analysis, we consider two generators with equal cost coefficients and equal penalty parameter, i.e., $c_1 = c_2 = c = 0.1\$/MW^2$ and $\delta_1 = \delta_2 = \delta$. We also consider an uncertain net inelastic load (demand minus renewable generation), sampled 100,000 times from a normal distribution[1] $\tilde{d} \sim N(150, 15)$. Moreover, we fix the value of the constant slope $b^d = \frac{1}{c} = 10$.

First, we examine the impact of the penalty parameters on the day-ahead load allocation at the equilibria, as given in Eqs. (48a) and (52a), respectively. Fig. 3 illustrates the resulting allocations - (a) at the competitive equilibrium and (b) at the Nash equilibrium - as we vary the penalty parameters, $\delta \in [10^{-2}, 10^2]$ and $\eta \in [10^{-2}, 10^2]$, respectively. Interestingly, at the competitive equilibrium (panel a), the load is consistently allocated above the expected value of net demand, $\mathbb{E}[\tilde{d}] = 150$ MW, resulting in low or even negative demand in the real-time market. Moreover, a small value of δ - representing more risk-averse generators - has a relatively stronger impact on load allocation than its counterpart η, which reflects the risk preference of the loads, as shown on the left edge of the panel (a) in Fig. 3. We observe a similar behavior, at the Nash equilibrium, where risk preference of generators again has a stronger effect on load allocation, as shown in the top-left corner of panel (b). However, in the case of risk-neutral loads, the day-ahead allocation decreases, and more demand is cleared in real-time. The situation worsens for small values of the penalty parameters - indicating increasingly risk-neutral participants - the day-ahead load allocation drops significantly and eventually vanishes, as shown in the bottom-left corner of panel (b) in Fig. 3. This results in a similar observation to Theorem 5 where load clears in the real-time market, a scenario that is typically undesirable for market operators.

In Fig. 4, we show the day-ahead clearing price as the penalty parameters δ and η are varied along the x-axis and y-axis, respectively. As before, we compare the resulting prices at the (a) competitive equilibrium and (b) Nash equilibrium. In both cases, the prices - given by Eqs. (48a) and (52a) - increase at a polynomial rate with increasingly risk-averse market participant behavior, as seen in the top-right corners of panels (a) and (b) in Fig. 4. Although the day-ahead prices are consistently higher than the expected real-time prices at the competitive equilibrium, this is not always true in the case of the Nash equilibrium. Interestingly, when generators are risk-averse (high δ) and loads are risk-neutral (low η), the day-ahead prices fall below the expected real-time prices -and may even become negative. Intuitively, in such a scenario, the load prefers to allocate less demand in the day-ahead market (as seen along the bottom edge of panel (b) in Fig. 3), while generators prefers to clear more demand in the day-ahead, resulting in lower prices. Moreover, a higher variance penalty (i.e., a large δ) discourages generators from relying solely on the real-time market, as doing so could expose them to higher profit variance.

We next analyze the relation between market power, measured as the ratio of aggregate generator profit at the Nash equilibrium and at the competitive equilibrium, and the penalty parameters. A similar trend is observed for the aggregate load payment. In Fig. 5, we show the ratio for demand $\tilde{d} \sim N(150, 15)$, sampled 100,000 times for different values of (a) δ as we very η, and (b) η as we very δ. In symmetric cases - i.e., (low δ, low η), (med δ, med η), and (high δ, high η) - the profit ratio remains relatively small compared to asymmetric cases, where one set of participants is more risk-averse than the other. Notably, the ratio becomes significantly higher in boundary scenarios, such as when generators are risk-neutral (low δ) and loads are risk-averse (high η), or vice versa. These cases indicate the presence of excessive market power, as shown on the right edge of panel (a) and the left edge of panel (b) in Fig. 5, respectively. Moreover, when load participants are risk-neutral, they tend to exert market power, resulting in a profit ratio below 1 - as shown by the blue scatter plot in panel (a) and the purple scatter plot in panel (b). Overall, the risk preference of generators has a relatively smaller impact on the overall market power.

[1] The data analysis on load data from New York ISO for 2023 indicated low skewness values, i.e., $\tilde{\mu}_3 \in [-1.5, 1.5]$. Hence, for simplicity, we assume a symmetric normal distribution to model the uncertainty in net demand.

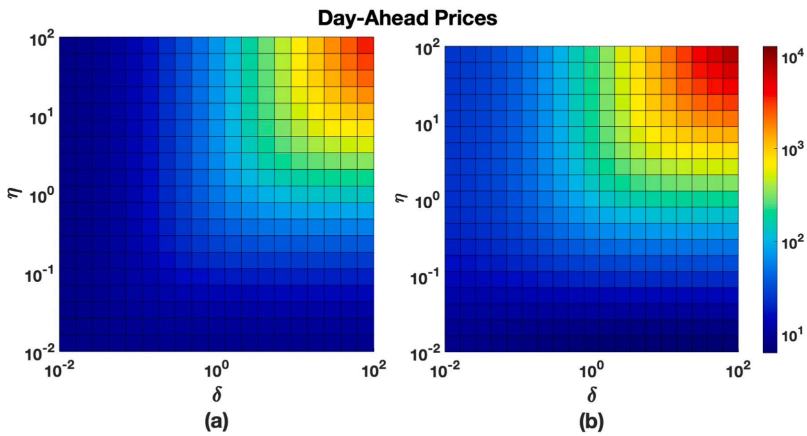

Fig. 4. Day-ahead price at (a) competitive equilibrium and (b) Nash equilibrium with respect to homogeneous penalty parameters δ (x-axis) and η (y-axis) associated with generators and loads.

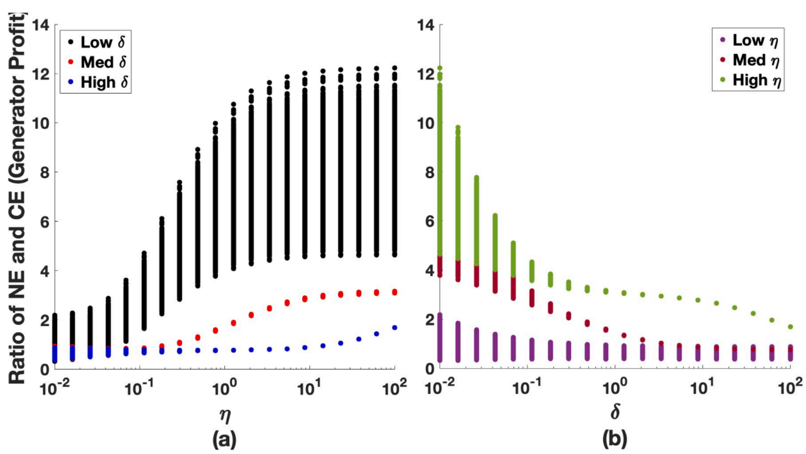

Fig. 5. Aggregate generator profit ratio (Nash Equilibrium (NE) to Competitive Equilibrium (CE)) for demand $\tilde{d} \sim \mathcal{N}(150, 15)$, sampled 100,000 times, with respect to homogeneous penalty parameters: (a) η for low, medium, and high values of δ, and (b) δ for low, medium, and high values of η.

7. Conclusions

We analyze strategic interaction in a two-stage settlement market - commonly used by many system operators - under system-level MPM policies, modeling generators that bid supply-function intercepts and loads that strategically allocate quantities across stages. Our focus is on default-bid substitution schemes in which noncompetitive generator offers are replaced with operator-estimated cost-based bids. Using a (no-mitigation) standard market benchmark, we show that day-ahead and simultaneous MPM policies (i.e., MPM in both market stages) substantially reduce generator market power compared to either real-time MPM policy, although both policies shift strategic leverage toward loads. These results demonstrate that system-level substitution rules can materially reshape incentives on both sides of the market.

Under a real-time MPM policy, strategic interaction in the day-ahead market shifts all demand to real time, yielding an undesirable market outcome. To test the robustness of this effect, we incorporate demand uncertainty through a variance-penalized expectation framework. Using variance as a measure of risk in uncertain decision making, we show that under low risk aversion, loads continue to allocate more demand to the real-time market, similar to the outcome in the deterministic model. However, as participants become more risk-averse, demand gradually shifts toward the day-ahead market, driving up day-ahead prices. Interestingly, an imbalance in risk, where one group of participants is significantly more risk-averse than the other, tends to favor generators, leading to an increase in market power.

CRediT authorship contribution statement

Rajni Kant Bansal: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Conceptualization; **Yue Chen:** Writing – review & editing, Validation, Supervision, Funding acquisition; **Pengcheng You:** Writing – review & editing, Validation, Supervision, Investigation, Funding acquisition; **Enrique Mallada:** Writing – review & editing, Validation, Supervision, Project administration, Investigation, Funding acquisition, Conceptualization.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Equilibrium comparison with slope function bid in a standard market

In this section, we compare the intercept function bidding with the conventional slope function bidding[2], a.k.a. linear supply function in a standard market (without the implementation of an MPM policy). Our goal is to further understand the impact of the functional form of the bid on the market power of respective participants. In the case of the slope function bidding, each generator submits a slope function in the day-ahead and the real-time markets, parameterized by $\hat{b}_j^d \in \mathbb{R}_{\geq 0}$, $\hat{b}_j^r \in \mathbb{R}_{\geq 0}$, respectively:

$$g_j^d = \hat{b}_j^d \lambda^d, \quad g_j^r = \hat{b}_j^r \lambda^r. \tag{A.1}$$

Here λ^d and λ^r denote the prices in the day-ahead and real-time market, respectively. We first characterize the competitive equilibrium in a standard two-stage market.

Theorem 12. *(You et al., 2022) A competitive equilibrium in a two-stage market exists and is explicitly given by*

$$\hat{b}_j^d + \hat{b}_j^r = \frac{1}{c_j}, \ \hat{b}_j^d \geq 0, \ \hat{b}_j^r \geq 0, \forall j \in \mathcal{G} \tag{A.2a}$$

$$d_l^d + d_l^r = d_l, \forall l \in \mathcal{L} \tag{A.2b}$$

$$\lambda^d = \lambda^r = \frac{d}{\sum_{j \in \mathcal{G}} c_j^{-1}} \tag{A.2c}$$

The resulting competitive equilibrium is efficient, i.e., it aligns with the social planner problem (5). Similar to the competitive equilibrium for intercept function bidding in Theorem 1, the resulting equilibrium in Theorem 12 exists non-uniquely. We next consider the case of price-anticipating participants and characterize the resulting Nash equilibrium.

Theorem 13. *(You et al., 2022) Assume strategic generators are homogeneous ($c_j := c, \ \forall j \in \mathcal{G}$). If there are at least three firms, i.e., $|\mathcal{G}| \geq 3$, a symmetric Nash equilibrium in a two-stage market exists with identical bids ($\hat{b}_j^v := \hat{b}_j^v, \ \forall j \in \mathcal{G}, v \in \{d, r\}$). Further, this equilibrium is unique, as:*

$$\hat{b}_j^d = \frac{|\mathcal{L}|(|\mathcal{G}|-1)+1}{|\mathcal{L}|(|\mathcal{G}|-1)} \frac{|\mathcal{G}|-2}{|\mathcal{G}|-1} \frac{1}{c}, \ \hat{b}_j^r = \frac{1}{|\mathcal{L}|+1} \frac{(|\mathcal{G}|-2)^2}{(|\mathcal{G}|-1)^2} \frac{1}{c} \tag{A.3}$$

$$d_l^d = \frac{|\mathcal{L}|(|\mathcal{G}|-1)+1}{|\mathcal{L}|(|\mathcal{L}|+1)(|\mathcal{G}|-1)} d, \ d_l^r = d_l - d_l^d \tag{A.4}$$

$$\lambda^d = \frac{|\mathcal{L}|}{|\mathcal{L}|+1} \frac{|\mathcal{G}|-1}{|\mathcal{G}|-2} \frac{c}{|\mathcal{G}|} d, \ \lambda^r = \frac{|\mathcal{G}|-1}{|\mathcal{G}|-2} \frac{c}{|\mathcal{G}|} d \tag{A.5}$$

Theorem 13 shows the existence of a unique symmetric Nash equilibrium. At the resulting equilibrium, loads allocate more demand in the day-ahead market to exploit lower prices. However, the load allocation at the Nash equilibrium in the intercept function in Theorem 3 is a function of market parameters b^d and b^r. Fig. A.1 plots the aggregate load allocation in the day-ahead market as the slope of the intercept function bid changes in the day-ahead and real-time markets. We assume 4 strategic homogeneous generators and 4 strategic loads are participating in a standard two-stage market setting. The mix of individual inelastic demand bids is given by $d_l = [0.2, 25.6, 106.6, 199.6]^T MW$ with total aggregate inelastic demand $d = 332MW$. We assume a cost coefficient $c_j = 0.1\$/MW^2, \ \forall j \in \mathcal{G}$ corresponding to the cost coefficients from the IEEE 300-bus system (Zimmerman & Murillo-Sanchez, 2019) for homogeneous generators. The aggregate allocation in the day-ahead market (normalized with the total inelastic demand) can be increased by the operator with the help of appropriate slope parameters.

Fig. A.2 plots the normalized aggregate profit at the Nash equilibrium in the intercept function bid market mechanism in a standard market w.r.t the day-ahead and real-time slope parameters. As discussed in

[2] For ease of comparison between the two bidding mechanisms, we say a generator submits an intercept function or a slope function when it bids intercept or slope of the supply function, respectively

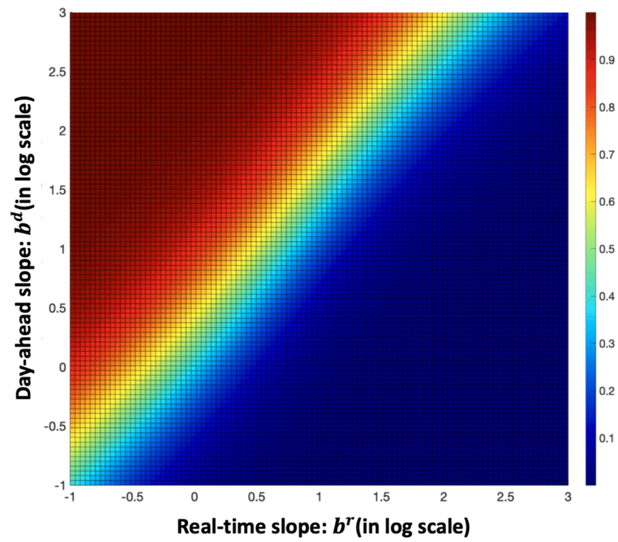

Fig. A.1. Normalized load allocation in the day-ahead stage in intercept function bid-based standard market.

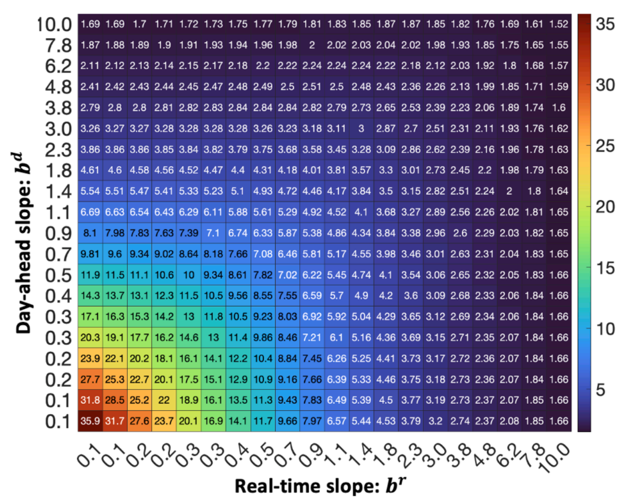

Fig. A.2. Total profit at Nash equilibrium normalized with competitive equilibrium in intercept function bid-based standard market.

row 2 of Table 2, the market power of generators in the standard market is relatively low for either end of the spectrum, i.e., most of the demand is allocated in either day-ahead or real-time market. Furthermore, high values of slope parameters b^d (b^r) for a total allocation of demand in day-ahead (real-time) further normalize the market power of generators at Nash equilibrium, as shown in the top left and the bottom right part of Fig. A.2.

Fig. A.3 compares the (normalized) aggregate profit at Nash equilibrium in the standard market without any mitigation policy. We perturb the value of the slope parameter for the intercept function bid to understand the impact of model parameters, i.e.,

$$b^d = b^r = b, b \in \{(1+\gamma)^{-1}c^{-1}, c^{-1}, (1-\gamma)^{-1}c^{-1}\},$$

where $\gamma = 0.1$. For the sake of comparison between the two market settings, we evaluate the Nash equilibrium with the assumption that $|\mathcal{G}| > 2$ in the market. The aggregate profit is normalized with the profit at competitive equilibrium levels. In the slope function bid-based market mechanism, there is a shift in the market power between loads and generators, e.g., loads win the competition for a relatively large number of generators in the market and vice versa. In particular, for a small number of loads and a large number of generators, loads exercise market

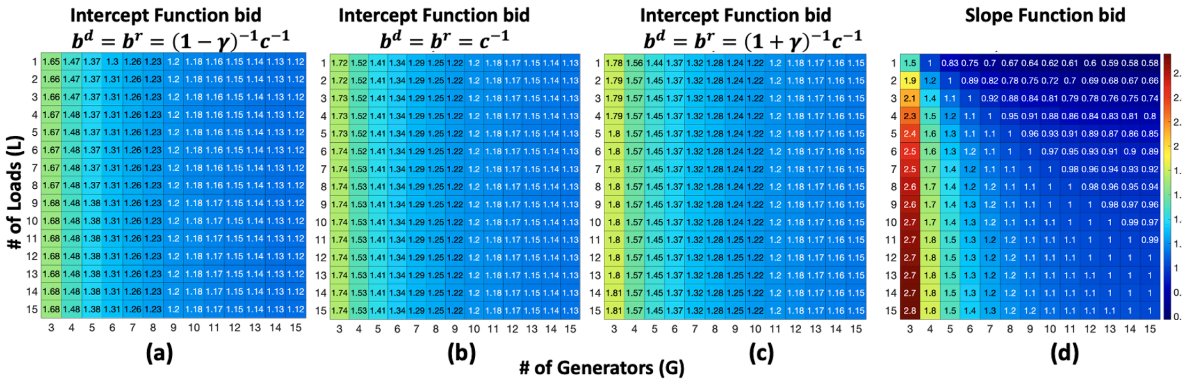

Fig. A.3. Aggregate generators' profit at Nash equilibrium (NE) normalized with competitive equilibrium (CE) in a standard market for Intercept function bid (a) with parameters $b^d = b^r = (1-\gamma)^{-1}c^{-1}$, (b) with parameters $b^d = b^r = c^{-1}$, (c) with parameters $b^d = b^r = (1+\gamma)^{-1}c^{-1}$, and (d) Slope function bid.

power with lower payments at the expense of increased competition between generators. Similarly, a decrease in the number of generators and an increase in the number of loads favors generators in the market, as shown in panel (d) in Fig. A.3. However, generators always win the competition with higher profits at the Nash equilibrium in the intercept function bid based market mechanism, as shown in panel (b) in Fig. A.3. Moreover, such behavior, where generators always win the competition, exists regardless of slope parameter values in the intercept function bid, as shown in row 2 of Table 2 and panels (a),(c) in Fig. A.3.

Appendix B. Proof of Theorem 1

Under price-taking behavior, the individual problem for loads (13) is a linear program with the closed-form solution given by:

$$\begin{cases} d_l^d = \infty, d_l^r = -\infty, d_l^d + d_l^r = d_l, \text{ if } \lambda^d < \lambda^r \\ d_l^d = -\infty, d_l^r = \infty, d_l^d + d_l^r = d_l, \text{ if } \lambda^d > \lambda^r \\ d_l^d + d_l^r = d_l, \quad \text{if } \lambda^d = \lambda^r \end{cases} \tag{B.1}$$

where loads prefer lower price in the market. The individual problem for generators (16) requires:

$$\begin{cases} \beta_j^d = \infty, \beta_j^r = -\infty, \beta_j^d + \beta_j^r = \frac{b^d + b^r - c_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}} d, \text{ if } \lambda^d < \lambda^r \\ \beta_j^d = -\infty, \beta_j^r = \infty, \beta_j^d + \beta_j^r = \frac{b^d + b^r - c_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}} d, \text{ if } \lambda^d > \lambda^r \\ \beta_j^d + \beta_j^r = \frac{b^d + b^r - c_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}} d, \text{ if } \lambda^d = \lambda^r \end{cases} \tag{B.2}$$

where generators prefer higher prices in the market and seek to maximize profit. At the competitive equilibrium the intercept function (6),(8) and individual optimal solution (B.1),(B.2) holds simultaneously and this is only possible if the market price is equal in the two stages. Thus a set of competitive equilibria exists.

Appendix C. Proof of Theorem 2

Given the parameter $(\beta_j^d, g_j^d, d - d^d)$ from market-clearing in the day-ahead market, each generator j maximizes their profit (14) for the optimal decision β_j^r with complete knowledge of the market clearing in the real-time stage as characterized below:

$$\sum_{j\in\mathcal{G}} g_j^r = d^r \implies \sum_{j\in\mathcal{G}} (b^r\lambda^r - \beta_j^r) = d^r \implies \lambda^r = \frac{d^r + \beta^{r,\mathcal{G}}}{b^r|\mathcal{G}|} \tag{C.1}$$

where $\beta^{r,\mathcal{G}} = \sum_{j\in\mathcal{G}} \beta_j^r$. Given the parameter $(\beta_j^d, g_j^d, d - d^d)$, substituting (C.1) in the individual problem (12) gives the concave strategic individual problem of generators, i.e., the real-time subgame problem:

$$\max_{\beta_j^r} \left(\frac{d^r + \beta^{r,\mathcal{G}}}{b^r|\mathcal{G}|}\right)\left(b^r\frac{d^r + \beta^{r,\mathcal{G}}}{b^r|\mathcal{G}|} - \beta_j^r\right) + \lambda^d g_j^d - \frac{c_j}{2}\left(g_j^d + b^r\left(\frac{d^r + \beta^{r,\mathcal{G}}}{b^r|\mathcal{G}|}\right) - \beta_j^r\right)^2 \tag{C.2}$$

Hence, taking the derivative of (C.2) with respect to bid β_j^r we get:

$$\frac{\partial \pi_j}{\partial \beta_j^r} = \frac{1}{b^r|\mathcal{G}|}\left(\frac{d^r + \beta^{r,\mathcal{G}}}{|\mathcal{G}|} - \beta_j^r\right) - \frac{|\mathcal{G}|-1}{|\mathcal{G}|}\left(\frac{d^r + \beta^{r,\mathcal{G}}}{b^r|\mathcal{G}|}\right) + c_j\left(g_j^d + \frac{d^r + \beta^{r,\mathcal{G}}}{|\mathcal{G}|} - \beta_j^r\right)\frac{|\mathcal{G}|-1}{|\mathcal{G}|} = 0$$

$$\implies \frac{1}{b(|\mathcal{G}|-1)} g_j^r - \lambda^r + c_j\left(g_j^d + g_j^r\right) = 0 \tag{C.3}$$

where we substitute (8) and (C.1). The Eq. (C.3) is the required KKT condition of the convex dispatch problem (18), with λ^r as the dual variable of the constraint (18b).

Appendix D. Proof of Theorem 3

From the KKT conditions of the augmented convex social planner problem (18), we have the relation between price and generator dispatch in the real-time stage λ^r as

$$g_j^r = \frac{\lambda^r - c_j g_j^d}{C_j} \implies \sum_{j\in\mathcal{G}} g_j^r = \sum_{j\in\mathcal{G}} \frac{\lambda^r - c_j g_j^d}{C_j} \tag{D.1}$$

where $C_j := \left(\frac{1}{b^r(|\mathcal{G}|-1)} + c_j\right)$. Substituting (9) in the Eq. (D.1), we get:

$$d^r = \sum_{j\in\mathcal{G}} \frac{\lambda^r - c_j g_j^d}{C_j} \implies \lambda^r = \frac{d^r + \sum_{j\in\mathcal{G}} \frac{c_j}{C_j} g_j^d}{\sum_{j\in\mathcal{G}} C_j^{-1}} \tag{D.2}$$

Substituting (D.2) in (D.1) we get

$$g_j^r = \frac{d^r + \sum_{k\in\mathcal{G}} \frac{c_k}{C_k} g_k^d}{C_j \sum_{k\in\mathcal{G}} C_k^{-1}} - \frac{c_j}{C_j} g_j^d \tag{D.3}$$

From the market-clearing in the day-ahead stage (7), we have the following relation

$$\implies \sum_{j\in\mathcal{G}} \left(b^d\lambda^d - \beta_j^d\right) = \sum_{l\in\mathcal{L}} d_l^d \implies$$

$$\lambda^d = \frac{d^d + \beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}, \; g_j^d = b^d\frac{d^d + \beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|} - \beta_j^d \tag{D.4}$$

where $\beta^{d,\mathcal{G}} = \sum_{j\in\mathcal{G}} \beta_j^d$. Substituting (D.2)–(D.4) in the individual profit (14), we get,

$$\max_{\beta_j^d} \frac{d^d + \beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}\left(\frac{d^d + \beta^{d,\mathcal{G}}}{|\mathcal{G}|} - \beta_j^d\right) + \left(\frac{d^r + \sum_{m\in\mathcal{G}} \frac{c_m}{C_m}\left(\frac{d^d + \beta^{d,\mathcal{G}}}{|\mathcal{G}|} - \beta_m^d\right)}{C_j \sum_{k\in\mathcal{G}} C_k^{-1}}\right)^2$$

$$-\frac{c_j}{C_j}\frac{d^r+\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_m\right)}{\sum_{k\in\mathcal{G}}C_k^{-1}}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-b^d_j\right)$$
$$-\frac{c_j}{2}\left(\left(1-\frac{c_j}{C_j}\right)\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_j\right)+\frac{d^r+\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_m\right)}{C_j\sum_{k\in\mathcal{G}}C_k^{-1}}\right)^2 \tag{D.5}$$

Writing the first order condition and taking the derivative of (D.5) wrt β^d_j we have

$$\implies \frac{1}{b^d|\mathcal{G}|}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_j\right)+\frac{d^d+\beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}\left(\frac{1}{|\mathcal{G}|}-1\right)$$
$$+\frac{2}{C_j}\left(\frac{d^r+\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_m\right)}{\sum_{k\in\mathcal{G}}C_k^{-1}}\right)\left(\frac{\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\frac{1}{|\mathcal{G}|}-\frac{c_j}{C_j}}{\sum_{k\in\mathcal{G}}C_k^{-1}}\right)$$
$$-\frac{c_j}{C_j}\left(\frac{\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\frac{1}{|\mathcal{G}|}-\frac{c_j}{C_j}}{\sum_{k\in\mathcal{G}}C_k^{-1}}\right)\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_j\right)$$
$$-\frac{c_j}{C_j}\frac{d^r+\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_m\right)}{\sum_{k\in\mathcal{G}}C_k^{-1}}\left(\frac{1}{|\mathcal{G}|}-1\right)$$
$$-c_j\left(\left(1-\frac{c_j}{C_j}\right)\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-b^d_j\right)+\frac{d^r+\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-b^d_m\right)}{C_j\sum_{k\in\mathcal{G}}C_k^{-1}}\right)$$
$$\left(\left(1-\frac{c_j}{C_j}\right)\left(\frac{1}{|\mathcal{G}|}-1\right)+\frac{1}{C_j}\frac{\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\frac{1}{|\mathcal{G}|}-\frac{c_j}{C_j}}{\sum_{k\in\mathcal{G}}C_k^{-1}}\right)=0 \tag{D.6}$$

Assuming generators are homogeneous, i.e. $c_j := c,\ \forall j\in\mathcal{G}$ and we solve for symmetric equilibrium in the market, i.e., $\beta^d_j := \beta^d,\ \forall j\in\mathcal{G}$, the Eq. (D.6) can be rewritten as :

$$\implies \beta^d = b^d c\frac{d}{|\mathcal{G}|}+b^d c\frac{d^r}{|\mathcal{G}|}\left(1-\frac{c}{C}\right)-\frac{d^d}{|\mathcal{G}|}\frac{|\mathcal{G}|-2}{|\mathcal{G}|-1} \tag{D.7}$$

Recall $C=\left(\frac{1}{b^r(|\mathcal{G}|-1)}+c\right)$. Similarly, substituting (D.2)–(D.4) in the individual payment problem (15), we get a convex optimization problem,

$$\min_{d^d_l}\ \frac{d^d+\beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}d^d_l+\frac{d-d^d+\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_m\right)}{\sum_{k\in\mathcal{G}}C_k^{-1}}(d_l-d^d_l) \tag{D.8}$$

Taking the derivative of (D.8) we have

$$\implies \frac{d^d_l}{b^d|\mathcal{G}|}+\frac{d^d+\beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}+\frac{-1+\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\frac{1}{|\mathcal{G}|}}{\sum_{k\in\mathcal{G}}C_k^{-1}}(d_l-d^d_l)$$
$$-\frac{d-d^d+\sum_{m\in\mathcal{G}}\frac{c_m}{C_m}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_m\right)}{\sum_{k\in\mathcal{G}}C_k^{-1}}=0 \tag{D.9}$$

Assume generators are homogeneous, i.e. $c_j := c,\ \forall j\in\mathcal{G}$. We first sum over $l\in\mathcal{L}$ and solve for the case of symmetric bid participation of generators by rewriting the Eq. (D.9) as,

$$\implies d^d = -\frac{|\mathcal{G}|}{|\mathcal{L}|+1}\frac{|\mathcal{L}|\beta^d_j+b^dC\frac{-(|\mathcal{L}|+1)+\frac{c}{C}}{|\mathcal{G}|}d}{1+\frac{b^d}{b^r(|\mathcal{G}|-1)}} \tag{D.10}$$

Solving the Eqs. (6),(8),(D.2)–(D.4),(D.7), and (D.10) simultaneously for the equilibrium, we get the unique Nash equilibrium. Thus the symmetric Nash equilibrium exists uniquely.

Appendix E. Proof of Theorem 4

Under price-taking behavior, the individual problem for loads (25) is a linear program with the closed-form solution given by:

$$\begin{cases} d^d_l=\infty, d^r_l=-\infty, d^d_l+d^r_l=d_l, & \text{if } \lambda^d<\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\\ d^d_l=-\infty, d^r_l=\infty, d^d_l+d^r_l=d_l, & \text{if } \lambda^d>\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\\ d^d_l+d^r_l=d_l, & \text{if } \lambda^d=\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\end{cases} \tag{E.1}$$

where loads prefers lower price in the market. The individual problem for generators (24) requires:

$$\begin{cases} \beta^d_j=\infty, & \text{if } \lambda^d<\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\\ \beta^d_j=-\infty, & \text{if } \lambda^d>\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\\ \beta^d_j\in\mathbb{R}, & \text{if } \lambda^d=\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\end{cases} \tag{E.2}$$

where generators prefer higher prices in the market and seek to maximize profit. At the competitive equilibrium the day-ahead supply function (6), real-time true dispatch condition (22), real-time clearing prices (23), and the individual optimal solution (E.1),(E.2) holds simultaneously and this is only possible if the market price is equal in the two stages. Thus a set of competitive equilibria exists.

Appendix F. Proof of Theorem 5

From the market-clearing in the day-ahead stage (7), we have the following relation

$$\implies \sum_{j\in\mathcal{G}}\left(b^d\lambda^d-\beta^d_j\right)=\sum_{l\in\mathcal{L}}d^d_l$$
$$\implies \lambda^d=\frac{d^d+\beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|},\quad g^d_j=b^d\frac{d^d+\beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}-\beta^d_j \tag{F.1}$$

Substituting the real-time true dispatch condition (22), real-time clearing prices (23), day-ahead dispatch and day-ahead prices (F.1) in the individual problem of generator (27), we get:

$$\max_{\beta^d_j}\ \left(\frac{d^d+\beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}-\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\right)$$
$$\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_j\right)+\frac{c_j^{-1}}{2}\left(\frac{d}{\sum_{j\in\mathcal{G}}c_j^{-1}}\right)^2 \tag{F.2}$$

where $\beta^{d,\mathcal{G}}=\sum_{j\in\mathcal{G}}\beta^d_j$. Taking the derivative of (F.2) wrt β^d_j and writing the first-order condition:

$$\frac{1}{b^d|\mathcal{G}|}\left(\frac{d^d+\beta^{d,\mathcal{G}}}{|\mathcal{G}|}-\beta^d_j\right)$$
$$+\left(\frac{d^d+\beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}-\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\right)\left(\frac{1}{|\mathcal{G}|}-1\right)=0 \tag{F.3}$$

Summing the Eq. (F.3) over the set of generators, i.e., $j\in\mathcal{G}$ we get

$$\implies \frac{1}{b^d|\mathcal{G}|}d^d-\left(\frac{d^d+\beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}-\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\right)(|\mathcal{G}|-1)=0$$
$$\implies \beta^{d,\mathcal{G}}=\frac{b^d|\mathcal{G}|}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}d-\frac{(|\mathcal{G}|-2)}{(|\mathcal{G}|-1)}d^d$$
$$\implies \beta_j=b^d\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}-\frac{|\mathcal{G}|-2}{|\mathcal{G}|}\frac{1}{(|\mathcal{G}|-1)}d^d \tag{F.4a}$$

Similarly, substituting the real-time clearing prices (23) and day-ahead prices (F.1) in the individual problem of generator (28), we get:

$$\min_{d^d_l}\ \left(\frac{d^d+\beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|}\right)d^d_l+\left(\frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}\right)(d_l-d^d_l) \tag{F.5}$$

Writing the first-order condition of the convex optimization problem (F.5), we get

$$\frac{d_l^d + d^d + \beta^{d,\mathcal{G}}}{b^d|\mathcal{G}|} - \frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}} = 0 \tag{F.6}$$

Summing the Eq. (F.6) over $l \in \mathcal{L}$, we get

$$\implies d^d = \frac{|\mathcal{L}|}{|\mathcal{L}|+1}\frac{b^d|\mathcal{G}|}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}}d - \frac{|\mathcal{L}|}{|\mathcal{L}|+1}\beta^{d,\mathcal{G}} \tag{F.7}$$

At the equilibrium the Eqs. (2),(23),(F.1),(F.4a), and (F.7) must hold simultaneously. Solving them simultaneously, we get the unique equilibrium. This completes the proof.

Appendix G. Proof of Theorem 6

Under price-taking behavior, the individual problem for loads (13) is a linear program with the closed-form solution given by:

$$\begin{cases} d_l^d = \infty, d_l^r = -\infty, d_l^d + d_l^r = d_l, \text{ if } \lambda^d < \lambda^r \\ d_l^d = -\infty, d_l^r = \infty, d_l^d + d_l^r = d_l, \text{ if } \lambda^d > \lambda^r \\ d_l^d + d_l^r = d_l, \quad \text{if } \lambda^d = \lambda^r \end{cases} \tag{G.1}$$

where loads prefer lower price in the market. Further solving the individual bidding problem for generators in real-time market (32) by taking the derivative of the concave profit function w.r.t β_j^r:

$$-\lambda^r + c_j\left(\frac{(c_j+\epsilon_j)^{-1}d^d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}} + b^r\lambda^r - \beta_j^r\right) = 0 \tag{G.2}$$

Substituting (8), (30), and (31) in (G.2), we get

$$\implies -\lambda^r + c_j(g_j^d + g_j^r) = 0 \implies \sum_{j\in\mathcal{G}}\frac{1}{c_j}\lambda^r = \sum_{j\in\mathcal{G}} g_j = d \implies \lambda^r = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{G.3}$$

At the competitive equilibrium the conditions (30),(31),(G.1)–(G.3) must hold simultaneously and this is only possible if the market price are equal in the two stages, i.e.,

$$\lambda^r = \lambda^d = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}; d_l^d + d_l^r = d_l, \forall l \in \mathcal{L}; d^d = \frac{1}{1+\epsilon}d; d^r = \left(1 - \frac{1}{1+\epsilon}\right)d$$

$$g_j^d = \frac{1}{c_j}\frac{1}{1+\epsilon}\frac{d}{\sum_{k\in\mathcal{G}} c_k^{-1}},\ g_j^r = \frac{1}{c_j}\left(1 - \frac{1}{1+\epsilon}\right)\frac{d}{\sum_{k\in\mathcal{G}} c_k^{-1}}$$

WLOG, here we assume $\epsilon_j = \epsilon c_j, \forall j \in \mathcal{G}$, for a constant parameter $\epsilon \geq 0$.

Appendix H. Proof of Theorem 7

First, we find the relation between price and generator dispatch in real-time by formulating a subgame equilibrium in real-time. Given the parameter $(g_j^d, d - d^d)$ from market-clearing in the day-ahead market, each generator j maximizes their profit (14) for the optimal β_j^r:

$$\sum_{j\in\mathcal{G}} g_j^r = d^r \implies \sum_{j\in\mathcal{G}}(b\lambda^r - \beta_j^r) = d^r \implies \lambda^r = \frac{d^r + \sum_{j\in\mathcal{G}}\beta_j^r}{bG} \tag{H.1}$$

Substituting (H.1) in the individual problem (12), the individual problem of generators, is:

$$\max_{\beta_j\geq 0}\left(\frac{d^r+\beta^{\mathcal{G}}}{bG}\right)\left(b\frac{d^r+\beta^{\mathcal{G}}}{bG} - \beta_j\right) + \lambda^d g_j^d - \frac{c_j}{2}\left(g_j^d + b\left(\frac{d^r+\beta^{\mathcal{G}}}{bG}\right) - \beta_j\right)^2 \tag{H.2}$$

Writing the first-order necessary and sufficient conditions of the concave optimization problem (H.2), and subsequently substituting (8) and (H.1), we get

$$\lambda^r = \frac{d^r + \sum_{j\in\mathcal{G}}\frac{c_j}{C_j}g_j^d}{\sum_{k\in\mathcal{G}} C_k^{-1}},\ g_j^r = \frac{d^r + \sum_{m\in\mathcal{G}}\frac{c_m}{C_m}g_m^d}{C_j\sum_{k\in\mathcal{G}} C_k^{-1}} - \frac{c_j}{C_j}g_j^d \tag{H.3}$$

where $C_j = \frac{1}{b^r(|\mathcal{G}|-1)} + c_j$. Substituting (30),(31) in the expression (H.3) we get

$$\lambda^r = \frac{d^r}{\sum_{k\in\mathcal{G}} C_k^{-1}} + \frac{d^d}{\sum_{k\in\mathcal{G}} c_k^{-1}},\ g_j^r = \frac{1}{C_j}\frac{d^r}{\sum_{k\in\mathcal{G}} C_k^{-1}} \tag{H.4}$$

Substituting (31) and (H.4) in the individual problem of load l (15) we get

$$\min_{d_l^d}\frac{(1+\epsilon)d^d}{\sum_{j\in G} c_j^{-1}}d_l^d + \left(\frac{d-d^d}{\sum_{k\in\mathcal{G}} C_k^{-1}} + \frac{d^d}{\sum_{k\in\mathcal{G}} c_k^{-1}}\right)(d_l - d_l^d) \tag{H.5}$$

Therefore taking the derivative of the convex individual problem (H.5) wrt d_l^d we get,

$$\epsilon\frac{d^d + d_l^d}{\sum_{j\in G} c_j^{-1}} - \frac{d-d^d}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d_l}{\sum_{j\in\mathcal{G}} c_j^{-1}} + \frac{d_l^d - d_l}{\sum_{j\in\mathcal{G}} C_j^{-1}} = 0 \tag{H.6}$$

Summing over $l \in \mathcal{L}$ we get

$$d^d = \frac{\sum_{j\in\mathcal{G}} c_j^{-1} - \frac{1}{L+1}\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1} + \epsilon\sum_{j\in\mathcal{G}} C_j^{-1}}d \tag{H.7}$$

Appendix I. Proof of Theorem 8

Under price-taking behavior, the individual problem for loads (38) is a linear program with the closed-form solution given by:

$$\begin{cases} d_l^d = \infty, d_l^r = -\infty, d_l^d + d_l^r = d_l, \text{ if } \lambda^d < \frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}} \\ d_l^d = -\infty, d_l^r = \infty, d_l^d + d_l^r = d_l, \text{ if } \lambda^d > \frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}} \\ d_l^d + d_l^r = d_l, \quad \text{if } \lambda^d = \frac{d}{\sum_{k\in\mathcal{G}}(c_k+\epsilon_k)^{-1}} \end{cases} \tag{I.1}$$

where loads prefer lower prices in the market. At the competitive equilibrium, the day-ahead true dispatch condition (37a), real-time true dispatch condition (37b), and the individual optimal solution (I.1) must hold simultaneously. This is only possible if the market price is equal in the two stages, i.e.,

$$d_l^d = d_l,\ d_l^r = 0; \lambda^d = \lambda^r$$

Appendix J. Proof of Theorem 9

Writing the first-order condition of the convex optimization problem (40) and taking the derivative wrt d_l^d we get:

$$\left(\frac{d^d}{\sum\limits_{j\in\mathcal{G}}(c_j+\epsilon_j^d)^{-1}} - \frac{d}{\sum\limits_{j\in\mathcal{G}}(c_j+\epsilon_j^r)^{-1}}\right) + \left(\frac{d_l^d}{\sum\limits_{j\in\mathcal{G}}(c_j+\epsilon_j^d)^{-1}}\right) = 0 \tag{J.1a}$$

$$\implies d^d = \frac{L}{L+1}\frac{\sum_{j\in\mathcal{G}}(c_j+\epsilon_j^d)^{-1}}{\sum_{j\in\mathcal{G}}(c_j+\epsilon_j^r)^{-1}}d \implies d_l^d = \frac{1}{L+1}\frac{\sum_{j\in\mathcal{G}}(c_j+\epsilon_j^d)^{-1}}{\sum_{j\in\mathcal{G}}(c_j+\epsilon_j^r)^{-1}}d \tag{J.1b}$$

Substituting (J.1) in Eq. (37), we have

$$\lambda^d = \frac{L}{L+1}\frac{d}{\sum_{j\in\mathcal{G}}(c_j+\epsilon_j^r)^{-1}},\ \lambda^r = \frac{d}{\sum_{j\in\mathcal{G}}(c_j+\epsilon_j^r)^{-1}}$$

Appendix K. Proof of Theorem 10

Substituting the profit function (44) in the optimization problem (45), we get:

$$\max_{\beta_j^d} -(\lambda^d - \mathbb{E}[\lambda^r])\beta_j^d$$
$$+\delta_j\left((\lambda^d - \mathbb{E}[\lambda^r])^2(b^d\lambda^d - \beta_j^d)^2 - \frac{1}{c_j}(\lambda^d - \mathbb{E}[\lambda^r])\mathbb{E}[(\lambda^r)^2]\beta_j^d\right)$$

$$-\delta_j\left(\mathbb{E}[(\lambda^d-\lambda^r)^2](b_d\lambda^d-\beta_j^d)^2-\frac{1}{c_j}\mathbb{E}[(\lambda^d-\lambda^r)(\lambda^r)^2]\beta_j^d\right)+\vartheta_l \tag{K.1}$$

where

$$\vartheta_l := b^d\lambda^d(\lambda^d-\mathbb{E}[\lambda^r])+\frac{1}{2c_j}\mathbb{E}[(\lambda^r)^2] + \delta_j\left(\frac{1}{4c_j^2}\left(\mathbb{E}[(\lambda^r)^2]\right)^2+\frac{1}{c_j}(\lambda^d-\mathbb{E}[\lambda^r])b^d\lambda^d\mathbb{E}[(\lambda^r)^2]\right) - \delta_j\left(\frac{1}{4c_j^2}\mathbb{E}[(\lambda^r)^4]+\frac{1}{c_j}\mathbb{E}[(\lambda^d-\lambda^r)(\lambda^r)^2]b^d\lambda^d\right) \tag{K.2a}$$

$$\lambda^r=\frac{\tilde{d}}{\sum_{j\in\mathcal{G}}c_j^{-1}} \tag{K.2b}$$

Writing the second-order derivative of the objective function (K.1), we have:

$$\delta_j\left(2(\lambda^d-\mathbb{E}[\lambda^r])^2\right)-\delta_j\left(2\mathbb{E}[(\lambda^d-\lambda^r)^2]\right)=-2\delta_j Var(\lambda^d-\lambda^r)\le 0 \tag{K.3a}$$

Therefore this is a concave optimization problem and the first-order necessary and sufficient conditions is given by:

$$-(\lambda^d-\mathbb{E}[\lambda^r])+\delta_j\left(-2(\lambda^d-\mathbb{E}[\lambda^r])^2(b^d\lambda^d-\beta_j^d)-\frac{1}{c_j}(\lambda^d-\mathbb{E}[\lambda^r])\mathbb{E}[(\lambda^r)^2]\right) - \delta_j\left(-2\mathbb{E}[(\lambda^d-\lambda^r)^2](b_d\lambda^d-\beta_j^d)-\frac{1}{c_j}\mathbb{E}[(\lambda^d-\lambda^r)(\lambda^r)^2]\right)=0 \tag{K.4a}$$

$$\implies \beta_j^d=\frac{1}{2\delta_j Var(\lambda^r)} \left(\mathbb{E}[\lambda^r]-\lambda^d+\delta_j\left(2b^dVar(\lambda^r)\lambda^d+\frac{1}{c_j}\mathbb{E}[\lambda^r]\mathbb{E}[(\lambda^r)^2]-\frac{1}{c_j}\mathbb{E}[(\lambda^r)^3]\right)\right) \tag{K.4b}$$

Similarly, substituting (46) in the optimization problem (47), the individual optimization problem of each load l is given by:

$$\min_{d_l^d}\ (\lambda^d-\mathbb{E}[\lambda^r])d_l^d-\eta_l\left((\lambda^d-\mathbb{E}[\lambda^r])^2(d_l^d)^2+2(\lambda^d-\mathbb{E}[\lambda^r])\mathbb{E}[\lambda^r\tilde{d}_l]d_l^d\right) + \eta_l\left(\mathbb{E}[(\lambda^d-\lambda^r)^2](d_l^d)^2+2d_l^d\mathbb{E}[(\lambda^d-\lambda^r)\lambda^r\tilde{d}_l]\right)+\varrho_l \tag{K.5}$$

where

$$\varrho_l := \mathbb{E}[\lambda^r\tilde{d}_l]-\eta_l(\mathbb{E}[\lambda^r\tilde{d}_l])^2+\eta_l\mathbb{E}[(\lambda^r)^2\tilde{d}_l^2] \tag{K.6}$$

Writing the second-order derivative of the objective function (K.5), we have:

$$-\eta_l\left(2(\lambda^d-\mathbb{E}[\lambda^r])^2\right)+\eta_l\left(2\mathbb{E}[(\lambda^d-\lambda^r)^2]\right)=2\eta_l Var(\lambda^d-\lambda^r)\ge 0 \tag{K.7}$$

Therefore, this is a convex optimization problem. Writing the first order necessary and sufficient condition, we have

$$(\lambda^d-\mathbb{E}[\lambda^r])-\eta_l\left(2(\lambda^d-\mathbb{E}[\lambda^r])^2d_l^d+2(\lambda^d-\mathbb{E}[\lambda^r])\mathbb{E}[\lambda^r\tilde{d}_l]\right) + \eta_l\left(2\mathbb{E}[(\lambda^d-\lambda^r)^2]d_l^d+2\mathbb{E}[(\lambda^d-\lambda^r)\lambda^r\tilde{d}_l]\right)=0 \tag{K.8a}$$

$$\implies d_l^d=-\frac{1}{2\eta_l Var(\lambda^r)}\left((\lambda^d-\mathbb{E}[\lambda^r])-\eta_l\left(-2\mathbb{E}[\lambda^r]\mathbb{E}[\lambda^r\tilde{d}_l]+2\mathbb{E}[(\lambda^r)^2\tilde{d}_l]\right)\right) \tag{K.8b}$$

Now, at the equilibrium, the Eqs. (6), (23), (K.4b), and (K.8b) must satisfy simultaneously. Also, using the definition of Skewness (Groeneveld & Meeden, 1984), we have

$$\tilde{\mu}_3=\frac{E[\tilde{d}^3]-3\mu\sigma^2-\mu^3}{\sigma^3} \implies \frac{\mathbb{E}[\tilde{d}^3]-\mathbb{E}[\tilde{d}]\mathbb{E}[\tilde{d}^2]}{Var(\tilde{d})}=\tilde{\mu}_3\sigma+2\mu \tag{K.9a}$$

Solving the Eqs. (6), (23), (K.4b), and (K.8b) simultaneously, we have a unique equilibrium.

Appendix L. Proof of Theorem 11

Substituting (22), and (23) in (42), we get the individual problem of generator j as:

$$\max_{\beta_j^d} -(\lambda^d-\mathbb{E}[\lambda^r])\beta_j^d + \delta_j\left((\lambda^d-\mathbb{E}[\lambda^r])^2(b^d\lambda^d-\beta_j^d)^2-\frac{1}{c_j}(\lambda^d-\mathbb{E}[\lambda^r])\mathbb{E}[(\lambda^r)^2]\beta_j^d\right) - \delta_j\left(\mathbb{E}[(\lambda^d-\lambda^r)^2](b_d\lambda^d-\beta_j^d)^2-\frac{1}{c_j}\mathbb{E}[(\lambda^d-\lambda^r)(\lambda^r)^2]\beta_j^d\right)+\vartheta_l \tag{L.1}$$

where

$$\vartheta_l := b^d\lambda^d(\lambda^d-\mathbb{E}[\lambda^r])+\frac{1}{2c_j}\mathbb{E}[(\lambda^r)^2] + \delta_j\left(\frac{1}{4c_j^2}\left(\mathbb{E}[(\lambda^r)^2]\right)^2+\frac{1}{c_j}(\lambda^d-\mathbb{E}[\lambda^r])b^d\lambda^d\mathbb{E}[(\lambda^r)^2]\right) - \delta_j\left(\frac{1}{4c_j^2}\mathbb{E}[(\lambda^r)^4]+\frac{1}{c_j}\mathbb{E}[(\lambda^d-\lambda^r)(\lambda^r)^2]b^d\lambda^d\right) \tag{L.2a}$$

$$\lambda^d=\frac{d^d+\sum_{j\in\mathcal{G}}\beta_j^d}{b^d|\mathcal{G}|},\quad \lambda^r=\frac{\tilde{d}}{\sum_{j\in\mathcal{G}}c_j^{-1}} \tag{L.2b}$$

Writing the second-order derivative of the objective function (L.1), we have

$$-\frac{2}{|\mathcal{G}|b^d}-\delta_j\left(-2Var(\lambda^r)(\frac{1}{|\mathcal{G}|}-1)\right)+\frac{2}{|\mathcal{G}|}\frac{1}{|\mathcal{G}|b^d}= -\left(1-\frac{1}{|\mathcal{G}|}\right)\left[\frac{2}{|\mathcal{G}|b^d}+2\delta_j Var(\lambda^r)\right]<0 \tag{L.3a}$$

Therefore, this is a strict concave optimization problem. Writing the first-order necessary and sufficient conditions, we get:

$$\left(\frac{1}{|\mathcal{G}|}-1\right)(\lambda^d-\mathbb{E}[\lambda^r])+\omega_j b^d\lambda^d + \left(\frac{1}{|\mathcal{G}|}-1\right)\frac{\delta_j}{c_j}\left(\mathbb{E}[(\lambda^r)^3]-\mathbb{E}[\lambda^r]\mathbb{E}[(\lambda^r)^2]\right)=\omega_j\beta_j^d \tag{L.4}$$

where we define

$$\omega_j := \left(\frac{1}{b^d|\mathcal{G}|}+2\delta_j Var(\lambda^r)\right) \tag{L.5}$$

Similarly, substituting (22), and (23) in (43), we get the individual problem of load l as:

$$\min_{d_l^d}\ (\lambda^d-\mathbb{E}[\lambda^r])d_l^d-\eta_l\left((\lambda^d-\mathbb{E}[\lambda^r])^2(d_l^d)^2+2(\lambda^d-\mathbb{E}[\lambda^r])\mathbb{E}[\lambda^r\tilde{d}_l]d_l^d\right) + \eta_l\left(\mathbb{E}[(\lambda^d-\lambda^r)^2](d_l^d)^2+2d_l^d\mathbb{E}[(\lambda^d-\lambda^r)\lambda^r\tilde{d}_l]\right)+\varphi \tag{L.6}$$

where

$$\varphi := \mathbb{E}[\lambda^r\tilde{d}_l]-\eta_l(\mathbb{E}[\lambda^r\tilde{d}_l])^2+\eta_l\mathbb{E}[(\lambda^r)^2\tilde{d}_l^2]$$

$$\lambda^d=\frac{d^d+\sum_{j\in\mathcal{G}}\beta_j^d}{b^d|\mathcal{G}|},\quad \lambda^r=\frac{\tilde{d}}{\sum_{j\in\mathcal{G}}c_j^{-1}} \tag{L.7}$$

Writing the first order necessary and sufficient condition of the convex optimization problem:

$$(\lambda^d-\mathbb{E}[\lambda^r])+\left(\frac{1}{b^d|\mathcal{G}|}+2\eta_l Var(\lambda^r)\right)d_l^d - 2\eta_l\left(\mathbb{E}[(\lambda^r)^2\tilde{d}_l]-\mathbb{E}[\lambda^r]\mathbb{E}[\lambda^r\tilde{d}_l]\right)=0 \tag{L.8}$$

where,

$$\kappa_l := \frac{1}{b^d|\mathcal{G}|}+2\eta_l Var(\lambda^r) \tag{L.9}$$

At the equilibrium (6), (7), (23), (L.4), and (L.8) must hold simultaneously. Solving them, we get a unique Nash equilibrium.

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